THE

NATIONAL ARITHMETIC,

ON THE

INDUCTIVE SYSTEM;

COMBINING THE

ANALYTIC AND SYNTHETIC METHODS,

TOGETHER WITH THE CANCELLING SYSTEM;

FORMING A

COMPLETE MERCANTILE ARITHMETIC.

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New Stereotype Edition,

REVISED, ENLARGED, AND MUCH IMPROVED.

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PREFACE.

The National Arithmetic has now been before the public for nearly twelve years, and has met with an acceptance far beyond the original expectation of the author. The demand for it has constantly increased, and such has been the encouragement which both the author and the publisher have received from teachers of the highest character, and from the public generally, that the work has been thoroughly revised and very considerably enlarged, particularly in the department of demonstration, and is now presented in a form which, it is believed, will greatly increase its value. In the work of revision and enlargement, the author has availed himself of important suggestions from many practical teachers, and has had the direct assistance of gentlemen intimately acquainted, not only with the business of teaching Arithmetic, but also with the higher branches of Mathematics. His own labors in this work have been hardly less than in the original preparation of the book, and he is confident that the improvements introduced into the present edition will be seen and appreciated by all who may compare it with preceding editions.

It has been the author's privilege, for more than thirty years, to be engaged in the business of instruction. He has been acquainted with the methods of communicating knowledge which were formerly practised, and has endeavoured to make himself familiar with all the improvements in this respect which distinguish the present age from the past. The present work is offered to the public, as one constructed on a plan which appears to the author better adapted to meet the wants of the times than any other now in use. The end to be sought in the study of Arithmetic he regards as twofold, — a practical knowledge of numbers and the art of calculation, and the discipline of the mental powers; and the present work, it is believed, will be found fitted to these two objects. It is intended to be comprehensive in its principles, and sufficiently extensive in its details; and while the road to a knowledge of

the science is not designed to be unnecessarily steep and rugged, the author does not desire to relieve the learner of all occasion for effort, nor make him feel that the "Hill of Science" is no hill at all, but only a fiction of former ages. The author's idea is, that, in order to become a thorough and accomplished arithmetician, one must *study*, and the National Arithmetic proposes no substitute for mental exertion. Still, it is not designed to be difficult beyond the necessities of the case, and no pupil, who is faithful to himself, will, it is thought, find reason to complain that enough is not done by way of suitable illustration to facilitate his progress.

It is the opinion of some teachers, that no rules should be furnished the pupil to aid him in performing arithmetical questions, but that every pupil should form his own rules by the process of *induction*. But the author's experience has led him to a different conclusion, nor does he think that the insertion of proper rules, in a work like the present, interferes in the least with the necessity of study, or a thorough knowledge of the different numerical processes.

The National Arithmetic is intended to be complete in itself; but the smaller works of the author will prepare the pupil for an easy entrance upon the study of it. The learner can omit the more difficult parts of the present work until he reviews it, if thought advisable by the teacher.

A few rules, which are omitted in some works on Arithmetic at the present day, the author has thought best to retain,— such as Practice, Progression, Position, Permutation, &c. For, though these rules may not in themselves be of great practical utility, yet, as they are well adapted to improve the reasoning powers, and give interest to the higher departments of arithmetical science, it is deemed desirable to place them within reach of the student.

In closing these prefatory remarks, the author would earnestly recommend that the pupil be required to give a minute and thorough analysis of every question he performs, at least until he has proved himself familiar with all the principles involved in the rule under consideration, and also the manner of their application. He would further recommend a frequent and thorough review of the parts of the work which the pupil has gone over, the exercise having respect mainly to the *principles* involved in the preceding rules and examples.

Bradford Seminary, September 1, 1847.

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1922
1652
270 years

INTRODUCTION.

HISTORY OF ARITHMETIC.

THE question, who was the inventor of Arithmetic, or in what age or among what people did it originate, has received different answers. In ordinary history we find the origin of the science attributed by some to the Greeks, by some to the Chaldeans, by some to the Phœnicians, by Josephus to Abraham, and by many to the Egyptians. The opinion, however, rendered most probable, if not absolutely certain, by modern investigations is, that Arithmetic, properly so called, is of Indian origin, — that is, that the science received its first definite form and became the regular germ of modern Arithmetic in the regions of the East.

It is evident, from the nature of the case, that some knowledge of numbers and of the art of calculation was necessary to men in the earliest periods of society, since without this they could not have performed the simplest business transactions, even such as are incidental to an almost savage state. The question, therefore, as to the invention of Arithmetic deserves to be considered only as it respects the origin of the science as we now have it, and which, as all scholars admit, has reached a surprising degree of perfection. And in this sense the honor of the invention must be awarded to the Hindoos.

The history of the various methods of Notation, or the different means by which numbers have been expressed by signs or characters, is one of much interest to the advanced and curious scholar, but the brevity of this sketch allows us barely to touch upon it here. Among the ancient nations which possessed the art of writing, it was a natural and common device to employ letters to denote what we express by our numeral figures. Accordingly we find, that, with the Hebrews and Greeks, the first letter of their respective alphabets was used for 1, the second for 2, and so on to the number 10, — the latter, however, inserting one new character to denote the number 6, and evidently in order that their notation might coincide with that of the Hebrews, the sixth letter of the Hebrew alphabet having no corresponding one in the Greek.

The Romans, as is well known, employed the letters of their alphabet as numerals. Thus, I denotes 1; V, 5; X, 10; L, 50; C, 100; D, 500; and M, 1000. The intermediate numbers were expressed by a repetition of these letters in various combinations; as II for 2; VI for 6; XV for 15; IV for 4; IX for 9, &c. They fre-

quently expressed any number of thousands by the letter or letters denoting so many units, with a line drawn above; thus, $\overline{V}$, 5,000; $\overline{VI}$, 6,000; $\overline{X}$, 10,000; $\overline{L}$, 50,000; $\overline{C}$, 100,000; $\overline{M}$, 1,000,000.

In the classification of numbers, as well as in the manner of expressing them, there has been a great diversity of practice. While we adopt the *decimal* scale and reckon by tens, other nations have adopted the *vicenary*, reckoning by twenties; others the *quinary*, reckoning by fives; and others the *binary*, reckoning by twos. The adoption of one or another of these scales has been so general, that they have been regarded as natural, and accounted for by referring them to a common and natural cause. The reason for assuming the binary scale probably lay in the use of the two hands, which were employed as counters in computing; that for employing the quinary, in a similar use of the five fingers on either hand; while the decimal and vicenary scales had respect, the former to the ten fingers on the two hands, and the latter to the ten fingers combined with the ten toes on the naked feet, which were as familiar to the sight of a rude, uncivilized people as their fingers. — It is an interesting circumstance that in the common name of our numeral figures, digits (*digiti*) or fingers, we preserve a memento of the reason why ten characters and our present decimal scale of numeration were originally adopted to express all numbers, even of the highest order.

It is now almost universally admitted that our present numeral characters, and the method of estimating their value in a tenfold ratio from right to left, have decided advantages over all other systems, both of notation and numeration, that have ever been adopted. There are those who think that a *duodecimal* scale, and the use of twelve numeral figures instead of ten, would afford increased facility for rapid and extensive calculations, but most mathematicians are satisfied with the present number of numerals and the scale of numeration which has attained an adoption all but universal.

It was long supposed, that for our modern Arithmetic the world was indebted to the Arabians. But this, as we have seen, was not the case. The Hindoos at least communicated a knowledge of it to the Arabians, and, as we are not able to trace it beyond the former people, they must have the honor of its invention. They do not, however, claim this honor, but refer it to the Divinity, declaring that *the invention of nine figures, with device of place, is to be ascribed to the beneficent Creator of the universe.*

But though the invention of modern Arithmetic is to be ascribed to the Hindoos, the honor of introducing it into Europe belongs unquestionably to the Arabians. It was they who took the torch from the East and passed it along to the West. The precise period, however, at which this was done, it is not easy to determine. It is evident that our numeral characters and our method of computing by them were in common use among the Arabians about the middle of the tenth century, and it is probable that a knowledge of them was soon afterwards communicated to the inhabitants of Spain and gradually to those of the other European countries. Their general adoption in Europe would not seem to have been earlier than the twelfth or thirteenth century.

The science of Arithmetic, like all other sciences, was very limited and imperfect at the beginning, and the successive steps by which it has reached its present extension and perfection have been taken at long intervals and among different nations. It has been developed by the necessities of business, by the strong love of certain minds for mathematical science and numerical calculation, and by the call for its higher offices by other sciences, especially that of Astronomy. In its progress we find that the Arabians discovered the method of proof by casting out the 9's, and that the Italians early adopted the practice of separating numbers into periods of six, for the purpose of enumeration. To facilitate the process of multiplication, this latter people also introduced, probably from the writings of Boethius, the long neglected Table of Pythagoras.

The invention of the Decimal Fraction was a great step in the advancement of arithmetical science, and the honor of it has generally been given to Regiomontanus, about the year 1464. It appears, however, more properly to belong to Stevinus, who in 1582 wrote an express treatise on the subject. The credit of first using the decimal point, by which the invention became permanently available, is given by Dr. Peacock to Napier, the inventor of Logarithms; but De Morgan says that it was used by Richard Witt as early as 1613, while it is not shown that Napier used it before 1617. Circulating Decimals received but little attention till the time of Dr. Wallis, the author of the Arithmetic of Infinites. Dr. Wallis died at Oxford, in 1703.

The greatest improvement which the art of computation ever received was the invention of Logarithms, the honor of which is unquestionably due to Baron Napier, of Scotland, about the end of the sixteenth or the commencement of the seventeenth century.

The oldest treatises on Arithmetic now known are the 7th, 8th, 9th, and 10th books of Euclid's Elements, in which he treats of proportion and of prime and composite numbers. These books are not contained in the common editions of the great geometer, but are found in the edition by Dr. Barrow, the predecessor of Sir Isaac Newton in the mathematical chair at Cambridge. Euclid flourished about 300 B. C.

The next writer on Arithmetic mentioned in history is Nicomachus, the Pythagorean, who wrote a treatise relating chiefly to the distinctions and divisions of numbers into classes, as plain, solid, triangular, &c. He is supposed to have lived near the Christian era.

The next writer of note is Boethius, the Roman, who, however, copied most of his work from Nicomachus. He lived at the beginning of the sixth century, and is the author of the well-known work on the Consolation of Philosophy.

The next writer of eminence on the subject is Jordanus, of Namur, who wrote a treatise about the year 1200, which was published by Joannes Faber Stapulensis in the fifteenth century, soon after the invention of Printing.

The author of the first *printed* treatise on Arithmetic was Pacioli, or, as he is more frequently called, Lucas de Burgo, an Italian monk, who in 1484 published his great work, entitled *Summa de Arithmetica*, &c., in which our present numerals appear under very nearly their modern form.

In 1522, Bishop Tonstall published a work on the Art of Computation, in the Dedication of which he says that he was induced to study Arithmetic to protect himself from the frauds of money-changers and stewards, who took advantage of the ignorance of their employers. In his preparation for this work, he professes to have read all the books which had been published on this subject, adding, also, that there was hardly any nation which did not possess such books.

About the year 1540, Robert Record, Doctor in Physic, printed the first edition of his famous Arithmetic, which was afterward augmented by John Dee, and subsequently by John Mellis, and which did much to advance the science and practice of Arithmetic in England in its early stages. This work, which is now quite a curiosity, effectually destroys the claim to originality of some things of which authors much more modern have obtained the credit. In it we find the celebrated case of a will, which we have in the Miscellaneous Questions of Webber and Kinne, and which, altered in language and the time of making the testament, is the 11th Miscellaneous Question in the present work. This question is, by his own confession, older than Record, and is said to have been famous since the days of Lucas de Burgo. In Record it occurs under the "Rule of Fellowship." Record was the author of the first treatise on Algebra in the English language.

In 1556, a complete work on Practical Arithmetic was published by Nicolas Tartaglia, an Italian, and one of the most eminent mathematicians of his time.

From the time of Record and Tartaglia, works on Arithmetic have been too numerous to mention in an ordinary history of the science. De Morgan, in his recent work (*Arithmetical Books*), has given the names of a large number, with brief observations upon them, and to this the inquisitive student is referred for further information in regard both to writers and books on this subject since the invention of Printing. It is remarkable that De Morgan knew next to nothing of any American works on Arithmetic. He mentions the "American Accountant" by William Milns, New York, 1797, and gives the name of Pike (probably Nicholas Pike) among the names of which he had heard in connection with the subject. He had also seen the Memoir of Zerah Colburn. Of the compilation of Webber and the original works of Walsh and Warren Colburn, he seems to have been entirely ignorant.

The various signs or symbols, which are now so generally used to abridge arithmetical as well as algebraical operations, were introduced gradually, as necessity or convenience taught their importance. The earliest writer on Algebra after the invention of Printing was Lucas de Burgo, above mentioned, and he uses *p* for plus and *m* for minus, and indicates the powers by the first two letters, in which he was followed by several of his successors. After this, Steifel, a German, who in 1544 published a work entitled *Arithmetica Integra*, added considerably to the use of signs, and, according to Dr. Hutton, was the first who employed $+$ and $-$ to denote addition and subtraction. To denote the root of a quantity he also used our present sign $\surd$, originally *r*, the initial of the word *radix*, root. The sign $=$ to denote

equality was introduced by Record, the above-named English mathematician, and for this reason, as he says, that "noe 2 thynges can be moar equalle," namely, than two parallel lines. It is a curious circumstance that this same symbol was first used to denote subtraction. It was also employed in this sense by Albert Girarde, who lived a little later than Record. Girarde dispensed with the *vinculum* employed by Steifel, as in $\overline{3+4}$, and substituted the parenthesis $(3+4)$, now so generally adopted. The first use of the St. Andrew's cross, $\times$, to signify multiplication is attributed to William Oughtred, an Englishman, who in 1631 published a work entitled *Clavis Mathematicæ*, or Key of Mathematics.

It was intended to notice several other works, ancient and modern, but the length to which this sketch has already extended forbids it We must not, however, omit to mention two American works, which have done much for the cause of practical Arithmetic in this country. These are the large work of Nicholas Pike, first published about 1787, and the little unpretending "First Lessons" in Arithmetic, by Warren Colburn. From the former of these many later authors have borrowed much that is useful, and the latter has exerted an influence on the method of studying Arithmetic greater, perhaps, than any other modern production. No better elementary work than that of Colburn has ever, it is believed, appeared in any language.

We had thought of alluding to the ancient philosophic Arithmetic, and the elevated ideas which many of the early philosophers had of the science and properties of numbers. But a word must here suffice. Arithmetic, according to the followers of Plato, was not to be studied "with gross and vulgar views, but in such a manner as might enable men to attain to the contemplation of numbers; not for the purpose of dealing with merchants and tavern-keepers, but for the improvement of the mind, considering it as the path which leads to the knowledge of truth and reality." These transcendentalists considered perfect numbers, compared with those which are deficient or superabundant, as the images of the virtues, which, they allege, are equally remote from excess and defect, constituting a mean between them; as in the case of true courage, which, they say, lies midway between audacity and cowardice, and of liberality, which is a mean between profusion and avarice. In other respects, also, they regard this analogy as remarkable: perfect numbers, like the virtues, are "few in number and generated in a constant order; while superabundant and deficient numbers are, like vices, infinite in number, disposable in no regular series, and generated according to no certain and invariable law."

We conclude this brief sketch in the earnest hope that the noble science of numbers may ere long find some devoted friend who shall collect, arrange, and bring within the reach of ordinary students, much more fully than we have done, the scattered details of its long-neglected history.

ARITHMETICAL SIGNS.

$=$ Sign of equality; as 12 inches $=$ 1 foot signifies that 12 inches are equal to one foot.

$+$ Sign of addition; as $8+6=14$ signifies that 8 added to 6 is equal to 14.

$-$ Sign of subtraction; as $8-6=2$, that is, 8 less 6 is equal to 2.

$\times$ Sign of multiplication; as $7\times 6=42$, that is, 7 multiplied by 6 is equal to 42.

$\div$ Sign of division; as $42\div 6=7$, that is, 42 divided by 6 is equal to 7.

$\frac{12}{8}$ Numbers placed in this manner imply that the *upper* number is to be divided by the *lower* one.

: :: : Signs of proportion; thus, $2:4::6:12$, that is, 2 has the same ratio to 4 that 6 has to 12; and such numbers are called *proportionals*.

$\overline{12-3+4}=13$. Numbers placed in this manner show that 3 is to be taken from 12, and 4 added to the remainder. The line at the top is called a *vinculum*, and connects all the numbers over which it is drawn.

9^2 implies that 9 is to be raised to the second power; that is, multiplied by itself.

8^3 implies that 8 is to be multiplied into its square, or to be raised to the third power.

$\sqrt{}$ This sign prefixed to any number shows that the square root is to be extracted.

$\sqrt[3]{}$ This sign prefixed to a number shows that the cube root is to be extracted.

Sometimes roots are designated by fractional indices, thus; $9^{\frac{1}{2}}$ denotes the square root of 9; $27^{\frac{1}{3}}$ denotes the cube root of 27.

( ) [ ] Parentheses and brackets are often used instead of a vinculum. Thus, $(7-3)\times 5=60\div 3$.

☞ An edition of this work, *without answers*, is published for the accommodation of those teachers who prefer that the pupil should not have access to them.

A Key, containing solutions and explanations, is also published for the convenience of teachers.

ARITHMETIC.

SECTION I.

ARITHMETIC is the science of numbers, and the art of computing by them.

The operations of Arithmetic are performed principally by Addition, Subtraction, Multiplication, and Division.

NUMERATION.

NUMERATION teaches to express the value of numbers, either by words or characters.

Numbers in Arithmetic are expressed by the ten following characters, which are called numeral figures; viz. 1 (*one*), 2 (*two*), 3 (*three*), 4 (*four*), 5 (*five*), 6 (*six*), 7 (*seven*), 8 (*eight*), 9 (*nine*), 0 (*cipher*, or *nothing*).

The first nine of these figures are called *significant*, as distinguished from the cipher, which is of itself insignificant.

Besides this value of the numerical figures, they have another value, dependent on the place which they occupy, when connected together. This is illustrated by the following table and its explanation.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1
	9	8	7	6	5	4	3	2
		9	8	7	6	5	4	3
			9	8	7	6	5	4
				9	8	7	6	5
					9	8	7	6
						9	8	7
							9	8
								9

Here any figure occupying the first place, reckoning from right to left, denotes only its simple value or number of units. But the figure standing in the second place denotes *ten* times its simple value; that occupying the third place, a *hundred* times its simple value, and so on to any required number of places; the value of any figure being always increased tenfold by its removal one place to the left.

Thus, in the number 1834, the 4 in the first place denotes only four units, or simply 4; the 3 in the second place signifies three tens, or thirty; the 8 in the third place signifies eighty tens, or eight hundred; and the 1 in the fourth place, one thousand; so that the whole number is read thus,—one thousand eight hundred thirty-four.

Although the cipher has no value of itself, when standing alone, yet, being joined to the right hand of significant figures, it increases their value in a tenfold proportion; thus, 5 signifies simply five, while 50 denotes five tens, or fifty; 500, five hundred, and so on.

NOTE.—The idea of number is the latest and most difficult to form. Before the mind can arrive at such an abstract conception, it must be familiar with that process of classification, by which we successively remount from individuals to species, from species to genera, from genera to orders. The savage is lost in his attempts at numeration, and significantly expresses his inability to proceed by holding up his expanded fingers or pointing to the hair of his head. See Lacroix

NUMERATION TABLE.

The following is the French method of enumeration, and is in general use in the United States and on the continent of Europe.

In order to enumerate any number of figures by this method, they should be separated by commas into divisions of three figures each, as in the annexed table. Each division will be known by a different name. The first three figures, reckoning from right to left, will be so many units, tens, and hundreds, and the next three so many thousands, and the next three so many millions, &c.

123,	Vigintillions.
456,	Novemdecillions.
789,	Octodecillions.
123,	Septendecillions.
456,	Sexdecillions.
789,	Quindecillions.
123,	Quatuordecillions.
456,	Tridecillions.
789,	Duodecillions.
123,	Undecillions.
456,	Decillions.
789,	Nonillions.
123,	Octillions.
456,	Septillions.
789,	Sextillions.
123,	Quintillions.
456,	Quadrillions.
789,	Trillions.
123,	Billions.
456,	Millions.
789,	Thousands.
123	Units.

The value of the numbers in the annexed table, expressed in words, is One hundred twenty-three vigintillions, four hundred fifty-six novemdecillions, seven hundred eighty-nine octodecillions, one hundred twenty-three septendecillions, four hundred fifty-six sexdecillions, seven hundred eighty-nine quindecillions, one hundred twenty-three quatuordecillions, four hundred fifty-six tridecillions, seven hundred eighty-nine duodecillions, one hundred twenty-three undecillions, four hundred fifty-six decillions, seven hundred eighty-nine nonillions, one hundred twenty-three octillions, four hundred fifty-six septillions, seven hundred eighty-nine sextillions, one hundred twenty-three quintillions, four hundred fifty-six quadrillions, seven hundred eighty-nine trillions, one hundred twenty-three billions, four hundred fifty-six millions, seven hundred eighty-nine thousands, one hundred twenty-three units.

NUMERATION TABLE.

317,	Thousands.
897;	Tridecillions.
431,	Thousands.
032;	Duodecillions.
639,	Thousands.
864;	Undecillions.
361,	Thousands.
316;	Decillions.
461,	Thousands.
315;	Nonillions.
123,	Thousands.
675;	Octillions.
816,	Thousands.
131;	Septillions.
123,	Thousands.
456;	Sextillions.
123,	Thousands.
614;	Quintillions.
315,	Thousands.
131;	Quadrillions.
398,	Thousands.
832;	Trillions.
563,	Thousands.
871;	Billions.
351,	Thousands.
615;	Millions.
123,	Thousands.
561.	Units.

The following is the old English method of enumeration, but it has become almost obsolete in this country. In order to enumerate any number of figures by this method, they should be separated by semicolons into *divisions* of six figures each, and each division separated in the middle by a comma, as in the annexed table. Each division will be known by a different name. The first three figures, in each division, reckoning from right to left, will be so so many units, tens, and hundreds of the name belonging to the division, and the three on the left will be so many thousands of the same name. The value of the numbers in the annexed table, expressed in words, is Three hundred and seventeen thousand, eight hundred and ninety-seven tridecillions; four hundred and thirty-one thousand, thirty-two duodecillions; six hundred thirty-nine thousand, eight hundred sixty-four undecillions; three hundred sixty-one thousand, three hundred sixteen decillions; four hundred sixty-one thousand, three hundred fifteen nonillions; one hundred twenty-three thousand, six hundred seventy-five octillions; eight hundred sixteen thousand, one hundred thirty-one septillions; one hundred twenty-three thousand, four hundred fifty-six sextillions; one hundred twenty-three thousand, six hundred fourteen quintillions; three hundred fifteen thousand, one hundred thirty-one quadrillions; three hundred ninety-eight thousand, eight hundred thirty-two trillions; five hundred sixty-three thousand, eight hundred seventy-one billions; three hundred fifty-one thousand, six hundred fifteen millions; one hundred twenty-three thousand five hundred sixty-one.

NOTE.—The student must be familiar with the names, from units to tridecillions, and from tridecillions to units, so that he may repeat them with facility either way.

Let the following numbers be written in words:—

706
313,461
604,021
3,607,005
607,081,107
470,803,020
7,801,410,909
322,172,517,101
607,100,001,070
407,000,010,703,801
200,070,007,801,000
670,812,000,170,063,891
478,127,815,016,666,060,707
800,800,800,800,800,800,800,800
127,081,061,071,081,010,009,007,007
407,144,140,070,060,700,007,101,800,808

Let the following numbers be written in figures:*—

1. Twenty-nine.
2. Four hundred and seven.
3. Twenty-three thousand and seven.
4. Five millions and twenty-seven.
5. Seven millions, two hundred five thousand and five.
6. Two billions, two hundred seven millions, six hundred four thousand and nine.
7. One hundred five billions, nine hundred nine millions, three hundred eight thousand two hundred and one.
8. Nine quintillions, eight billions and forty-six.
9. Fifteen quintillions, thirty-one millions and seventeen.
10. Five hundred seven septillions, two hundred three trillions, fifty-seven millions and eighteen.
11. Nine nonillions, forty-seven trillions, seven billions, two millions, three hundred ninety-two.
12. Fifteen duodecillions, ten trillions, one hundred twenty-seven billions, twenty-six millions, three hundred twenty thousand four hundred twenty-six.

* To express numbers by *figures*, begin at the left hand with the highest order mentioned, and, proceeding to units, write in each successive order the figure which denotes the given number in that order. If any of the intervening orders are not mentioned in the given number, supply their places with ciphers.

SECTION II.

ADDITION.

ADDITION is the collecting of numbers to find their sum.

1. A man has three farms; the first contains 378 acres, the second 586 acres, and the third 168 acres. How many acres are there in the three farms?

OPERATION.
Acres.
378
586
168
1132

In this question, the units are first added, and their sum is found to be 22; in 22 units there are two tens and two units. The two units are written under the column of units, and the 2 (tens) are carried to be added with the tens, which are found to amount to 23 tens = 2 hundreds and 3 tens. The 3 is written under the column of tens, and the 2 (hundreds) is carried to the column of the hundreds, which amount to 11 = 1 thousand 1 hundred. The whole of which is set down. Hence the propriety of the following

RULE.

Write units under units, tens under tens, &c. Then begin at the bottom and add the units upwards, and, if the amount be less than ten, set it down under the column of units; but if the amount be ten or more, write down the unit figure, and add the figure denoting the number of tens to the column of tens. Thus proceed, till every column of figures is added, writing down on the left the sum total of the left-hand column, and the result will be the sum of the whole as required.

PROOF.

Begin at the top and add all the columns *downwards*, in the same manner as they were before added *upwards*; then, if the two sums agree, the work may be presumed to be correct.

2.	3.	4.	5.	6.	7.
Dollars.	Bushels	Pecks.	Tons.	Miles.	Acres.
15	76	765	126	969	6789
26	48	381	384	872	5832
18	59	872	876	446	4671
91	81	315	243	392	8907
150	264	2333	1629	2679	26199

12345

8.	9.	10.	11.	12.	13.	14.
Barrels.	Pounds.	Acres.	Cents.	Eagles.	Rods.	Poles.
123	678	456	789	456	781	889
456	901	789	987	781	175	776
789	278	127	123	197	564	432
341	633	815	321	715	337	876
1709	2490	2187	2220	2149	1857	2973

15.	16.	17.	18.	19.	20.
Ounces.	Inches.	Rods.	Furlongs.	Cords.	Feet.
7891	3256	6789	1234	4567	4561
3245	7890	1234	5678	8912	7890
6789	1234	5678	9012	3456	7658
1234	5678	6543	3456	7891	8888
5678	7801	1234	7891	4567	9199
24837	25859	21478	27271	29393	38196

21.	22.	23.	24.	25.	26.
Hogsheads.	Furlongs.	Miles.	Dollars.	Casks.	Pence.
1789	6781	7890	1785	7895	4371
6543	1371	1070	5678	5678	1699
2177	8715	4437	9137	7186	1098
8915	6371	6789	8171	5176	8816
6781	1234	5378	1888	4321	6171
4325	7171	1234	1919	4127	7185

27.	28.	29.	30.	31.
Shillings.	Tons.	Miles.	Trees.	Loads.
78956	12345	34567	76717	56789
32167	87655	78901	77777	12345
41328	34517	32199	67890	67819
45678	65483	17188	71444	34567
13853	79061	88888	47474	71888
71667	20939	12345	16175	33197

32	33.	34.	35.
Acres.	Roods.	Poles.	Yards.
789516	451237	1234567	789123
377895	813715	8901234	456789
378567	679919	5678901	987654
832156	787651	3456789	357913
789567	637171	5432115	245678
813138	813785	7177444	999999

36.	37.	38.	39.	40.
123456	876543	789012	987654	678953
789012	789112	345678	456112	467631
345678	345678	901234	222333	117777
901234	965887	789037	456789	888888
567890	445566	891133	987654	444444
987654	788743	477666	321178	667679
321032	399378	557788	123456	998889
765437	456789	888878	789561	671236

41. Tons.	42. Pounds.	43. Ounces.	44. Grains.
783256	12004	30530	276605
7128	32	31643	3980839
39	1	26798	4183478
432815	7836	28578	31881050
99	100	34383	3837156
67851	46	29340	4801393
125	3	283649	5067696
641236	6176	300000	5640426
801	32	264088	4344737
4328	91876	357477	1937678

45. Add the following numbers, 763, 4663, 37, 49763, 6178, and 671. Ans. 62075.

46. A butcher sold to A 369 lbs. of beef, to B 169 lbs., to C 861 lbs., to D 901 lbs., to E 71 lbs., and to F 8716 lbs.; what did they all receive? Ans. 11087 lbs.

47. A owes to one creditor 596 dollars, to another 3961, to another 581, to another 6116, to another 469, to another 506, to another 69381, and to another 1261. What does he owe them all? Ans. $ 82871.

48. If a boy earn 17 cents a day, how much will he earn in 7 days? Ans. 119 cts.

49. If a man's wages be 19 dollars per month, what are they per year? Ans. $ 228.

50. If a boy receive a present every New Year's day of 1783 dollars, how much money will he possess, when he is 21 years old? Ans. $ 37443.

51. How many inhabitants were there in Essex county, Mass., in 1840, Haverhill having 4336, Amesbury 2471, Andover 5207, Beverly 4689, Bradford 2222, Boxford 942, Danvers 5020, Essex 1450, Georgetown 1540, Gloucester 6350, Hamilton 818,

Ipswich 3000, Lynn 9369, Lynnfield 707, Manchester 1355, Marblehead 5575, Methuen 2251, Middleton 657, Newbury 3789, Newburyport 7161, Rockport 2650, Rowley 1203, Salem 15082, Salisbury 2739, Saugus 1098, Topsfield 1059, Wenham 689, West Newbury 1560? Ans. 94,989.

52. How many were the members of Congress in 1846, there being 2 Senators from each State, and Maine sending 7 Representatives, New Hampshire 4, Massachusetts 10, Rhode Island 2, Connecticut 4, Vermont 4, New York 34, New Jersey 5, Pennsylvania 24, Delaware 1, Maryland 6, Virginia 15, North Carolina 9, South Carolina 7, Georgia 8, Alabama 7, Mississippi 4, Louisiana 4, Tennessee 11, Kentucky 10, Ohio 21, Indiana 10, Illinois 7, Missouri 5, Arkansas 1, Michigan 3, Florida 1, Texas 2? Ans. 282.

53. According to the census of 1840, Maine had 501,793 inhabitants, New Hampshire 284,574, Massachusetts 737,699, Rhode Island 108,830, Connecticut 309,978, Vermont 291,948 New York 2,428,921, New Jersey 373,306, Pennsylvania 1,724,033, Delaware 78,085, Maryland 469,232, District of Columbia 43,712, Virginia 1,239,797, North Carolina 753,419, South Carolina 594,398, Georgia 691,392, Kentucky 779,828, Tennessee 829,210, Ohio 1,519,467, Indiana 685,866, Mississippi 375,651, Missouri 383,702, Illinois 476,183, Louisiana 352,411, Alabama 590,756, Michigan 212,267, Arkansas 97,574, Florida 54,477, Wisconsin 30,945, Iowa 43,112, and on board U. S. vessels 6,100. What was the whole number of inhabitants? Ans. 17,068,666.

SECTION III.

SUBTRACTION.

SUBTRACTION teaches to find the difference between two numbers by taking the less from the greater.

OPERATION.

From	935
Take	673
	262

In this question, we take 3 units from 5 units and 2 units remain, which we write down under units, as the first figure in the answer. In attempting to take the 7 tens from 3 tens we find a difficulty, as 7 cannot be taken from 3. We therefore borrow 1 (hundred) from the 9 (hundred), which being equal to 10 tens, we add it to the 3 tens in the upper line, making 13 tens; from which we take 7 tens, and 6 tens re-

main, which we write down under the place of tens. We then proceed to the hundreds. As we have borrowed 1 from the 9 hundreds, the 9 is too large by 1. We must therefore take the 6 (hundreds) from 8 hundreds and there will remain 2 (hundreds). We therefore write down the 2 in the place of hundreds. Or, because the 9 is too large by 1, we may add 1 to the 6, and say 7 from 9 and 2 will remain. Hence the following

RULE.

Place the less number under the greater; units under units, tens under tens, &c. Begin with the units, and if the lower figure be smaller than the one above it, write the difference below. But, if the upper figure be less than the lower, then add ten to the upper one, and write the difference between the sum thus obtained and the lower figure. Then carry or add one to the lower figure of the next column, and proceed as before, till all the numbers are subtracted, and the result will be the difference.

NOTE.—The upper number is called the Minuend, from the Latin word *minuendum*, signifying *to be made less;* and the lower one the Subtrahend, from *subtrahendum, to be taken away.* The result is the Remainder.

PROOF.

Add the remainder to the subtrahend, and, if their sum be like the minuend, the work may be considered correct.

	2.	3.	4.	5.	6.
	£.	Cwt.	Tons.	Miles.	Inches.
Minuend,	79	86	469	876315123	11630078
Subtrahend,	24	25	183	177897638	1919179
	55	61	286	698417485	9710899

	7.	8.	9.	10.
	Miles.	Gallons.	Minutes.	Pecks.
From	7654	7116	6178	4567
Take	1978	1997	1769	1978

	11.	12.	13.	14.
	Barrels.	Degrees.	Furlongs.	Tons.
From	765116	56789	56781	71678
Take	716669	10091	39109	18819

	15.	16.	17.	18.
	Hogsheads.	Bushels.	Yards.	Pounds.
From	611000	617853	7111111	999000
Take	199999	190909	909009	199919

	19. Roods.	20. Acres.	21. Poles.	22. Cords.
From	100200	511799	610000	789111
Take	98761	419109	166666	171670

	23. Dollars.	24. Eagles.	25. Guineas.
From	10000000	99999999	888888
Take	9099019	1000919	99999

	26. Seconds.	27. Hours.
From	100200300400500	600700800900
Take	90807060504039	191818917185

	28. Months.	29. Days.	30. Weeks.
From	61567101	1000000	10000000
Take	91678	1	9999999

31. From	6767851	take	81715
32. From	761619161	take	916781
33. From	31671675	take	361784
34. From	16781321	take	100716
35. From	1002007000	take	5971621
36. From	91611237	take	6718538
37. From	4637561	take	4171135
38. From	88895651	take	3147618
39. From	1111111	take	99999
40. From	7163878	take	11001
41. From	8887771	take	81106
42. From	1379156	take	76716
43. From	3671652	take	36

Sum of the remainders, 2004466259.

44. Sir Isaac Newton was born in the year 1642, and he died in 1727; how old was he at the time of his decease?

Ans. 85 years.

45. Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1441?

Ans. 111 years.

46. The mariner's compass was invented in Europe in the

year 1302; how long was this before the discovery of America by Columbus, which happened in 1492? Ans. 190 years.

47. What number is that, to which if 6956 be added, the sum will be one million? Ans. 993044.

48. A man bought an estate for seventeen thousand five hundred and sixty-five dollars, and sold it for twenty-nine thousand three hundred and seventy-five dollars. Did he gain or lose, and how much? Ans. Gained $11810.

49. Bought a pair of oxen for 85 dollars, a horse for 126 dollars, three cows at 25 dollars apiece; and sold the whole for three hundred dollars; how much did I gain? Ans. $14.

50. Bonaparte was declared emperor in 1804; how many years since?

51. The union of the government of England and Scotland was in the year 1603; how long was it from this period to the time of the declaration of the independence of the United States? Ans. 173 years.

52. Jerusalem was taken and destroyed by Titus in the year 70; how long was it from this period to the time of the first Crusade, which was in the year 1096? Ans. 1026 years.

Section IV.

MULTIPLICATION.

Multiplication is the repetition of a number any proposed number of times. It consists of three parts, the Multiplicand, or number to be multiplied; the Multiplier, or number by which to multiply; and the result, which is called the Product. The Multiplicand and Multiplier are called *factors*.

RULE.

Place the larger number uppermost for the multiplicand, and the smaller number under it for a multiplier, arranging units under units, tens under tens, &c. Then multiply each figure of the multiplicand by each figure of the multiplier, beginning with the right-hand figure, and carrying for every ten as in addition. If the multiplier consists of more than one figure, the right-hand figure of each product must be placed directly under the figure of the multiplier that produces it, which will cause the successive products to recede each one place to the left. The sum of the several products will be the whole product required.

Note 1. — When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the

significant figures only, remembering to set the first figure of the product directly under the figure of the multiplier that produces it. See Ex. 15.

NOTE. 2. — If there are ciphers at the right hand either of the multiplier or multiplicand, or of both, they may be neglected to the close of the operation, when they must be annexed to the product.

PROOF.

The correctness of the result in Multiplication may be conveniently ascertained in three ways; viz., by Division, by Multiplication, or by casting out the nines.

According to the first method,* divide the product by the multiplier; and, if the work is right, the quotient will be equal to the multiplicand.

According to the second method, take the multiplier for the multiplicand and the multiplicand for the multiplier, and proceed according to the rule for multiplication; and, if the work be right, the product will be the same as by the former operation.

According to the third method, begin at the left hand of the multiplicand, and add together its successive figures towards the right, till the sum obtained equals or exceeds the number 9. If it *equals* it, drop the nine, and begin to add again at this point, and proceed till you obtain a sum equal to or greater than nine. If it *exceeds* nine, drop the nine as before, and carry the excess to the next figure, and then continue the addition as before. Proceed in this way till you have added all the figures in the multiplicand and rejected all the nines contained in it, and write the final excess at the right hand of the multiplicand. Proceed in the same manner with the multiplier, and write the final excess under that of the multiplicand. Multiply these *excesses* together and place the excess of nines in their product under the other excesses. Then proceed to find the excess of nines in the product obtained by the original operation, and, if the work be right, the excess thus found will be equal to the excess contained in the product of the above excesses of the multiplicand and multiplier. See Example 15.

NOTE. — This method of proof, though perhaps sufficiently sure for common purposes, is not always a test of the correctness of an operation. Cases will sometimes occur in which the excesses above named will be equal, when the work is not right.

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* As the pupil is presumed not to be acquainted with Division, he will pass over this method of proof for the present. It is placed here as a method important to be known, and because there seems to be no better place for it, though it presupposes an acquaintance with a rule yet to be learned.

TABLE OF PYTHAGORAS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576

EXAMPLES.

1.	2.	3.	4.
678956324	36785678	123456789	987654321
3	7	6	9
2036868972	257499746	740740734	8888888889

5.	6.	7.	8.
789123	1234567	989898	3789588
4	5	2	8

9.	10.	11.	12.
678954	616783	789563	789567
24	36	57	98
2715816	3700698	5526941	6316536
1357908	1850349	3947815	7106103
16294896	22204188	45005091	77377566

13.	14.	15.
3678543	67854	612346 = 4
4567	10234	430049 = 2
25749801	271416	5511114 = 8
22071258	203562	2449384
18392715	135708	1837038
14714172	67854	2449384
16799905881	694417836	263338784954 = 8

16.	17.	18.	19.
678567	4567895	6785000	478763000
8007	60004	32000	12000

20. Multiply 75432 by 47. Ans. 3545304.
21. Multiply 76785316 by 7615. Ans. 584720181340.
22. Multiply 67853000 by 8765. Ans. 594731545000.
23. Multiply 38123450 by 31243. Ans. 1191090948350.
24. Multiply 40670007 by 10002. Ans. 406781410014.
25. Multiply 31235678 by 10203. Ans. 318697622634.
26. Multiply 76786321 by 30070. Ans. 2308964672470.
27. Multiply 317160070 by 700500. Ans. 222170629035000.
28. Multiply 467325812 by 167000. Ans. 78043410604000.
29. Multiply 6176777 by 22222. Ans. 137260338494.
30. Multiply 123456789 by 987654321. Ans. 121932631112635269.
31. Multiply 7060504 by 30204. Ans. 213255462816.
32. Multiply 6017853 by 800070. Ans. 4814703649710.
33. Multiply 5000700 by 530071. Ans. 2650726049700.
34. Multiply 88888 by 4444. Ans. 395018272.
35. Multiply 123000 by 78000. Ans. 9594000000.

36. Multiply 7008005 by 10008. Ans. 70136114040.
37. Multiply 4001100 by 40506. Ans. 162068556600.
38. Multiply 6716700 by 808070. Ans. 5427563769000.
39. Multiply 987648 by 481007. Ans. 475065601536.
40. Multiply 18711000 by 470. Ans. 8794170000.
41. Multiply 10000 by 7000. Ans. 70000000.
42. Multiply 101010101 by 2020202. Ans. 204060808060402.
43. Multiply 70000 by 10000. Ans. 700000000.
44. Multiply 800008 by 9009. Ans. 7207272072.
45. Multiply 900900 by 70070. Ans. 63126063000.
46. Multiply 4807658 by 706007. Ans. 3394240201606.
47. Multiply 16789001 by 10080. Ans. 169233130080.
48. Multiply 304050607 by 3011101. Ans. 915527086788307.
49. Multiply 908007004 by 500123. Ans. 454115186861492.
50. Multiply 2003007001 by 6007023.
Ans. 12032109124168023.
51. Multiply 9000006 by 9000006. Ans. 81000108000036.
52. Multiply 1152921504606846976 by 1152921504606846976.
Ans. 1329227995784915872903807060280344576.

53. What will 27 oxen cost at 35 dollars each?
Ans. $ 945.

54. What will 365 acres of land cost at 73 dollars per acre?
Ans. $ 26645.

55. What will 97 tons of iron cost at 57 dollars a ton?
Ans. $ 5529.

56. What will 397 yards of cloth cost at 7 dollars per yard?
Ans. $ 2779.

57. What will 569 hogsheads of molasses cost at 37 dollars per hogshead? Ans. $ 21053.

58. If a man travel 37 miles in one day, how far will he travel in 365 days? Ans. 13505 miles.

59. If one quire of paper have 24 sheets, how many sheets are in a ream, which consists of 20 quires? Ans. 480 sheets.

60. If a vessel sails 169 miles in one day, how far will she sail in 144 days? Ans. 24336 miles.

61. What will 698 barrels of flour cost at 7 dollars a barrel?
Ans. $ 4886.

62. What will 376 lbs. of sugar cost at 13 cents a pound?
Ans. 4888 cts.

63. What will 97 lbs. of tea cost at 93 cents a pound?
Ans. 9021 cts.

64. If a regiment of soldiers consists of 1128 men, how many men are there in an army of 53 regiments?
Ans. 59784.

65. What will an ox weighing 569 pounds amount to at 8 cents a pound? Ans. 4552 cts.

66. If a barrel of cider can be bought for 93 cents, what will 75 barrels cost? Ans. 6975 cts.

67. If in a certain factory 786 yards of cloth are made in one day, how many will be made in 313 days? Ans. 246018 yds.

68. A certain house contains 87 windows, and each window has 32 squares of glass; how many squares are there in the whole house? Ans. 2784 squares.

69. There are 407 wagons each loaded with 30009 pounds of coal; how many pounds are there in the whole?

Ans. 12213663 pounds.

70. Multiply three hundred and seventy-five millions two hundred and ninety-six thousand three hundred and twenty-one, by seventy-nine thousand and twenty-four.

Ans. 29657416470704.

71. What would be the cost of 687 fothers of lead at 73 dollars a fother? Ans. $ 50151.

Section V.

DIVISION.

The object of Division is to find how many times one number is contained in another.

Division consists of three principal parts; the Dividend, or number to be divided; the Divisor, or number by which we divide; and the Quotient, which shows how many times the dividend contains the divisor.

When the dividend contains the divisor an exact number of times, the quotient is expressed by a whole number. But when this is not the case, there will be a remainder, when the division has reached its limit, and this remainder placed above the divisor, with a horizontal line between them, will form a fraction, and should be written at the right hand of the quotient, and will be a part of it. See Example 2d, and note.

1. The Remainder may be considered a fourth term in Division, and it will always be of the same denomination with the dividend.

For the sake of convenience, Division has been divided into two kinds, Long and Short.

2. All questions in which the divisor is not more than 12 may be conveniently performed by Short Division; all others are better performed by Long Division.

SHORT DIVISION.

EXAMPLE.

1. Divide 948 dollars equally among 4 men.

	Dividend.
Divisor	4)948
Quotient	237

In performing this question, inquire how many times 4, the divisor, is contained in 9, which is 2 times, and 1 remaining; write the 2 under the 9 and suppose 1, the remainder, to be placed before the next figure of the dividend, 4, and the number will be 14. Then inquire how many times 4, the divisor, is contained in 14. It is found to be 3 times and 2 remaining. Write the 3 under the 4, and suppose the remainder, 2, to be placed before the next figure of the dividend, 8, and the number will be 28. Inquire again how many times 28 will contain the divisor. It is found to be 7 times, which we place under the 8. Thus we find each man receives 237 dollars.

From the above illustration, we deduce the following

RULE.

Write down the dividend and place the divisor on the left, with a curved or perpendicular line drawn between them. Draw also a horizontal line under the dividend, then observe how many times the divisor is contained in the first figure or figures of the dividend (beginning at the left hand), and place the quotient figure directly under the right-hand figure of the part of the dividend that was taken. If there be no remainder, proceed to inquire how many times the divisor is contained in the next figure of the dividend, and set down the result at the right hand of the quotient figure already obtained, or directly under the figure of the dividend, and continue the work in this manner until the whole dividend is divided. But if there be a remainder either in the first or any subsequent division, imagine the number denoting it to be placed directly before the next figure of the dividend, and ascertain the number of times the divisor is contained in the number thus formed, and place the*

* If this figure be smaller than the divisor, it cannot contain it, and the figure to be placed in the quotient will be a cipher. Sometimes, as when we divide by 11 or 12, we may have two successive ciphers in the quotient, as when the divisor is 12 and the next two figures are 1's or 1 and 0. We are then obliged to proceed to a third figure in the dividend, before we can effect a proper division.

quotient figure underneath, as before. Proceed in this way until every part of the dividend is thus divided, and the result will be the quotient sought.

EXAMPLES.

2.	3.	4.
3)67856336	5)123456789	6)98786356
$22618778\frac{2}{3}$*	$24691357\frac{4}{5}$	

5.	6.	7.
7)126711001	8)33445567	9)1234567

8.	9.	10.
10)178985	11)1667789	12)9167856

				Quotients.	Rem.
11. Divide	67893536	by	2	33946768	
12. Divide	316789311	by	3	105596437	
13. Divide	567895326	by	4	141973831	2
14. Divide	123456789	by	5	24691357	4
15. Divide	671678953	by	6	111946492	1
16. Divide	166336711	by	7	23762387	2
17. Divide	161331793	by	8	20166474	1
18. Divide	161677678	by	9	17964186	4
19. Divide	363895678	by	11	33081425	3
20. Divide	164378956	by	12	13698246	4
21. Divide	78950077	by	3		1
22. Divide	678956671	by	4		3
23. Divide	667788976	by	5		1
24. Divide	777777777	by	6		3
25. Divide	888888888	by	7		6
26. Divide	999999999	by	8		7
27. Divide	100000000	by	7		2

* From this and subsequent examples it will be seen that *fractions* arise from *division*, and are parts of a unit; that the *denominator* of the fraction represents the divisor, and shows into how many parts the given number or quantity is divided, and the *numerator*, being the *remainder*, shows how many units of the given quantity or *dividend* remain undivided. By writing the numerator over the denominator in the form of a fraction, we signify that it is to be divided by the denominator; and when placed at the right hand of the whole number in the quotient, the fraction becomes a part of the quotient, and, as such, is as much less than a unit, as the numerator is less than the denominator.

LONG DIVISION.

CASE I.

EXAMPLE.

1. A prize, valued at $3978, is to be equally divided among 17 men. What is the share of each?

OPERATION.

		Dividend.	
Divisor.	17)	3978 (	234 *Quotient.*
		34	17
		57	1638
		51	234
		68	3978 *Proof.*
		68	
		00 *Remainder.*	

The object of this question is to find how many times 3978 will contain 17, or how many times must 17 be subtracted from 3978, until nothing shall remain. We first inquire, how many times the first two figures of the dividend will contain the divisor; that is, how many times 39 will contain 17. Having found it to be 2 times, we write 2 in the quotient and multiply the divisor, 17, by it, and place their product 34 under 39, from which we subtract it, and find the remainder to be 5, to which we annex the next figure of the dividend, 7. And having found that 57 will contain the divisor 3 times, we write 3 in the quotient, multiply it by 17, and place the product 51 under 57, from which we subtract it, and to the remainder, 6, we annex the next figure of the dividend, 8, and inquire how many times 68 will contain the divisor, and find it to be 4 times. And having placed the product of 4 times 17 under 68, we find there is no remainder, and that 3978 will contain 17, the divisor, 234 times; that is, each man will receive 234 dollars. To prove our work is right, we reason thus. If one man receives 234 dollars, 17 men will receive 17 times as much, and 17 times 234 are 3978, the same as the dividend; and this operation is effected by multiplying the divisor by the quotient. The student will now see the propriety of the following

RULE

Place the divisor and dividend as under the preceding rule, and draw a curved or perpendicular line on the right of the dividend. Then observe how many figures of the dividend, counting from left to right, must be taken to contain the divisor one or more times, but never exceeding nine times, and ascertain how many times these figures will

contain the divisor, placing the quotient figure on the right hand of the dividend. Then multiply the divisor by this quotient figure, and place the product in order under the figures of the dividend that were taken. Subtract this product from the part of the dividend above it, and to the difference bring down and annex the next figure of the dividend. Divide this number by the divisor, and place the quotient figure on the right of the one already found. Multiply the divisor by the quotient figure last found, and subtract the product from the number last divided, and bring down and annex as before, till the last figure of the dividend is taken; and the several figures on the right of the dividend will be the quotient required. The difference between the number last divided and the last product will be the remainder, which, with the divisor, will form a fraction, as under the preceding rule.

NOTE 1. — It will often happen, that, after a figure is brought down and annexed to a remainder, the number will not contain a divisor. In such a case, a cipher is to be placed in the quotient, and the next figure to be brought down and annexed, and thus till the number formed shall be large enough to contain the divisor. Sometimes it will be necessary thus to place several ciphers in succession in the quotient.

NOTE 2. — The proper remainder is in all cases *less* than the divisor; and if, at any time, the subtraction named in the rule gives a remainder *larger* than the divisor, we discover at once, that an error has been committed in the division, and that the quotient figure must be increased.

PROOF.

Division may be proved by Multiplication, by Addition, by casting out the 9's, or by Division.

By the first method, we multiply the quotient by the divisor, adding to the product the remainder, and the result, if the work be right, is equal to the dividend.

By the second method, we add up the several products of the several quotient figures by the divisor, together with the remainder, and the result, if the work is right, is like the dividend. See Example 2.

To prove Division by casting out the 9's, we find the excess of 9's in the divisor and also in the quotient, and multiply these excesses together and find the excess in their product. We then subtract the remainder from the dividend, and find the excess of 9's in the difference, which, if the work is right, will be equal to the excess found in the product of the excesses above named. See Example 3.

To prove Division by Division itself, we subtract the remainder from the dividend, and divide the difference by the quotient, and, if the work is right, the result will be equal to the original divisor. See Example 4.

EXAMPLES.

```
        2.                              3.
97)147856(1524              328)678767(2069
   97*                          656
   508                           2276            Proof.
   485*                          1968              5
    235                           3087           4 × 8
    194*                          2952               5
     416                           135
     388*                        678632
      28*
  147856
```

* NOTE. — The asterisks show the numbers to be added.

```
          4.                              Proof.
72)37895(526                            37895
   360                                     23
    189                            526)37872(72
    144                                3682
     455                                1052
     432                                1052
      23
```

				Quotients.	Rem.
5. Divide	6756785	by	35	193051	
6. Divide	789636	by	46	17166	
7. Divide	7967848	by	52	153227	44
8. Divide	16785675	by	61	275175	
9. Divide	675753	by	39	17327	
10. Divide	5678910	by	82	69255	
11. Divide	6716394	by	94	71451	
12. Divide	1167861	by	135	8650	111
13. Divide	7861783	by	87	90365	28
14. Divide	1678567	by	365	4598	297
15. Divide	87635163	by	387	226447	174
16. Divide	34567890	by	6789	5091	5091
17. Divide	78911007	by	36712	2149	16919
18. Divide	78963167	by	45671	1728	43679
19. Divide	671616589	by	61476	10924	52765
20. Divide	471361876	by	36789	12812	21208
21. Divide	300700801	by	10037	29959	2318
22. Divide	10000000	by	9999	1000	1000
23. Divide	199999999	by	123456		1279

				Rem.
24. Divide	6716789513	by	7816789	2167762
25. Divide	1613716131	by	3151638	77475
26. Divide	121932631112635269	by	123456789	
27. Divide	213255467083	by	30204	4267
28. Divide	4814703652065	by	800070	2355
29. Divide	2650726050934	by	530071	1234
30. Divide	395020613	by	4444	2341
31. Divide	9594004321	by	78000	4321
32. Divide	162068563389	by	40506	6789
33. Divide	5427563776896	by	808070	7896
34. Divide	475065610503	by	481007	8967
35. Divide	8794170278	by	470	278
36. Divide	70006876	by	7000	6876
37. Divide	204060808062747	by	2020202	2345
38. Divide	700003456	by	10000	3456
39. Divide	7207276639	by	9009	4567
40. Divide	63126068678	by	70070	5678
41. Divide	3394240208391	by	706007	6785
42. Divide	169233137936	by	10080	7856
43. Divide	9155270867968874	by	3011101	8567
44. Divide	454115186870257	by	500123	8765
45. Divide	12032109124169380	by	6007023	1357

CASE II.

To divide by any number with ciphers annexed.

Cut off the ciphers from the divisor, and the same number of figures from the right hand of the dividend. Then divide the remaining figures of the dividend by the remaining figures of the divisor, and the result will be the quotient. To complete the work, annex to the last remainder found by the operation the figures cut off from the dividend, and the whole will form the true remainder.

EXAMPLE.

1. Divide 36378967 by 31000.

```
31,000)36378,967(1173
       31
       --
        53
        31
        --
        227
        217
        ---
         108
          93
         -----
         15967 Remainder.
```

				Quotients.	Rem.
2. Divide	32100	by	6000	5	2100
3. Divide	3167810	by	160000	19	127810
4. Divide	12345678	by	1400000	8	1145678
5. Divide	1637851	by	500000	3	137851
6. Divide	3678953	by	326100	11	91853
7. Divide	41111111	by	1100000	37	411111

CASE III.

To divide by a unit with ciphers annexed.

Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor, and the figures on the left hand of the separatrix will be the quotient, and those on the right hand the remainder.

				Quotients.	Rem.
1. Divide	123456789	by	10	12345678	9
2. Divide	987654321	by	100	9876543	21
3. Divide	122112347800	by	1000	122112347	800
4. Divide	89765432156	by	1000000	89765	432156

CASE IV.

To divide by a composite number, that is, a number produced by the multiplication of two or more numbers.

Divide the dividend by any one of the factors, and the quotient thus found by another, and thus proceed till every factor has been made a divisor, and the last quotient will be the true quotient required.

NOTE. — To find the true remainder, we multiply the last remainder by the last divisor but one, and to the product add the next preceding remainder; we multiply this sum by the next preceding divisor, and to the product add the next preceding remainder; and so on, till we have gone through all the divisors and remainders to the first.

This rule will be better understood by the pupil, after he has become acquainted with fractions.

EXAMPLES.

1. Divide 47932 by 72.

```
9)47932
 8)5325   7
   665    5 = 52
```

As 72 is equal to 9 times 8, we first divide the dividend by 9, and the quotient thence arising by 8; and to find the true remainder, we multiply the last remainder, 5, by the first divisor, 9, and to the product add the first remainder, 7; and find the amount to be 52, the true remainder.

2. Divide 5371 by 192.

$$\begin{array}{l} 4)\underline{5371} \\ 6)\underline{1342-3} \\ 8)\underline{223-4} \\ \quad 27-7=187 \end{array}$$

We find 192 equal to the product of 4 times 6 times 8, $= 4 \times 6 \times 8 = 192$. We therefore divide by these *factors*, as in the last example. To find the true remainder, we multiply the last remainder, 7, by the last divisor but one, 6; and to the product add the last remainder but one, 4; this sum we multiply by the first divisor, 4; and to the product add the first remainder, 3; and find the amount to be 187.

				Quotients.	Rem.
3. Divide	7691	by	$24 = 4 \times 6$	320	11
4. Divide	8317	by	$27 = 3 \times 9$	308	1
5. Divide	3116	by	$81 = 9 \times 9$	38	38
6. Divide	61387	by	$121 = 11 \times 11$	507	40
7. Divide	19917	by	$144 = 12 \times 12$	138	45
8. Divide	91746	by	$336 = 6 \times 7 \times 8$	273	18
9. Divide	376785	by	$315 = 5 \times 7 \times 9$	1196	45

Section VI.

CONTRACTIONS IN MULTIPLICATION.

I. To multiply by 25.

Rule. — *Annex two ciphers to the multiplicand, and divide it by 4, and the quotient is the product required.*

Rationale. — By annexing two ciphers, we increase the multiplicand one hundred times, and by dividing this number by 4, the result will be an increase of the multiplicand only twenty-five times, because 25 is one fourth of 100.

1. Multiply 785643 by 25.

OPERATION.

$$\begin{array}{l} 4)\underline{78564300} \\ \quad 19641075 \text{ Product.} \end{array}$$

2. Multiply 9876543 by 25. Ans.

3. Multiply 47110721 by 25. Ans.

II. To multiply by $33\frac{1}{3}$.

Rule. — *Annex two ciphers to the multiplicand, and divide it by 3, and the quotient is the product required.*

Rationale.—As in the last case, by annexing two ciphers, we increase the multiplicand one hundred times; and by dividing the number by 3, we only increase the multiplicand thirty-three and one third times, because $33\frac{1}{3}$ is one third of 100.

4. Multiply 87138942 by $33\frac{1}{3}$.

OPERATION.

$$\begin{array}{r} 3)\underline{8713894200} \\ 2904631400 \text{ Product.} \end{array}$$

5. Multiply 66666993 by $33\frac{1}{3}$. Ans.
6. Multiply 12336723 by $33\frac{1}{3}$. Ans.

III. To multiply by 125.

RULE.—*Annex three ciphers to the multiplicand, and divide by 8, and the quotient is the product.*

NOTE.—By annexing three ciphers, the number is increased one thousand times; and, by dividing by 8, the quotient will be only one eighth of 1000, that is, 125 times.

7. Multiply 12345678 by 125.

OPERATION.

$$\begin{array}{r} 8)\underline{12345678000} \\ 1543209750 \text{ Product.} \end{array}$$

IV. To multiply by any number of 9's.

RULE.—*Annex as many ciphers to the multiplicand as there are 9's in the multiplier, and from this number subtract the number to be multiplied, and the remainder is the product required.*

8. Multiply 87654 by 999.

OPERATION.

$$\begin{array}{r} 87654000 \\ \underline{87654} \\ 87566346 \text{ Product.} \end{array}$$

By annexing three ciphers, we make the number one thousand times larger. If from this number, with the ciphers annexed, we subtract the multiplicand, we make the product one thousandth part less; that is, the product will be only 999 times the multiplicand. Q. E. D.

9. Multiply 7777777 by 9999. Ans. 77769992223.
10. Multiply 5555 by 999999. Ans. 5554994445.

NOTE.—To multiply by any number of 3's, proceed as above and divide the product by 3; but if it be required to multiply by 6's, proceed as above and then multiply the product by 2, and divide the result by 3, and the quotient is the product.

11. Multiply 987654 by 333333.

OPERATION.

```
  987654000000
        987654
3)987653012346
  329217670782 Product, Ans.
```

12. Multiply 32567895 by 3333. Ans.
13. Multiply 876543 by 66666. Ans.

OPERATION.

```
  87654300000
       876543
  87653423457
            2
3)175306846914
  58435615638 Product, Ans.
```

14. Multiply 345678 by 6666666. Ans.

V. When the multiplier can be separated into periods, which are multiples of one another, the operation may be contracted in the following manner.

15. Multiply 112345678 by 288144486.

OPERATION.

```
          112345678
          288144486
          674074068 = the product by 6.
        5392592544  = the foregoing product × by 8 for 48.
     16177777632    = the last product × by 3 for 144.
32355555264         = the last product × by 2 for 288.
32371787641631508 Product.
```

SECTION VII.

CONTRACTIONS IN DIVISION.

I. To divide by 5.

RULE. — *Multiply the dividend by 2, and the product, except the last figure at the right, is the quotient.*

NOTE. — The remainder will be tenths.

1. Divide 67895 by 5. Ans. 13579.

OPERATION.

```
 67895
     2
13579,0 Quotient.
```

II. To divide by 25.

RULE. — *Multiply the dividend by 4, and the product, except the last two figures at the right, is the quotient.*

NOTE. — The two figures at the right are hundredths.

2. Divide 8765887 by 25. Ans. $350635\frac{48}{100}$.

OPERATION.

```
 8765887
       4
350635,48 Quotient.
```

III. To divide by $33\frac{1}{3}$.

RULE. — *Multiply the dividend by 3, and the product, except the last two figures at the right, is the quotient, and the last two are hundredths.*

3. Divide 876735 by $33\frac{1}{3}$. Ans. $26302\frac{5}{100}$.

OPERATION.

```
 876735
      3
26302,05 Quotient.
```

IV. To divide by 125.

RULE. — *Multiply the dividend by 8, and the product, except the last three figures, is the quotient, and these last three figures will be thousandths.*

4. Divide 1234567 by 125. Ans. $9876\frac{536}{1000}$.

OPERATION.

```
1234567
      8
9876,536 Quotient.
```

5. Divide 8786789 by 125. Ans.
6. Divide 1234567 by 125 Ans.

V. A short method of performing Long Division.

7. Divide 16294896 by 24. Ans. 678954.

OPERATION.

24)16294896(678954, Ans.
14482606
181229
169129
121
121

This method differs from the common way by placing the right-hand figure of every product immediately under the dividend.

8. Divide 3545304 by 47. Ans. 75432.
9. Divide 45005091 by 57. Ans. 789563.

VI. To divide by any number of 9's, when their number is not less than half the number of places that will be in the quotient, and when there is no remainder.

RULE. — *Annex as many ciphers to the dividend, as there are 9's in the divisor. Then write the proper dividend under the number thus found, and subtract it from the number to which ciphers have been annexed; and, as many places of the remainder at the right hand as there were ciphers annexed, are so many figures for the right hand of the quotient; and, for the remaining numbers of the quotient, a competent number must be taken from the left hand of the above remainder.*

10. Divide 123332544 by 999.

OPERATION.

123332544000
123332544
123,209211,456
123456 Quotient, Ans.

By examining the dividend and divisor, we know there will be 6 places in the quotient. We therefore take three of these figures from the right hand of the remainder for the three right-hand figures of the quotient, and the other three we take from the left hand of the remainder.

11. Divide 12332655 by 999. Ans. 12345.
12. Divide 987551235 by 9999. Ans. 98765.
13. Divide 9123456779876543211 by 999999999.
Ans. 9123456789.

SECTION VIII.

MISCELLANEOUS EXAMPLES.

1. What number multiplied by 1728 will produce 1705536? Ans. 987.

2. If a garrison of 987 men are supplied with 175686 pounds of beef, how much will there be for each man? Ans. 178 lbs.

3. In one dollar there are 100 cents; how many dollars in 697800 cents? Ans. $ 6978.

4. In one pound there are 16 ounces; how many pounds are in 111680 ounces? Ans. 6980 lbs.

5. A dollar contains 6 shillings; how many dollars are in 5868 shillings? Ans. $ 978.

6. The President of the United States receives a salary of $ 25,000; what does he receive per month? Ans. $ $2083\frac{1}{3}$.

7. A man receiving $ 96 for 8 months' labor, what does he receive for 1 month? Ans. $ 12.

8. The distance from Haverhill to Boston is 30 miles; and, if a man travel 6 miles an hour, how long will he be in going this distance? Ans. 5 hours.

9. The annual revenue of a gentleman being $ 8395, how much per day is that equivalent to, there being 365 days in a year? Ans. $ 23.

10. The car on the Liverpool railroad goes at the rate of 65 miles an hour; how long would it take to pass round the globe, the distance being about 25,000 miles? Ans. $384\frac{8}{13}$ hours.

11. How much sugar at $ 15 per cwt. may be bought for $ 405? Ans. 27 cwt.

12. In 6789560 shillings how many pounds, there being 20 shillings in a pound? Ans. 339478 pounds.

13. The Bible contains 31,173 verses; how many must be read each day, that the book may be read through in a year? Ans. $85\frac{148}{365}$ verses.

14. In 123456720 minutes how many hours? Ans. 2057612 hours.

15. A gentleman possessing an estate of $ 66,144, bequeathed one fourth to his wife, and the remainder was to be divided between his 4 children; what was the share of each? Ans. $ 12,402.

16. A man disposed of a farm containing 175 acres at $ 87 per acre; of the avails he distributed $ 1234 for charitable purposes; $ 197 was expended for the purchase of a horse and chaise; the remainder was divided between 6 gentlemen and 8 ladies, and each lady was to receive twice as much as a gentleman; what was the share of each?

Ans. $ 627 for a gentleman, and $ 1254 for a lady.

17. If there are 160 square rods in an acre, how many acres are in 1086240 square rods? Ans. 6789 acres.

18. If 144 square inches make one square foot, how many square feet in 14222160 square inches? Ans. 98765 feet.

19. What number is that, which being multiplied by 24, the product divided by 10, the quotient multiplied by 2, 32 subtracted from the product, the remainder divided by 4, and 8 subtracted from the quotient, the remainder shall be 2?
Ans. 15.

20. What is the difference between half a dozen dozen, and six dozen dozen? Ans. 792.

21. Bought of F. Johnson 8 barrels of flour at $ 7 per barrel, and 3 hundred weight of sugar at $ 8 per hundred. What was the amount of his bill? Ans. $ 80.

22. Sold S. Jenkins my best horse for $ 75, my second-best chaise for $ 87, a good harness for $ 31. He has paid me in cash $ 38, and has given me an order on Peter Parker for $ 12. How many dollars remain my due? Ans. $ 143.

23. T. Webster has sold his wagon to J. Emerson for $ 85. He is to receive his pay in wood at $ 5 per cord. How many cords will it require to balance the value of the wagon?
Ans. 17 cords.

24. Purchased a farm of 500 acres for $ 17,876. I sold 127 acres of it at $ 47 an acre, 212 acres at $ 96 an acre, and the remainder at $ 37 an acre. What did I gain by my bargain?
Ans. $ 14,402.

25. A tailor has 938 yards of broadcloth; how many cloaks can be made of the cloth, if it require 7 yards to make one cloak? Ans. 134 cloaks.

26. Bought 97 barrels of molasses at $ 5 a barrel. Gave 17 barrels to support the poor, and the remainder was sold at $ 8 a barrel. Did I gain or lose, and how much?
Ans. $ 155 gain.

27. There are 12 pence in one shilling; required the number of pence in 671 shillings. Ans. 8052 pence.

28. Twelve inches make one foot in length; required the number of inches in 5280 feet, it being the length of a mile.
Ans. 63360 inches.

29. In one pound avoirdupois there are 16 ounces; required the ounces in 1728 pounds. Ans. 27648 ounces.

30. Required the number of shillings in 8136 pence.
Ans. 678 shillings.

31. It requires 1728 cubic inches to make one cubic foot required the number of cubic inches in 3787 cubic feet.
Ans. 6543936 inches.

SECTION IX.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

UNITED STATES MONEY.

10 Mills	make	1 Cent,	marked	c.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$.
10 Dollars	"	1 Eagle,	"	E.

Mills.		Cents.		Dimes.		Dollars.		Eagle
10	=	1						
100	=	10	=	1				
1000	=	100	=	10	=	1		
10000	=	1000	=	100	=	10	=	1

ENGLISH MONEY.

4 Farthings	make	1 Penny,	marked	d.
12 Pence	"	1 Shilling,	"	s.
20 Shillings	"	1 Pound,	"	£.
21 Shillings sterling	"	1 Guinea,	"	G.
28 Shillings N. E.	"	1 Guinea,	"	G.

NOTE. — One pound sterling is equal to $ 4.44$\frac{4}{9}$, exchange value.

		d.		s.		£.
4	=	1				
48	=	12	=	1		
960	=	240	=	20	=	1

FRENCH MONEY.

100 Centimes make 1 Franc = .186 dollar.

TROY WEIGHT.

24 Grains	make	1 Pennyweight,	marked	dwt.
20 Pennyweights	"	1 Ounce,	"	oz.
12 Ounces	"	1 Pound,	"	lb.

gr.		dwt.		oz.		lb.
24	=	1				
480	=	20	=	1		
5760	=	240	=	12	=	1

By this weight are weighed gold, silver, and jewels.

NOTE. — "The original of all weights used in England was a grain or corn of wheat, gathered out of the middle of the ear; and, being well dried, 32 of them were to make one pennyweight, 20 pennyweights one ounce, and 12 ounces one pound. But in later times, it was thought sufficient to divide the same pennyweight into 24 equal parts, still called grains, being the least weight now in common use; and from hence the rest are computed."

APOTHECARIES' WEIGHT.

20 Grains	make	1 Scruple,	marked sc. or ℈
3 Scruples	"	1 Dram,	" dr. or ʒ
8 Drams	"	1 Ounce,	" oz. or ℥
12 Ounces	"	1 Pound,	" lb. or ℔

gr.		sc.		dr.		oz.		lb.
20	=	1						
60	=	3	=	1				
480	=	24	=	8	=	1		
5760	=	288	=	96	=	12	=	1

Apothecaries mix their medicines by this weight; but buy and sell by Avoirdupois. The pound and ounce of this weight are the same as in Troy Weight.

AVOIRDUPOIS WEIGHT.

16 Drams	make	1 Ounce,	marked oz.
16 Ounces	"	1 Pound,	" lb.
28 Pounds	"	1 Quarter,	" qr.
4 Quarters	"	1 Hundred Weight,	" cwt.
20 Hundred Weight	"	1 Ton,	" ton.

dr.		oz.		lb.		qr.		cwt.		ton.
16	=	1								
256	=	16	=	1						
7168	=	448	=	28	=	1				
28672	=	1792	=	112	=	4	=	1		
573440	=	35840	=	2240	=	80	=	20	=	1

By this weight are weighed almost every kind of goods, and all metals except gold and silver. By a late law of Massachusetts, the cwt. contains 100 lbs. instead of 112 lbs.

A ton is reckoned at the custom-houses of the United States at 2240 lbs.

LONG MEASURE.

3 Barleycorns, or 12 Lines	make	1 Inch,	marked in.
12 Inches	"	1 Foot,	" ft.
3 Feet	"	1 Yard,	" yd.
6 Feet	"	1 Fathom,	" fth.
5½ Yards, or 16½ Feet	"	1 Rod, or Pole,	" rd.
40 Rods	"	1 Furlong,	" fur.
8 Furlongs	"	1 Mile,	" m.
3 Miles	"	1 League	" lea.
69½ Miles nearly	"	1 Degree,	" Deg. or °
360 Degrees	"	1 Circle of the Earth.	

in.		ft.		yd.		rd.		fur.		m.
12	=	1								
36	=	3	=	1						
198	=	16½	=	5½	=	1				
7920	=	660	=	220	=	40	=	1		
63360	=	5280	=	1760	=	320	=	8	=	1

CLOTH MEASURE.

2¼	Inches	make	1 Nail	marked	na.
4	Nails	"	1 Quarter of a yard,	"	qr.
4	Quarters	"	1 Yard,	"	yd.
3	Quarters	"	1 Ell Flemish,	"	E. F.
5	Quarters	"	1 Ell English,	"	E. E.
4	Quarters 1⅕ inch	"	1 Ell Scotch,	"	E. S.

SQUARE MEASURE.

144	Square inches	make	1 Square foot,	marked	ft.
9	Square feet	"	1 Square yard,	"	yd.
30¼	Square yards	"	1 Square rod or pole,	"	p.
272¼	Square feet	"	1 Square rod or pole,	"	p.
40	Square rods or poles	"	1 Rood,	"	R.
4	Roods	"	1 Acre,	"	A.
640	Acres	"	1 Square mile,	"	S. M.

in.	ft.	yd.	p.	R.	A.	S.M.
144 =	1					
1596 =	9 =	1				
39204 =	272¼ =	30¼ =	1			
1568160 =	10890 =	1210 =	40 =	1		
6272640 =	43560 =	4840 =	160 =	4 =	1	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1

DRY MEASURE.

2	Pints	make	1 Quart,	marked	qt.
4	Quarts	"	1 Gallon,	"	gal.
2	Gallons	"	1 Peck,	"	pk.
4	Pecks	"	1 Bushel,	"	bu.
36	Bushels	"	1 Chaldron,	"	ch.

pts.	gal.	pk.	bu.	ch.
8 =	1			
16 =	2 =	1		
64 =	8 =	4 =	1	
2304 =	288 =	144 =	36 =	1

NOTE.—This measure is applied to all goods that are not liquid and are sold by measure, as corn, fruit, salt, coals, &c. A Winchester Bushel is 18½ inches in diameter, and 8 inches deep. The standard Gallon Dry Measure contains 268⅘ cubic inches.

ALE AND BEER MEASURE.

2	Pints	make	1 Quart,	marked	qt.
4	Quarts	"	1 Gallon,	"	gal.
32	Gallons	"	1 Barrel,	"	bar.
54	Gallons	"	1 Hogshead,	"	hhd.
2	Hogsheads	"	1 Butt,	"	butt.
2	Butts	"	1 Tun,	"	tun.

pts.		qt.		gal.		bar.		hhd.		butt.
2	=	1								
8	=	4	=	1						
256	=	128	=	32	=	1				
432	=	216	=	54	=	$1\frac{18}{36}$	=	1		
864	=	432	=	108	=	3	=	2	=	1

NOTE. — By a law of Massachusetts, the Barrel for cider and beer shall contain 32 gallons, but in some other States it is of different capacity. The Ale Gallon contains 282 cubic or solid inches.

Milk is sold by the Beer Gallon.

WINE MEASURE.

4 Gills	make	1 Pint,	marked	pt.
2 Pints	"	1 Quart,	"	qt.
4 Quarts	"	1 Gallon,	"	gal.
42 Gallons	"	1 Tierce,	"	tier.
63 Gallons, or 1½ Tierces	"	1 Hogshead,	"	hhd.
2 Tierces	"	1 Puncheon,	"	pun.
2 Hogsheads	"	1 Pipe or Butt,	"	pi
2 Pipes, or 4 Hhds.	"	1 Tun,	"	tun.

pts.		qt.		gal.		tier.		hhd.		pun.		pi.		tun.
2	=	1												
8	=	4	=	1										
336	=	168	=	42	=	1								
504	=	252	=	63	=	1½	=	1						
672	=	336	=	84	=	2	=	1⅓	=	1				
1008	=	504	=	126	=	3	=	2	=	1½	=	1		
2016	=	1008	=	252	=	6	=	4	=	3	=	2	=	1

NOTE. — The Wine Gallon contains 231 cubic inches. Water, wine, and spirits are measured and sold by this measure.

A cubic foot of distilled water weighs 158 ounces Avoirdupois.

The English Imperial Gallon contains 277¼ cubic inches, and weighs 10 lb. Avoirdupois, or 12 lb. 1 oz. 16 dwt. 16 gr. Troy. There is no legal measure in the United States for tierce, hogshead, puncheon, pipe, or butt.

OF TIME.

60 Seconds, or 60″	make	1 Minute,	marked	m.
60 Minutes	"	1 Hour,	"	h.
24 Hours	"	1 Day,	"	d.
7 Days	"	1 Week,	"	w.
4 Weeks	"	1 Month,	"	mo.
13 Months, 1 day, 6 hours, or 365 days, 6 hours,		1 Julian Year,	"	y.
12 Calendar months	"	1 Year,	"	y.

sec.		m.		h.		d.		w.		mo.		y.
60	=	1										
3600	=	60	=	1								
86400	=	1440	=	24	=	1						
604800	=	10080	=	168	=	7	=	1				
2419200	=	40320	=	672	=	28	=	4	=	1		
31557600	=	525960	=	8766	=	365¼					=	1

NOTE. — The true solar year is the time measured from the sun's leaving either equinox or solstice, to its return to the same again. A periodical year is the time in which the earth revolves round the sun, and is 365 d. 6 h. 9 m. 14¼ sec., and is often called the Sidereal year. The civil year is that which is in common use among the different nations of the world, and contains 365 days for three years in succession, but every fourth year contains 366 days. When any year can be divided by four, without any remainder, it is *leap year*, and has 366 days.

	w.	d.	h.		mo.	d.	h.		
Or,	52	1	6	=	13	1	6	=	1 Julian Year.

	d.	h.	m.	sec.		
But,	365	5	48	57	=	1 Solar Year.
And,	365	6	9	14¼	=	1 Sidereal Year.

The days in each month are as follows: — January, March, May, July, August, October, and December have 31 days each; April, June, September, and November have 30 days each; February has 28 days, excepting leap year, when it has 29.

CIRCULAR MOTION.

60 Seconds, or 60 ″	make	1 Prime minute,	marked	′
60 Minutes	"	1 Degree,	"	°
30 Degrees	"	1 Sign,	"	s.

12 Signs, or 360 Degrees, the whole great circle of the zodiac.

″		′		°		s.		
60	=	1						
3600	=	60	=	1				
108000	=	1800	=	30	=	1		
1296000	=	21600	=	360	=	12	=	zodiac.

MEASURING DISTANCES.

7$\frac{92}{100}$	Inches	make	1 Link.
25	Links	"	1 Pole.
100	Links	"	1 Chain.
10	Chains	"	1 Furlong.
8	Furlongs	"	1 Mile.

Inches.		Link.								
$7\frac{92}{100}$	=	1		Pole.						
192	=	25	=	1		Chain.				
792	=	100	=	4	=	1		Furlong.		
7920	=	1000	=	40	=	10	=	1		Mile.
63360	=	8000	=	320	=	80	=	8	=	1

SOLID MEASURE.

1728	Inches	make	1 Foot.
27	Feet	"	1 Yard.
40	Feet of round timber	"	1 Ton.
128	Feet, i. e. 8 in length, 4 in breadth, and 4 in height,	"	1 Cord of wood.

NOTE. — One ton of round timber, as usually surveyed, contains $50\frac{93}{100}$ solid feet.

MISCELLANEOUS TABLE.

A gallon of train oil	weighs	$7\frac{1}{2}$	pounds.
A stone of butcher's meat	"	8	"
A gallon of molasses	"	11	"
A stone of iron	"	14	"
A tod	"	28	"
A firkin of butter	"	56	"
A firkin of soap	"	94	"
A quintal of fish	"	100	"
A weigh	"	182	"
A sack	"	364	"
A puncheon of foreign prunes	"	1120	"
A last	"	4368	"
A fother of lead	"	$19\frac{1}{2}$	cwt.
A barrel of anchovies	"	30	pounds.
" raisins	"	112	"
" flour	"	196	"
" pork or beef	"	200	"
" soap	"	256	"
" shad or salmon in Connecticut or New York	"	200	"
" fish in Massachusetts	is	30	gallons.
" cider and beer	"	32	"
" herrings in England	"	32	"
" salmon or eels do.	"	42	"
8 bushels of salt, measured on board the vessel,	"	1	hogshead.
$7\frac{1}{2}$ do. measured on shore,	"	1	"

3 hoops	make	1 cast.
40 casts	"	1 hundred.
10 hundred	"	1 thousand.
12 units, or things,	"	1 dozen.
12 dozen	"	1 gross.
144 dozen	"	1 great gross.

SECTION X.

COMPOUND ADDITION.

WHEN numbers are applied to things, the measure or value of which is expressed by different denominations, they lose their abstract character, and become subject to restrictions, imposed upon them by the denomination to which they are applied. Thus, when we say *six cents*, *ten days*, or *three inches*, we have not only the idea of number, but also the idea of a certain value or measure, which subjects the number in connection with it to certain limitations. And, when used in such connections, we call numbers *denominate*. Thus in 6£. 4s. 7d. the numbers 6, 4, and 7 are *denominate numbers*, so called, because they are applied to express each a particular denomination.

When now we have several numbers of different denominations, which we wish to add together, we call the process by which this is done *Compound Addition*; which we define by saying,

That it consists in adding together two or more numbers of different denominations to find the sum total.

RULE.

Write all the given numbers of the same denomination under each other; as dollars under dollars, cents under cents, &c. Then add together the numbers of the lowest denomination and divide the sum by the number which it takes of that denomination to make one of the denomination next above it, and set the remainder directly under the column that has been added. Carry the quotient to the column of the next denomination, and add as before, dividing by the number which it takes of this denomination to make one of the denomination next above it, setting down the remainder and carrying the quotient as before, and thus proceed till the column of the highest denomination is added, under which place its whole sum, and the numbers expressing the several denominations will be the sum total required.

EXAMPLES.

UNITED STATES MONEY.

1.

$.	cts.	m.
325	67	3
186	35	8
161	89	9
987	15	8
891	61	6
176	81	3
2729	51	7

2.

$.	cts.	m.
28	15	6
16	16	3
63	81	5
14	61	6
38	74	5
16	16	8

3.

E.	$.	cts.	m.
71	3	41	5
61	6	82	6
16	1	96	2
41	7	82	1
54	8	36	3
41	9	48	5

ENGLISH MONEY.

4.

£.	s.	d.
471	16	9
147	17	8
613	13	11
115	11	7
41	19	6
48	12	2
1439	11	7

5.

£.	s.	d.
28	6	$9\frac{1}{2}$
15	16	$11\frac{1}{4}$
31	13	$10\frac{1}{2}$
14	16	9
17	17	$7\frac{1}{4}$
32	18	$8\frac{1}{4}$

6.

£.	s.	d.	qr.
31	17	9	2
16	16	6	1
16	11	11	1
19	19	9	3
61	17	1	3
14	14	4	2

TROY WEIGHT.

7.

lb.	oz.	dwt.	gr.
16	11	19	23
31	10	18	16
63	9	12	15
17	8	13	12
61	7	12	16
17	6	17	22
209	7	15	8

8.

lb.	oz.	dwt.	gr.
123	9	7	13
98	11	17	14
49	7	13	21
13	10	10	20
47	9	19	23
51	5	15	15

APOTHECARIES' WEIGHT.

9.

℔	℥	ʒ	℈	gr.
27	11	7	2	19
16	10	6	1	13
41	9	3	2	16
38	10	5	2	14
41	4	4	1	11
16	6	6	2	6
183	6	3	1	19

10.

℔	℥	ʒ	℈	gr.
37	9	6	1	18
14	4	4	2	11
61	6	3	2	6
41	4	7	2	16
39	8	4	1	12
51	11	7	2	19

AVOIRDUPOIS WEIGHT.

11.

Ton.	cwt.	qr.	lb.	oz.	dr.
61	19	3	27	15	15
63	13	3	16	11	11
51	12	3	17	7	6
61	16	1	11	12	12
13	13	3	12	13	15
71	18	2	13	14	14
324	15	2	16	12	9

12.

cwt.	qr.	lb.	oz.	dr.
61	2	11	11	14
16	3	15	15	11
41	3	13	9	9
38	2	11	10	10
42	1	9	8	13
31	3	27	11	12

LONG MEASURE.

13.

Deg.	m.	fur.	rd.	ft.	in.	br.
17	69	7	39	16	11	2
61	62	3	17	12	9	1
16	16	6	16	13	10	2
48	19	3	15	15	6	1
17	58	6	33	14	7	1
33	35	5	19	9	9	2
195	54½	1	24	½	7	0
	½=4			½=6		
195	54	5	24	1	1	0

14.

m.	fur.	rd.	yd.	ft.	in.	br.
69	7	31	5	2	11	1
16	6	16	4	1	6	2
61	7	32	3	2	10	1
73	3	16	4	2	9	2
19	4	14	1	1	8	2
75	5	25	5	2	7	1

NOTE.—As half a mile is equal to 4 furlongs, we add them to the 1 furlong, which make 5 furlongs. And as half a foot is equal to 6 inches, we add them to the 7 inches, which make 13 inches; and these are equal to 1 foot 1 inch. By the same method, we obtain the result in the 14th and 17th questions.

CLOTH MEASURE.

15.

yd.	qr.	na.	in.
37	3	3	2
61	3	1	1
13	2	2	2
32	1	1	1
61	2	2	2
22	1	3	0
229	3	3	1¼

16.

E.E.	qr.	na.	in.
671	1	1	1
161	3	3	2
617	3	1	2
178	3	2	1
717	2	1	2
166	3	2	1

LAND OR SQUARE MEASURE.

17.

A.	R.	p.	ft.	in.
761	3	37	260	125
131	2	16	135	112
613	1	14	116	131
161	3	13	116	123
321	2	31	97	96
47	3	19	91	48
2038	1	13	2	95

18.

A.	R.	p.	ft.
38	1	39	278
61	3	38	167
35	3	19	198
47	3	16	271
86	2	13	198
46	1	14	269

SOLID MEASURE.

19.

Ton.	ft.	in.
29	36	1279
69	19	1345
67	18	1099
71	14	1727
43	35	916
53	17	1719
335	23	1173

20.

Cord.	ft.	in.
61	127	1161
37	89	1711
61	98	1336
43	56	1678
91	119	1357
81	115	1129

WINE MEASURE.

21.

Tun.	hhd.	gal.	qt.	pt.
61	3	62	3	1
39	2	16	1	1
68	3	57	2	1
87	3	45	3	1
47	2	59	3	1
47	3	39	2	1
354	0	30	1	0

22.

hhd.	gal.	qt.	pt.
67	15	3	1
16	16	3	0
39	16	3	0
47	62	1	1
43	57	3	0
71	61	3	1

ALE AND BEER MEASURE.

23.

Tun.	hhd.	gal.	qt.
46	3	50	3
91	2	48	3
17	3	18	0
81	3	38	2
41	1	47	1
37	2	29	3
317	2	17	0

24.

hhd.	gal.	qt.	pt.
161	53	3	1
371	52	3	1
98	19	1	0
47	43	1	0
61	43	1	1
42	27	3	1

DRY MEASURE.

25.

bu.	pk.	qt.	pt.
37	3	5	1
61	3	7	1
32	2	2	0
71	1	6	1
61	1	3	1
32	3	3	1
298	0	4	1

26.

ch.	bu.	pk.	qt.
16	31	3	3
39	31	3	1
14	16	3	1
55	15	3	0
71	17	3	1
42	14	3	1

TIME.

27.

y.	mo.	w.	d.	h.	m.	s.
57	11	3	6	23	29	55
31	11	1	3	19	19	39
46	9	2	2	17	28	56
43	10	1	1	18	17	48
32	9	1	3	16	23	28
14	1	1	5	22	28	16
227	2	0	3	21	28	2

28.

y.	mo.	d.	h.
13	5	29	17
61	11	17	21
15	9	19	16
61	10	25	23
41	4	16	17
18	5	9	6

CIRCULAR MOTION.

29.

S.	°	′	″
4	29	59	59
6	17	17	29
11	16	56	58
9	13	46	51
5	27	16	42
2	25	17	17
5	10	35	16

30.

S.	°	′	″
11	11	16	51
6	6	6	16
9	14	56	56
3	29	29	49
9	17	18	58
6	13	13	52

NOTE. — We divide the sum of the signs, in these questions, by 12, and write down the remainder, because it is Circular Motion.

MEASURING DISTANCES.

31.

m.	fur.	ch.	p.	l.
17	7	9	3	24
16	3	4	1	15
27	4	6	2	17
18	6	3	3	21
61	7	7	2	16
17	1	8	2	19
160	0	1	1	12

32.

m.	fur.	ch.	p.	l.
27	4	3	1	21
29	3	1	3	23
67	3	3	1	19
21	7	1	3	16
16	7	9	3	13
31	4	8	1	20

SECTION XI.

COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference between two numbers of different denominations.

RULE.

Write the smaller compound number under the greater, in the order of the different denominations, as pounds under pounds, shillings under shillings, &c. Begin with the lowest denomination and subtract each lower number from the one above it, and write the difference underneath. If, in any denomination, the lower number be greater than the one above it, add to the upper number the number required of this denomination to make one of the next higher; and, from the number thus obtained, subtract the lower number and set down the remainder underneath. Carry one to the next denomination in the subtrahend, and proceed in like manner with the subtraction, till the operation has been performed in all the columns, setting down the entire difference between the upper and lower numbers of the highest denomination, and the result will be the difference required.

NOTE.—The reason for increasing the number of the minuend by the number required of a lower denomination to make one of the next higher, is precisely the same as that for which we add ten to a figure of the minuend, in Simple Subtraction. In both cases we add the number denoting the ratio between the denomination in question, and the next higher number of the minuend. In Simple Subtraction, the ratio of increase from right to left being uniformly tenfold, we add ten, while in the case of farthings, pence, and shillings, we add 4, 12, and 20.

EXAMPLES.

UNITED STATES MONEY.

1.

\$	cts.	m.
169	81	3
85	93	8
83	87	5

2.

\$	cts.	m.
681	16	7
189	43	8

ENGLISH MONEY.

3.

£.	s.	d.
87	16	$3\frac{1}{4}$
19	17	$9\frac{1}{2}$
67	18	$5\frac{3}{4}$

4.

£.	s.	d.
617	11	$5\frac{1}{2}$
181	15	$8\frac{1}{4}$

TROY WEIGHT.

5.

lb.	oz.	dwt.	gr.
71	3	12	15
16	10	17	20
54	4	14	19

6.

lb.	oz.	dwt.	gr.
58	5	12	10
19	9	17	21

APOTHECARIES' WEIGHT.

7.

℔	℥	ʒ	℈	gr.
71	1	3	1	14
18	6	7	2	19
52	6	3	1	14

8.

℔	℥	ʒ	℈	gr.
15	2	2	0	15
9	9	1	1	18

AVOIRDUPOIS WEIGHT.

9.

T.	cwt.	qr.	lb.	oz.	dr.
71	18	1	13	1	13
19	19	2	16	8	5
51	18	2	24	9	8

10.

cwt.	qr.	lb.	oz.
73	1	15	13
19	1	19	15

CLOTH MEASURE.

11.

yd.	qr.	na.	in.
67	1	1	1
18	2	2	2
48	2	2	$1\frac{1}{4}$

12.

E.E.	qr.	na.
51	2	3
19	3	1

LONG MEASURE.

13.

m.	fur.	rd.	ft.	in.	bar.
16	7	18	3	2	1
9	7	19	16	8	2
6	7	38	$2\frac{1}{2}$	5	2
			$\frac{1}{2} =$	6	0
6	7	38	2	11	2

14.

deg.	m.	fur.	rd.	yd.	ft.	in.	bar.
38	41	3	29	2	1	7	2
29	36	5	31	3	1	9	1

NOTE. — As half a foot is equal to 6 inches, we add them to the 5 inches, which make 11 inches. The same principle is adopted in the 14th, 15th, and 16th examples.

LAND OR SQUARE MEASURE.

15.

A.	R.	p.	ft.	in.
56	1	19	119	110
17	3	13	127	113
38	2	5	264	33

16.

A.	R.	p.	yd.	ft.	in.
13	1	15	19	1	17
9	3	16	30	5	19

SOLID MEASURE.

17.

Tons.	ft.	in.
49	13	1611
18	15	1719
30	37	1620

18.

Cords.	ft.	in.
361	47	1178
197	121	1617

WINE MEASURE.

19.

Tun.	hhd.	gal.	qt.	pt.
79	3	19	1	1
11	1	28	2	1
68	1	53	3	0

20.

hhd.	gal.	qt.	pt.
16	1	1	0
9	2	2	1

ALE AND BEER MEASURE.

21.

Tun.	hhd.	gal.	qt.	pt.
63	1	15	1	0
19	3	16	3	1
43	1	52	1	1

22.

hhd.	gal.	qt.
769	18	1
191	19	3

DRY MEASURE.

23.

ch.	bu.	pk.	qt.
56	2	1	1
38	3	1	2
17	34	3	7

24.

ch.	bu.	pk.	qt.
39	12	2	1
12	25	3	5

TIME.

25.

mo.	d.	h.	m.	s.
6	16	13	27	19
1	22	16	41	37
4	23	20	45	42

26.

y.	m.	w.	d.	h.	m.	s.
48	0	2	5	19	27	31
19	10	3	7	21	38	56

CIRCULAR MOTION.

27.

S.	°	′	″
6	11	12	48
9	8	15	56
9	2	56	52

28.

S.	°	′	″
4	19	41	22
1	22	19	28

MEASURING DISTANCES.

29.

m.	fur.	ch.	p.	l.
21	1	3	2	19
19	2	1	3	21
1	7	1	2	23

30.

m.	fur.	ch.	p.	l.
28	6	1	2	18
15	7	3	1	19

EXERCISES IN COMPOUND ADDITION AND SUBTRACTION.

1. What is the sum of 16£. 5s. 8d. 2qr. — 31£. 16s. 11d. 3qr. — 21£. 11s. 1qr. — 19£, 0s. 10d. 3qr. — 13£. 13s. 7d. 3qr. and 28£. 17s. 5d. 1qr.? Ans. 131£. 5s. 8¼d.

2. Bought of a London tailor a vest for 1£. 13s. 4d., a coat for 7£. 12s. 9d., pantaloons for 2£. 3s. 9d., and surtout for 9£. 8s. 0d.; what was the whole amount?

Ans. 20£. 17s. 10d.

3. Bought a silver tankard, weighing 1lb. 8oz. 17dwt. 14gr., a silver can, weighing 1lb. 2oz. 12dwt., a porringer, weighing 11oz. 19dwt. 20gr., and three dozen of spoons, weighing 1lb. 9oz. 15dwt. 10gr.; what was the whole weight?

Ans. 5lb. 9oz. 4dwt. 20gr.

4. What is the weight of a mixture of 3℔ 4℥ 2ʒ 2℈ 14gr. of aloe, 2℔ 7℥ 6ʒ 1℈ 13gr. of picra, and 1℔ 10℥ 1ʒ 2℈ 17gr. of saffron? Ans. 7℔ 10℥ 3ʒ 1℈ 4gr.

5. Add 32℔ 9℥ 1ʒ 2℈ 14gr.; 13℔ 7℥ 6ʒ 1℈ 13gr.; and 16℔ 11℥ 7ʒ 1℈ 12gr. together.

Ans. 63℔ 4℥ 7ʒ 2℈ 19gr.

6. Sold 4 loads of hay; the first weighed 27cwt. 3qr. 18lb.; second, 31cwt. 1qr. 15lb.; third, 19cwt. 1qr. 15lb.; and fourth, 38cwt. 2qr. 27lb.; what is the weight of the whole?

Ans. 117cwt. 1qr. 19lb.

7. Bought 5 pieces of broadcloth; the first contained 17yd. 3qr. 2na.; second, 13yd. 2qr. 1na.; the third, 87yd. 1qr. 3na.; the fourth, 27yd. 1qr. 2na.; and the fifth, 29yd. 1qr. 2na.; what was the whole quantity purchased?

Ans. 175yd. 2qr. 2na.

8. A pedestrian travelled, the first week, 371m. 3fur. 37rd. 5yd. 2ft. 10in.; the second week, 289m. 2fur. 18rd. 3yd. 1ft. 9in.; and the third week he travelled 399m. 7fur., 3ft. 11in.; how many miles will he have travelled?

Ans. 1060m. 5fur. 16rd. 5yd. 1ft.

9. A man has 3 farms; the first contains 186A. 3R. 14p.;

the second, 286A. 17p.; and the third, 115A. 2R.; how much do they all contain? Ans. 588A. 1R. 31p.

10. The Moon is 5s. 18° 14′ 17″ east of the Sun; Jupiter is 7s. 10° 29′ 28″ east of the Moon; Mars is 11s. 12° 11′ 56″ east of Jupiter; and Herschel is 7s. 18° 38′ 15″ east of Mars; how far is Herschel east from the Sun? Ans. 7s. 29° 33′ 56″.

11. I have 4 piles of wood; the first contains 7 cords, 76ft. 1671in.; the second, 16c. 28ft. 56in.; the third, 29c. 127ft. 1000 in.; and the fourth, 29c. 10ft. 1216in.; how much is there in all? Ans. 82c. 115ft. 487in.

12. A vintner sold at one time 73hhd. 43gal. 3qt. 1pt. of wine; at another, 27hhd. 3gal.; at another, 15hhd. 3qt. 1pt.; and at another, 161hhd. and 2qt.; how much did he sell in all? Ans. 276hhd. 48gal. 1qt.

13 A man has 3 sons; the first is 14y. 3mo. 2w. 5d. old; the second is 9y. 10mo. 3w. 4d. 23h. 12m. 15sec.; and the third is 2y. 1mo. 3w. 2d. 7m.; what is the sum of their ages? and how much older is the first than the second?

Ans. 26y. 3mo. 1w. 4d. 23h. 19m. 15sec.
" 4y. 5mo. 3w. 0d. 0h. 47m. 45sec.

NOTE. — When 4 weeks are reckoned as a month, it requires 13 months to make one year.

14. I have 73A. of land; if I should sell 5A. 3R. 1p. 7ft., how much should I have left? Ans. 67A. 0R. 38p. 265¼ft.

15. A. owes B. 100£.; what will remain due after he has paid him 3s. 6½d.? Ans. 99£. 16s. 5½d.

16. It is about 25,000 miles round the globe; if a man shall have travelled 43m. 17rd. 9in. how much will remain to be travelled? Ans. 24,956m. 7fur. 22rd. 15ft. 9in.

17. Bought 7 cords of wood; and 2 cords 78ft. having been stolen, how much remained? Ans. 4c. 50ft.

18. I have 15 yards of cloth; having sold 3yd. 2qr. 1na., what remains? Ans. 11yd. 1qr. 3na.

19. If a wagon loaded with hay weighs 43cwt. 2qr. 18lb., and the wagon is afterwards found to weigh 9cwt. 3qr. 23lb., what is the weight of the hay? Ans. 33cwt. 2qr. 23lb.

20. Bought a hogshead of wine, and by an accident 8gal. 3qt. 1pt. leaked out; what remains? Ans. 54gal. 1pt.

21. I had 10A. 3R. 10p. of land; and I have sold two house-lots, one containing 1A. 2R. 13p., the other, 2A. 2R. 5p.; how much have I remaining? Ans. 6A. 2R. 32p.

22. The Moon moves 13° 10′ 35″ in a solar day, and the

Sun 59′ 8″ 20‴; now supposing them both to start from the same point in the heavens, how far will the Moon have gained on the Sun in 24 hours? Ans. 12° 11′ 26″ 40‴.

23. A farmer raised 136bu. of wheat; if he sells 49bu. 2pk. 7qt. 1pt., how much has he remaining?

Ans. 86bu. 1pk. 0qt. 1pt.

24. If from a stick of round timber, containing 2T. 18ft. 1410in., there be taken 38ft. 1720in., how much will be left?

Ans. 1T. 19ft. 1418in.

25. If from 1lb. of ipecacuanha there be taken at one time 4℥ 2ʒ 13gr., and at another, 3℥ 1ʒ 2℈ 14gr., how much will be left? Ans. 4℥ 3ʒ 2℈ 13gr.

26. A brewer has in one cellar 18bbl. 3gal. 2qt. of beer, and in another, 13bbl. 1p.; what is the whole quantity, and how much more is in one cellar than the other?

Ans. 31bbl. 3gal. 2qt. 1pt.
" 5bbl. 3gal. 1qt. 1pt.

27. If from $ 100.00 there be paid at one time $ 17.28,5, at another time $ 10.00,5, and at another $ 37.15, how much will remain? Ans. $ 35.56.

Section XII.

REDUCTION.

The object of Reduction is to change the denomination of numbers without altering their value. It consists of two parts, Descending and Ascending. The former is performed by Multiplication, the latter by Division.

Reduction Descending teaches to bring numbers of a higher denomination to a lower; as, to bring pounds into shillings, or tons into hundred weights.

Reduction Ascending teaches to bring numbers of a lower denomination into a higher; as, to bring farthings into pence, or shillings into pounds.

REDUCTION DESCENDING.

EXAMPLE.

1. In 48£. 12s. 7d. 2qr., how many farthings?

£.	s.	d.	qr.
48	12	7	2
20			
972			
12			
11671			
4			
46686			

In this example, we multiply the 48£. by 20, because it takes 20 shillings to make a pound; and to this product we add the 12s. in the question. Then we multiply by 12, because it takes 12 pence to make one shilling; and to the product we add the 7 pence in the question. We then multiply by 4, the number of farthings in a penny, and to the product we add the 2 farthings, and the work is done.

*From the above example and illustration, we deduce the following

RULE.

Multiply the highest denomination given by the number required of the next lower denomination to make one of the denomination next above it, and add to the product thus obtained the corresponding denomination of the multiplicand. Proceed in this way, till the reduction is brought to the denomination required by the question.

* NOTE 1. — To multiply by ½, we divide the multiplicand by 2, and to multiply by ¼, we divide by 4.

NOTE. 2. — The answers to Reduction Descending will be found in the questions of Reduction Ascending.

2. In 127£. 15s. 8d. how many farthings?
3. In 28£. 19s. 11d. 3qr. how many farthings?
4. In 378£. how many pence?
5. How many grains in 28lb. 11oz. 12dwt. 15gr. troy?
6. In 17lb. 12dwt. troy, how many pennyweights?
7. If a silver tankard weigh 3lb. 11oz., how many grains will it be?
8. How many scruples in 23lb. apothecaries' weight?
9. If a load of hay weigh 3T. 16cwt. 2qr. 18lb., how mā ounces will it be?
10. Required the number of drachms in a hogshead of sugar, weighing 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
11. In 57yd. how many nails?
12. In 83947E.E. 4qr. how many nails?
13. In 2263E.F. 2qr. how many quarters?
14. How many feet in 79 miles?
15. How many inches in 396 furlongs?
16. How many inches from Haverhill to Boston, the distance being 30 miles?
17. How many barleycorns will it take to reach round the world?

18. In 403m. 7fur. 35rd. 2yd. 0ft. 0in. 1bar. how many barleycorns?

19. In 413Le. 2m. 2fur. 38rd. 1yd. 0ft. 7in. how many inches?

20. In 144m. 1fur. 8rd. 1yd. 1ft. how many feet?

21. How many inches in 1051yd. 2ft. 5in.?

22. In 3576fur. 12rd. 3yd. how many yards?

23. How many square feet in 25 acres?

24. How many square rods in 365 square miles?

25. The surface of the earth contains 196563942 square miles. What would it be in square inches?

26. Required the number of feet in 10A. 3R. 38p. 6yd. 5ft. 72in.

27. In 2R. 0p. 24yd. 3ft. how many inches?

28. In 1A. 3R. 34p. 27yd. 4ft. 54in. how many inches?

29. In 17 cords of wood, how many inches?

30. In 19 tons of round timber, how many inches?

31. How many cubic feet of wood in 128 cords?

32. In 4899hhd. 4gal. 3qt. how many quarts?

33. In 1224 tuns 1p. 1hhd. 19gal. 1qt. 0pt. 1gi. how many gills?

34. How many pints in 790p. 0hhd. 58gal. 0qt. 1pt.?

35. In 460 butts 1hhd. 31gal. of beer, how many gallons?

36. In 36hhd. 26gal. 3qt. 1pt. how many pints?

37. In 16 tons of round timber, how many inches?

38. How many seconds from the deluge, it being 2348 years B. C., to the year 1836?

39. How many days did the last war continue, it having commenced June 18, 1812, and ended Feb. 17, 1815?

40. How many pecks in 676 chaldrons?

41. In 657 cents, how many mills?

42. In 3165 dimes, how many mills?

43. In 63 dollars, how many cents?

44. In 27 eagles, how many mills?

REDUCTION ASCENDING.

EXAMPLE.

1. In 76789 farthings, how many pounds?

Ans. 79£. 19s. $9\frac{1}{4}$d.

OPERATION.

```
 4)76789
12)19197  1qr.
 20)1599  9d.
      79£. 19s. 9¼d.
```

We first divide by 4, because 4 farthings make a penny. We then divide by 12, because 12 pence make a shilling. Lastly we divide by 20, the number of shillings in a pound.

From the preceding illustration and example, we deduce the following

RULE.

Divide the lowest denomination given by the number which it takes of that denomination to make one of the next higher; so proceed, until it is brought to the denomination required. Any remainders occurring in the successive divisions will be of the same denominations with the dividends to which they respectively belong.

NOTE 1. — To divide by $5\frac{1}{2}$, we multiply the multiplicand by 2, and divide the product by 11; to divide by $16\frac{1}{2}$, we multiply by 2 and divide by 33; and to divide by $272\frac{1}{4}$, we multiply by 4 and divide by 1089.

NOTE 2. — The answers to Reduction Ascending are the questions in Reduction Descending.

NOTE 3. — When we divide by 11 or 33, and there is a remainder, we divide that remainder by 2 to get the *true* remainder; and, when we divide by 1089, we must divide the remainder by 4.

2. In 122672 farthings, how many pounds?
3. In 27839 farthings, how many pounds?
4. In 90720 pence, how many pounds?
5. In 166863 grains, how many pounds troy?
6. How many pounds in 4092 pennyweights?
7. How many pounds troy in 22560 grains?
8. In 6624 scruples, how many pounds?
9. In 137376 ounces, how many tons?
10. In 1660157 drachms, how many tons?
11. How many yards in 912 nails?
12. Required the ells English in 1678956 nails.
13. Required the ells Flemish in 6791 quarters.
14. Required the miles in 417120 feet.
15. Required the furlongs in 3136320 inches.
16. Required the miles in 1900800 inches.
17. How many degrees in 4755801600 barleycorns?
18. How many miles in 767789567 barleycorns?
19. How many leagues in 78653167 inches?
20. Required the miles in 761116 feet.
21. Required the yards in 37865 inches.
22. Required the furlongs in 786789 yards.
23. How many acres in 1089000 square feet?
24. How many square miles in 37376000 square rods?
25. How many square miles in 789,103,900,894,003,200 square inches?
26. In 478675 square feet, how many acres?
27. In 3167856 square inches, how many roods?
28. How many acres in 12345678 square inches?
29. How many cords in 3760128 cubic inches?

*30. How many tons of round timber in 1313280 cubic inches ?

31. How many cords in 16384 cubic feet ?

32. How many hogsheads of wine in 1234567 quarts ?

33. How many tuns of wine in 9877001 gills ?

34. In 796785 pints of wine, how many pipes ?

35. How many butts of beer in 49765 gallons ?

36. In 15767 pints, how many hogsheads of beer ?

37. In 1105920 inches, how many tons of round timber ?

38. In 132005440800 seconds, how many years ?

39. In 974 days, how many years and calendar months ?

40. How many chaldrons in 97344 pecks ?

41. How many cents in 6570 mills ?

42. How many dimes in 316500 mills ?

43. How many dollars in 6300 cents ?

44. How many eagles in 270000 mills ?

COMPOUND REDUCTION.

1. In 57£. 15s. how many dollars ? Ans. $ 192.50cts.

2. In 67£. 14s. 9d. how many crowns at 6s. 7d. each ? Ans. 205cr. 5s. 2d.

3. How many pounds and shillings in 678 dollars ? Ans. 203£. 8s.

4. How many ells English in 761 yards ? Ans. 608E. E. 4qr.

5. How many yards in 61 ells Flemish ? Ans. 45yd. 3qr.

6. How many bottles, that contain 3 pints each, will it take to hold a hogshead of wine ? Ans. 168.

7. How many steps, 2ft. 8in. each, will a man take in walking from Bradford to Newburyport, the distance being 15 miles ? Ans. 29700.

8. How many spoons, each weighing 2oz. 12dwt., can be made from 5lb. 2oz. 8dwt. of silver ? Ans. 24.

9. How many times will the wheel of a coach revolve, whose circumference is 14ft. 9in. in passing from Boston to Washington, the distance being 436 miles ? Ans. 156073$\frac{38}{177}$.

10. I have a field of corn, consisting of 123 rows, and each row contains 78 hills, and each hill has 4 ears of corn ; now if it take 8 ears of corn to make a quart, how many bushels does the field contain ? Ans. 149bu. 3pk. 5qt. 0pt.

11. If it take 5yd. 2qr. 3na. to make a suit of clothes, how many suits can be made from 182 yards ? Ans. 32.

12. A goldsmith wishes to make a number of rings, each

weighing 5dwt. 10gr., from 3lb. 1oz. 2dwt. 2gr. of gold; how many will there be? Ans. 137.

13. How many shingles will it take to cover the roof of a building, which is 60 feet long and 56 feet wide, allowing each shingle to be 4 inches wide and 18 inches long, and to lay one third to the weather? Ans. 20160.

14. There is a house 56 feet long, and each of the two sides of the roof is 25 feet wide; how many shingles will it take to cover it, if it require 6 shingles to cover a square foot?

Ans. 16800.

15. If a man can travel 22m. 3fur. 17rd. a day, how long would it take him to walk round the globe, the distance being about 25000 miles? Ans. $1114\frac{4822}{7177}$ days.

16. If a family consume 7lb. 10oz. of sugar in a week, how long would 10cwt. 3qr. 16lb. last them? Ans. 160 weeks.

17. Sold 3 tons 17cwt. 3qr. 18lb. of lead at 7d. a pound; what did the lead amount to? Ans. 254£. 10s. 2d.

18. What will 5cwt. 1qr. 10lb. of tobacco cost, at $4\frac{1}{2}$d. a pound? Ans. 11£. 4s. 3d.

19. What will 7 hogsheads of wine cost, at 9 cents a quart?

Ans. $ 158.76.

20. What will 15 hogsheads of beer cost, at 3 cents a pint?

Ans. $ 194.40.

21. What will 73 bushels of meal cost, at 2 cents a quart?

Ans. $ 46.72.

22. A merchant has 29 bales of cotton cloth; each bale contains 57 yards; what is the value of the whole at 15 cents a yard? Ans. $ 247.95.

23. A merchant bought 4 bales of cotton; the first contained 6cwt. 2qr. 11lb.; the second, 5cwt. 3qr. 16lb.; the third, 7cwt. 0qr. 7lb.; the fourth, 3cwt. 1qr. 17lb. He sold the whole at 15 cents a pound; what did it amount to? Ans. $ 385.65.

24. A merchant having purchased 12cwt. of sugar, sold at one time 3cwt. 2qr. 11lb. and at another time he sold 4cwt. 1qr. 15lb.; what is the remainder worth, at 15 cents per pound? Ans. $ 67.50.

25. Bought 4 chests of hyson tea; the weight of the first was 2cwt. 1qr. 7lb.; the second, 3cwt. 2qr. 15lb.; the third, 2cwt. 0qr. 20lb.; the fourth, 5cwt. 3qr. 17lb.; what is the value of the whole, at $37\frac{1}{2}$ cents a pound? Ans. $ $589.12\frac{1}{2}$.

26. Purchased a cargo of molasses, consisting of 87 hogsheads; what is the value of it, at 33 cents a gallon?

Ans. $ 1808.73.

27. From a hogshead of wine, 10gal. 1qt. 1pt. 3 gills leaked out. The remainder was sold at 6 cents a gill; to what did it amount? Ans. $ 100.86.

28. A man has 3 farms; the first containing 100A. 3R. 15rd.; the second, 161A. 2R. 28rd.; the third, 360A. 3R. 5rd. He gave his oldest son a farm of 112A. 3R. 30rd.; his second son a farm of 316A. 1R. 18rd.; his youngest son a farm of 168A. 3R. 13rd.; and sold the remainder of his land at 1 dollar and 35 cents a rod; to what did it amount?
Ans. $ 5436.45.

29. A grocer bought a hogshead of molasses, containing 87gal. 1qt., from which 13 gallons leaked out; what is the remainder worth, at 1 cent a gill? Ans. $ 23.76.

30. A man bought 4 loads of hay; the first weighing 25cwt. 0qr. 17lbs.; the second, 37cwt. 2qr. 17lb.; the third, 18cwt. 3qr. 14lb.; and the fourth, 37cwt. 1qr. 17lb.; what is the value of the whole, at 2 cents a pound? Ans. $ 266.74.

REDUCTION OF THE OLD NEW ENGLAND CURRENCY TO UNITED STATES MONEY.

The original currency of N. E. was pounds, shillings, pence, and farthings; but, on the adoption of the Constitution of the United States, it was changed to dollars, cents, and mills. It is frequently necessary to reduce the former to the present currency of the United States; for which we have the following

RULE.

If pounds only are given, annex three ciphers and divide by 3, and the quotient will be the sum required in cents.

If pounds and an even number of shillings are given, annex to the pounds half the number of shillings and two ciphers, and divide as before.

If the number of shillings be odd, take half of the largest even number of shillings and annex it to the pounds with the figure 5 and one cipher, instead of two as above, and proceed as in the former instances.

If pounds, shillings, pence, and farthings are given, annex to the pounds and shillings, as before, and find the number of farthings contained in the given pence and farthings, taking care to increase their number by 1, if they exceed 12, and by 2, if they exceed 36. Annex the number thus obtained to the pounds in such a way that the units of the farthings shall occupy the third place from the pounds, and divide by 3, as before, and the quotient will be the result in cents.

NOTE. — A demonstration of this rule will be found in Sec. XXIX.

EXAMPLES.

1. Reduce 162£. to United States Money.

$$\begin{array}{r} 3\overline{)162000} \\ \hline \$\,540.00 \end{array}$$

Ans. \$ 540.

2. Change 319£. 17s. to United States Money.

$$\begin{array}{r} 3\overline{)319850} \\ \hline \$\,1066.16\tfrac{2}{3} \end{array}$$

Ans. \$ 1066.16⅔.

3. Change 176£. 17s. 8¾d. to United States Money.

$$\begin{array}{r} 176850 \\ 36 \\ \hline 3\overline{)176886} \\ \hline \$\,589.62 \end{array}$$

Ans. \$589.62.

4.	Reduce	315£.	to	U. S.	Money.	Ans.	\$ 1050.00
5.	"	619£.	to	"	"	"	2063.33⅓
6.	"	166£.	to	"	"	"	553.33⅓
7.	"	318£.	to	"	"	"	1060.00
8.	"	101£.	to	"	"	"	336.66⅔
9.	"	144£.	to	"	"	"	480.00
10.	"	161£. 18s.	to	"	"	"	539.66⅔
11.	"	361£. 17s.	to	"	"	"	1206.16⅔
12.	"	99£. 11s.	to	"	"	"	331.83⅓
13.	"	100£. 9s.	to	"	"	"	334.83⅓
14.	"	661£. 7s.	to	"	"	"	2204.50
15.	"	47£. 11s.	to	"	"	"	158.50
16.	"	109£. 1s.	to	"	"	"	363.50
17.	"	16£. 17s. 6½d.	to	"	"	"	56.25⅔
18.	"	69£. 1s. 3¼d.	to	"	"	"	230.21⅓
19.	"	87£. 16s. 11d.	to	"	"	"	292.82
20.	"	14£. 7s. 7½d.	to	"	"	"	47.93⅔
21.	"	73£. 3s. 4½d.	to	"	"	"	243.89⅔
22.	"	47£. 12s. 10d.	to	"	"	"	158.80⅔
23.	"	187£. 5s. 0¾d.	to	"	"	"	624.17⅔
24.	"	10£. 0s. 3¼d.	to	"	"	"	33.38

SECTION XIII.

UNITED STATES MONEY.

THE denominations of Federal or United States Money being in the ratio of 10, 100, and 1000 to each other, operations in-

volving dollars, cents, and mills are performed, when the numbers have been properly set down, as under the rules for simple numbers.

ADDITION.

RULE.

Write dollars under dollars, cents under cents, and mills under mills, and then proceed as in Simple Addition, and the result will be obtained in the several denominations added.

NOTE. — In all operations of United States Money, it must be borne in mind that a cent is one hundredth of a dollar, and hence, in arranging a column of cents or annexing any number of cents to dollars, 1 cent, 2 cents, &c., must be written .01, .02, &c., which denote one hundredth, two hundredths, &c.

EXAMPLES.

1.	2.	3.	4.
\$ cts. m.	\$ cts. m.	\$ cts. m.	\$ cts. m.
375.87,5	78.19,3	171.01,3	861.07,3
671.12,7	18.01,4	382.09,4	516.71,6
387.14,3	91.03,8	999.90,0	344.67,3
184.18,9	16.81,7	155.06,8	617.81,4
147.75,8	81.47,6	48.15,3	169.97,3
63.07,2	43.18,4	49.61,9	810.42,6
1829.16,4			

5. Add the following sums, \$18.16,5, \$701.63, \$151.16,1, \$375.08,9, and \$471.01,7. Ans. \$1717.06,2.

6. Bought a horse for eighty-seven dollars nine cents, a pair of oxen for sixty-five dollars twenty cents, and six gallons of molasses for two dollars six cents five mills; what was the amount of my bill? Ans. \$154.35,5.

7. Sold a calf for three dollars eight cents, a bushel of corn for ninety-seven cents five mills, and three bushels of rye for three dollars five cents; what was the amount received?

Ans. \$7.10,5.

SUBTRACTION.

RULE.

Write the several denominations of the subtrahend under the corresponding ones of the minuend, and then proceed as in Simple Subtraction, and the result will be the difference in the several denominations subtracted.

EXAMPLES.

1.	2.	3.	4.
$ cts. m.	$ cts. m.	$ cts. m.	$ cts. m
871.16,1	478.47,7	167.16,3	163.16,7
89.91,8	199.99,1	98.09,7	9.09,8
781.24,3			

5. Bought a farm for $ 1728.90, and sold it for $ 3786.98, what did I gain by my bargain? Ans. $ 2058.08.

6. Gave $ 79.25 for a horse, and $ 106.87,5 for a chaise, and sold them both for $ 200; what did I gain? Ans. $ 13.87,5.

7. Bought a farm for $ 8967, and sold it for nine thousand eight hundred seventy-six dollars seventy-five cents; what did I gain? Ans. $ 909.75.

8. Bought a barrel of flour for $ 7.50, three bushels of rye for $ 2.75, three cords of wood at $ 5.25 a cord; I sold the flour for $ 6.18, the rye for $ 3.00, and the wood for $ 6.75 a cord; what was gained by the bargain? Ans. $ 3.43.

9. A young lady went a "shopping." Her father gave her a twenty-dollar bill. She purchased a dress for $ 8.16, a muff for $ 3.19, a pair of gloves for $ 1.12, a pair of shoes for $ 1.90, a fan for $ 0.19, and a bonnet for $ 3.08; how much money did she return to her father? Ans. $ 2.36.

MULTIPLICATION.

RULE.

With the dollars, cents, &c. for the multiplicand, proceed as in Simple Multiplication, and the result will be the product in the terms of the lowest denomination contained in the multiplicand. If the multiplicand consists of dollars only, the product will be dollars; if there are cents, either with or without dollars, the product will be cents, and the two right hand figures must be separated by the appropriate point. If there are mills, the product will be mills, and the three right hand figures must be pointed off. The figures on the left of the point will denote dollars, the next two following it will denote cents, and the third mills.

EXAMPLES.

1. What will 365 barrels of Genesee flour cost, at $ 5.75 a barrel?

```
 $ 5.75
    365
   2875
  3450
 1725
$ 2098.75 Ans.
```

2. What will 128 pounds of sugar cost, at 13 cents 7 mills a pound?

```
$ .13,7
   128
  ----
  1096
  274
 137
 ------
$ 17.53,6 Ans.
```

3. What will 126 pounds of butter cost, at 13 cents a pound? Ans. $ 16.38.

4. What will 63 pounds of tea cost, at 93 cents a pound? Ans. $ 58.59.

5. What will 43 tons of hay cost, at 13 dollars 75 cents a ton? Ans. $ 591.25.

6. If 1 pound of pork is worth 7 cents 3 mills, what are 46 pounds worth? Ans. $ 3.35,8.

7. If 1cwt. of beef cost 3 dollars 28 cents, what are 76cwt. worth? Ans. $ 249.28.

8. What will 96,000 feet of boards cost, at 11 dollars 67 cents a thousand? Ans. $ 1120.32.

9. If a barrel of cider be sold for 2 dollars 12 cents, what will be the value of 169 barrels? Ans. $ 358.28.

10. What will be the value of a hogshead of wine, containing 63gals., at 1 dollar 63 cents a gallon? Ans. $ 102.69.

11. Sold a sack of hops, weighing 396 pounds, at 11 cents 3 mills a pound; to what did it amount? Ans. $ 44.74,8.

12. Sold 19 cords of wood, at 5 dollars 75 cents a cord; to what did it amount? Ans. $ 109.25.

13. Sold 169 tons of timber, at 4 dollars 68 cents a ton; what did I receive? Ans. $ 790.92.

14. Sold a hogshead of sugar, weighing 465 pounds; to what did it amount, at 14 cents a pound? Ans. $ 65.10.

15. What will be the amount of 789 pounds of leather, at 18 cents a pound? Ans. $ 142.02.

16. What will be the expense of 846 pounds of sheet lead, at 5 cents 7 mills a pound? Ans. $ 48.22,2.

17. When potash is sold for 132 dollars 55 cents a ton, what will be the price of 369 tons? Ans. $ 48910.95.

18. What will 365 pounds of beeswax cost, at 18 cents 4 mills a pound? Ans. $ 67.16.

19. If 1 pound of tallow cost 7 cents 3 mills, what are 968 pounds worth? Ans. $ 70.66,4.

20. What will a chest of souchong tea be worth, containing 69 pounds, at 29 cents 9 mills a pound? Ans. $20.63,1.

21. If 1 drum of figs cost 2 dollars 75 cents, what will be the price of 79 drums? Ans. $217.25.

22. If 1 box of oranges cost 6 dollars 71 cents, what will be the price of 169 boxes? Ans. $1133.99.

23. Purchased 796 pounds of cocoa, at 11 cents 4 mills a pound; what did I have to pay? Ans. $90.74,4.

24. A farmer sold 691 bushels of wheat, at 1 dollar 25 cents a bushel; what did he receive for it? Ans. $863.75.

25. What are 97 pounds of madder worth, at 17 cents 6 mills a pound? Ans. $17.07,2.

26. A merchant sold 73 hogsheads of molasses, each containing 63 gallons, for 44 cents a gallon; how much money did he receive? Ans. $2023.56.

27. A drover has 169 sheep, which he values at 2 dollars 69 cents a head; what is the value of the whole drove? Ans. $454.61.

28. A farm containing 144 acres is valued at 69 dollars 74 cents 8 mills an acre; what is the amount of the whole? Ans. $10043.71,2.

29. An auctioneer sold 48 bags of cotton, each containing 397 pounds, at 13 cents 7 mills a pound; what is the value of the whole? Ans. $2610.67,2.

30. If 1 yard of broadcloth cost 5 dollars 67 cents, what will be the value of 48 yards? Ans. $272.16.

31. A wool-grower has 179 sheep, each producing 4 pounds of wool; what will be its value, at 59 cents 3 mills a pound? Ans. $424.58,8.

32. A house having 17 rooms requires 6 rolls of paper for each room; now, if each roll cost 1 dollar 17 cents, what will be the expense for all the rooms? Ans. $119.34.

33. What will 89 yards of brown sheeting cost, at 17 cents a yard? Ans. $15.13.

34. What will 47,000 of shingles cost, at 3 dollars 75 cents a thousand? Ans. $176.25.

35. Bought 47 hogsheads of salt, each containing 7 bushels, for 1 dollar 12 cents a bushel; what did it cost? Ans. $368.48.

36. What will a ton of hay cost, at 1 dollar 17 cents a hundred weight? Ans. $23.40.

37. If 1 foot of wood cost 63 cents, what will 39 cords cost? Ans. $196.56.

38. What will 163 buckets cost, at 1 dollar 21 cents a bucket? Ans. $ 197.23.

39. What will 78 barrels of shad cost, at 3 dollars 89 cents a barrel? Ans. $ 303.42.

40. If 1 pound of salmon cost 17 cents 5 mills, what will 789 pounds cost? Ans. $ 138.07,5.

41. Bought 163 grindstones, at 6 dollars 79 cents each; what did the whole cost? Ans. $ 1106.77.

42. Sold 49 green hides, at 1 dollar 95 cents each; what did they all amount to? Ans. $ 95.55.

43. If a man's wages be 1 dollar 19 cents a day, what are they for a year? Ans. $ 434.35.

44. Hired a horse and chaise to go a journey of 146 miles, at 16 cents a mile; what did it cost? Ans. $ 23.36.

45. If it be worth 3 dollars 68 cents to plough one acre of land, what would it be worth to plough 79 acres? Ans. $ 290.72.

46. What will 148 tons of plaster of Paris cost, at 2 dollars 28 cents a ton? Ans. $ 337.44.

47. Bought 79 tons of logwood, at 49 dollars 75 cents a ton; what did it cost? Ans. $ 3930.25.

48. Bought 5 gross bottles of castor oil, at 37 cents a bottle; what did it cost? Ans. $ 266.40.

49. Sold 19 dozen pair of men's gloves, at 47 cents a pair; to what did they amount? Ans. $ 107.16.

50. Bought a hogshead of wine for 97 cents a gallon, and sold it for 1 dollar 75 cents a gallon; what did I gain? Ans. $ 49.14.

51. Bought 75 barrels of flour, at 5 dollars 75 cents a barrel, and sold it at 6 dollars 37 cents; what did I gain? Ans. $ 46.50.

52. Bought 17 score of penknives, at 17 cents each; what did they all cost? Ans. $ 57.80.

53. What will 17½ tons of coal cost, at $ 9.62 per ton? Ans. $ 168.35.

54. What will 19 barrels of cider cost, at $ 1,37½ per barrel? Ans. $ 26.12,5.

DIVISION.

RULE.

With the sum given for the dividend, proceed as in Simple Division, and the result will be the quotient in the lowest denomination contained in the dividend.

The rule for pointing off cents and mills is the same as in Multiplication. If the dividend consist of dollars only, and be either smaller than the divisor, or not divisible by it without a remainder, annex two or three ciphers, as the case may require, and the quotient will be cents or mills accordingly.

1. If 97 bushels of wheat cost $ 147.82,8, what is the value of one bushel? Ans. $ 1.52,4.

OPERATION.

```
97)147.82,8($ 1.52,4
   97
   ---
   508
   485
   ---
    232
    194
    ---
     388
     388
```

2. Bought 1789 acres of land for $ 1699.55; what cost one acre? Ans. $ 0.95.

3. A trader sold 425 pounds of sugar for $ 51.00; what was the cost of one pound? Ans. $ 0.12.

4. When rye is sold at the rate of 628 bushels for $ 471.00, what is that a bushel? Ans. $ 0.75.

5. A merchant bought 329 yards of broadcloth for $ 904.75; what cost one yard? Ans. $ 2.75.

6. When a chest of tea containing 42 pounds can be bought for $ 31.50, what cost one pound? Ans. $ 0.75.

7. If it cost $1460 to support a family 365 days, what would be the expense per day? Ans. $ 4.00.

8. A shoe-dealer sold 125 cases of shoes for $ 2500; what was the cost per case? Ans. $ 20.00.

9. A flour-merchant sold 475 barrels of flour for $ 2018.75; what cost one barrel? Ans. $ 4.25.

10. Bought 42 barrels of pears for $ 73.50; what cost one barrel? Ans. $ 1.75.

11. If 1624 pounds of pork cost $ 97.44, what cost one pound? Ans. $ 0.06.

12. If 47000 shingles cost $ 176.25, what is the cost per thousand? Ans. $ 3.75.

13. Bought 148 tons of plaster of Paris for $ 337.44; what was it per ton? Ans. $ 2.28.

14. If 78 barrels of fish cost $ 303.42, what will one barrel cost? Ans. $ 3.89.

15. A farmer sold 691 bushels of wheat for $ 863.75; what was it per bushel? Ans. $ 1.25.

16. If a man earn $ 434.35 in a year, what is that per day? Ans. $ 1.19.

17. Sold 169 tons of timber for $ 790.92; what cost one ton? Ans. $ 4.68.

18. What cost one pound of leather, if 789 pounds cost $ 142.02? Ans. $ 0.18.

19. If 369 tons of potash cost $ 48910.95, what will be the price of one ton? Ans. $ 132.55.

20. Bought 47 hogsheads of salt, each hogshead containing 7 bushels, for $ 368.48; what cost one bushel? Ans. $ 1.12.

21. If 19 cords of wood cost $ 106.97, what cost one cord? Ans. $ 5.63.

22. When 19 bushels of salt can be bought for $ 30.87,5, what cost one bushel? Ans. $ 1.62,5.

23. If 17 chests of souchong tea, each weighing 59 pounds, cost $ 672.01, what cost one pound? Ans. $ 0.67.

24. Sold 73 tons of timber for $ 414.64; what did I receive per ton? Ans. $ 5.68.

25. Bought oil at the rate of 144 gallons for $ 234.00; what did I give per gallon? Ans. $ 1.62,5.

26. A landholder sold 47 acres of land for $ 1774.25; what did he receive per acre? Ans. $ 37.75

27. What is the price of one yard of broadcloth, if 163 yards cost $ 1106.77? Ans. $ 6.79.

28. If a farm, containing 144 acres, is valued at $ 10043.71,2, what is one acre worth? Ans. $ 69.74,8.

BILLS.

1. Boston, July 4, 1835.

Mr. James Dow,

Bought of Dennis Sharp,

17	yds.	Flannel,	at	.45 cts.
19	"	Shalloon,	"	.37 "
16	"	Blue Camlet,	"	.46 "
13	"	Silk Vesting,	"	.87 "
9	"	Cambric Muslin,	"	.63 "
25	"	Bombazine,	"	.56 "
17	"	Ticking,	"	.31 "
19	"	Striped Jean,	"	.16 "
				$ 61.33

Received payment,

Dennis Sharp.

2.

Haverhill, May 5, 1835.

Mr. Samuel Smith,

Bought of David Johnson,

13 lbs.	Tea,	at	.98	cts.
16 "	Coffee,	"	.15	"
36 "	Sugar,	"	.13	"
47 "	Cheese,	"	.09	"
12 "	Pepper,	"	.19	"
7 "	Ginger,	"	.17	"
13 "	Chocolate,	"	.61	"
			$ 35.45.	

Received payment,

David Johnson.

3.

Salem, February 29, 1835.

Mr. John Dow,

Bought of Richard Fuller,

17 yds.	Broadcloth,	at	$ 5.25
29 "	Cassimere,	"	1.62
60 "	Bleached Shirting,	"	.17
49 "	Ticking,	"	.27
18 "	Blue Cloth,	"	3.19
27 "	Habit do.	"	2.75
75 "	Flannel,	"	.61
36 "	Plaid Prints,	"	.75
49 "	Brown Sheeting,	"	.18
			$ 372.90.

Received payment,

Richard Fuller.

4.

Baltimore, January 20, 1835.

Mr. John Rilley,

Bought of James Somes,

10 pair	Boots,	at	$ 2.75
19 "	Shoes,	"	1.25
83 "	Hose,	"	1.29
47 lbs.	Ginger,	"	.17
91 "	Chocolate,	"	.39
47 "	Pepper,	"	.28
68 "	Flour,	"	.13
27 pair	Gloves,	"	1.39
			$ 258.98.

Received payment,

James Somes.

5. Philadelphia, June 11, 1835.

Mr. Moses Thomas,

Bought of Luke Dow,

27	National Spelling-Books,	at	$0.19
25	Parker's Composition,	"	.27
17	National Arithmetics,	"	.75
9	Greek Lexicons,	"	3.75
8	Ainsworth's Dictionaries,	"	4.50
27	Greek Readers,	"	2.25
18	Folio Bibles,	"	9.87
75	Leverett's Cæsar,	"	.31
67	Fisk's Greek Grammar,	"	.75
15	Folsom's Cicero's Orations,	"	1.12
			$423.09

Received payment,

Luke Dow, by
Timothy True.

6. Boston, June 26, 1835.

Dr. Enoch Cross,

Bought of Maynard & Noyes,

14	oz.	Ipecacuanha,	at	$0.67
23	"	Laudanum,	"	.89
17	"	Emetic Tartar,	"	1.25
25	"	Cantharides,	"	2.17
27	"	Gum Mastic,	"	.61
56	"	Gum Camphor,	"	.27
				$136.94.

Received payment,

Maynard & Noyes,
by Timothy Jones.

7. Newburyport, June 5, 1835.

Mr. John Somes,

Bought of Samuel Gridley,

7½	yds.	Broadcloth,	at	$4.50
16¾	lbs.	Coffee,	"	.16
18¼	"	Candles,	"	.25
30	"	Soap,	"	.17
3	"	Pepper,	"	.19
7½	"	Ginger,	"	.18
				$48.01¼.

Received payment,

Samuel Gridley.

8. Boston, May 1, 1835.

Mr. Benjamin Treat,

Bought of John True,

37 Chests	Green Tea,	at	$ 25.50
41 "	Black do.	"	16.17
40 "	Chests of Imperial Tea,	"	97.75
13 Crates	Liverpool Ware,	"	169.37
			$ 7718.28.

Received payment,

John True.

9. New York, July 11, 1835.

Mr. John Cummings,

Bought of Lord & Secomb.

97 bbl.	Genesee Flour,	at	$ 6.25
167 "	Philadelphia do.	"	5.95
87 "	Baltimore do.	"	6.07
196 "	Richmond do.	"	5.75
275 "	Howard St. do.	"	7.25
69 bu.	Rye,	"	1.16
136 "	Virginia Corn,	"	.67
68 "	North River do.	"	.76
169 "	Wheat,	"	1.37
76 Ton	Lehigh Coal,	"	9.67
89 "	Iron,	"	69.70
49	Grindstones,	"	3.47
39	Pitchforks,	"	1.61
197	Rakes,	"	.17
86	Hoes,	"	.69
78	Shovels,	"	1.17
187	Spades,	"	.85
91	Ploughs,	"	11.61
83	Harrows,	"	17.15
47	Handsaws,	"	3.16
35	Millsaws,	"	18.15
47 cwt.	Steel,	"	9.47
57 "	Lead,	"	6.83
			$ 17315.32.

Received payment,

Lord & Secomb.

SECTION XIV.

COMPOUND MULTIPLICATION.

CASE I.

COMPOUND MULTIPLICATION consists in multiplying numbers of different denominations by simple numbers.

1. What will 6 bales of cloth cost, at 7£. 12s. 7d. per bale?

£.	s.	d.
7	12	7
		6
45	15	6

In this question, we multiply 7d. by 6, and find the product to be 42d. This we divide by 12, the number of pence in a shilling, and find it contains 3s. and 6d. We write the 6d. under the pence, and carry 3 to the product of 6 times 12, and find the amount to be 75s., which we reduce to pounds by dividing them by 20, and find them to be 3£. 15s. We write down the shillings under the shillings, and carry 3 to the product of 6 times 7£.; and we thus find the answer to be 45£. 15s. 6d.

From the above illustration we deduce the following

RULE.

When the multiplier is less than 12, multiply by the multiplier and carry as in Compound Addition.

NOTE.—For the answers in Multiplication, see Section XV., in Division.

2. What cost 9yds. of cloth, at 1£. 3s. 8d. per yard? Ans. 10£. 13s. 0d.
3. What cost 7bbls. of flour, at 1£. 8s. 7½d. per barrel?
4. What cost 8lbs. of Cayenne pepper, at 7s. 9½d. per lb.?
5. Multiply 10yd. 3qr. 3na. by 5.
6. Multiply 3cwt. 1qr. 8lb. by 9.
7. Multiply 7T. 11cwt. 1qr. 20lb. by 5.
8. Multiply 7 days 15h. 35m. 18sec. by 10.
9. Multiply 18£. 16s. 7½d. by 4. Ans. 75£. 6s. 6d.
10. Multiply 15£. 11s. 8¾d. by 8. Ans. 124£. 13s. 10d.
11. Multiply 27£. 19s. 11½d. by 9. Ans. 251£. 19s. 7½d.
12. Multiply 19£. 5s. 7¼d. by 11. Ans. 212£. 1s. 7¾d.
13. Multiply 81£. 14s. 9d. by 8. Ans. 653£. 18s. 0d.
14. Multiply 15£. 18s. 5d. by 7. Ans. 111£. 8s. 11d.
15. Multiply 13£. 5s. 4¾d. by 12. Ans. 159£. 4s. 9d.
16. Multiply 17lb. 7oz. 13dwt. 13gr. by 9.
17. Multiply 15lb. 11oz. 19dwt. 15gr. by 7.

18. Multiply 16T. 12cwt. 3qr. 13lb. 12oz. by 11.
19. Multiply 13T. 3cwt. 1qr. 14lb. 13oz. by 8.
20. Multiply 2℔ 5℥ 5ʒ 1℈ 16$\frac{1}{8}$gr. by 8.
21. Multiply 47yd. 3qr. 2na. 2in. by 7.
22. Multiply 17m. 7fur. 36rd. 13ft. 7in. by 12.
23. Multiply 16deg. 39m. 3fur. 39rd. 5yd. 2ft. by 9.
24. Multiply 16deg. 20m. 7fur. 12rd. 8ft. 11in. 1$\frac{1}{6}$bar. by 6.
25. Multiply 16A. 2R. 4p. 19yd. 7ft. 79in. by 11.
26. Multiply 7 cords 116ft. 1629in. by 4.
27. Multiply 29hhd. 61gal. 3qt. 1pt. 3gi. by 7.
28. Multiply 3 tuns 3hhd. 56gal. 2qt. by 9.
29. Multiply 7hhd. 5gal. 2qt. 1pt. by 8.
30. Multiply 19bu. 2pk. 7qt. 1pt. by 6.
31. Multiply 36ch. 18bu. 3pk. 7qt. by 7.
32. Multiply 13y. 316d. 15h. 27m. 39sec. by 8.

33. If a man gives each of his 9 sons 23A. 3R. 19$\frac{3}{8}$p., what do they all receive?

34. If 12 men perform a piece of labor in 7h. 24m. 36sec., how long would it take 1 man to perform the same task?

35. If 1 bag contain 3bu. 2pk. 4qt., what quantity do 8 bags contain?

CASE II.

When the multiplier is more than 12, and is a composite number, that is, a number which is the product of two or more numbers, the question is performed as in the following

EXAMPLE.

36. What will 42 yards of cloth cost, at 6s. 9d. a yard?

£.	s.	d.	
0	6	9	
		6	
2	0	6	= price of 6 yds.
		7	
14	3	6	= price of 42 yds.

In this example, we find that 6 multiplied by 7 will produce the quantity 42 yards. We therefore multiply 6s. 9d. first by the 6, and then its product by 7; and the last product, 14£. 3s. 6d. is the answer or price of the 42 yards.

The pupil will now see the propriety of the following

RULE.

Multiply by one of the factors of the composite number, and the product thus obtained by the other.

37. What will 16 yards of velvet cost, at 3s. 8d. per yard?

38. What will 72 yards of broadcloth cost, at 19s. 11d. per yard?

39. What will 84 yards of cotton cost, at 1s. 11d. per yard?

40. Bought 90 hogsheads of sugar, each weighing 12cwt. 2qr. 11lb.; what was the weight of the whole?

41. What cost 18 sheep, at 5s. 9½d. a piece?

42. What cost 21 yards of cloth, at 9s. 11d. per yard?

43. What cost 22 hats, at 11s. 6d. each?

44. If 1 share in a certain stock be valued at 13£. 8s. 9½d., what is the value of 96 shares?

45. If 1 spoon weigh 3oz. 5dwt. 15gr., what is the weight of 120 spoons?

46. If a man travel 24m. 7fur. 4rd. in 1 day, how far will he go in 1 month?

47. If the earth revolve 0° 15′ per minute, how far per hour?

48. Multiply 39A. 3R. 17p. 30yd. 8ft. 100in. by 32.

49. If a man be 2d. 5h. 17m. 19sec. in walking 1 degree, how long would it take him to walk round the earth, allowing 365¼ days to a year?

CASE III.

When the multiplier is such a number as cannot be produced by the product of two or more numbers, we should proceed as in the following

EXAMPLE.

50. What is the value of 53 tons of iron, at 18£. 17s. 11d. a ton?

£.	s.	d.	
18	17	11	
		5	
94	9	7	= price of 5 tons.
		10	
944	15	10	= price of 50 tons.
56	13	9	= price of 3 tons.
1001	9	7	= price of 53 tons.

£.	s.	d.	
18	17	11	
		3	
56	13	9	= price of 3 tons.

Because 53 is a *prime* number, that is, it cannot be produced by the product of any two numbers; we therefore find a convenient composite number less than the given number, viz. 50, which may be produced by multiplying 5 by 10. Having found the price of 50 tons by the last Case, we then find the price of the 3 remaining tons by Case I., and add it to the former, making the value of the whole quantity 1001£. 9s. 7d.

The pupil will hence perceive the propriety of the following

RULE.

Take for successive multipliers two or more numbers, whose continued product will be nearest the proper multiplier, and then find the value of the remainder by Case I., and the sum of the last two products will be the answer.

51. What will 57 gallons of wine cost, at 8s. 3¼d. per gallon?

52. Bought 29 lots of wild land, each containing 117A. 3R. 27p.; what were the contents of the whole?

53. Bought 89 pieces of cloth, each containing 37yd. 3qr. 2na. 2in.; what was the whole quantity?

54. Bought 59 casks of wine, each containing 47gal. 3qt. 1pt.; what was the whole quantity?

55. If a man travel 17m. 3fur. 13rd. 14ft. in one day, how far will he travel in a year?

56. If a man drink 3gal. 1qt. 1pt. of beer in a week, how much will he drink in 52 weeks?

57. There are 17 sticks of timber, each containing 37ft. 978in.; what is the whole quantity?

58. There are 17 piles of wood, each containing 7 cords 98 cubic feet; what is the whole quantity?

59. Multiply 2hhd. 19gal. 0qt. 1pt. by 39.

60. Multiply 3bu. 1pk. 4qt. 1pt. 1gi. by 53.

61. Multiply 16ch. 7bu. 2pk. 0qt. 0pt. by 17.

BILLS.

1. London, July 4, 1835.

Dow, Vance, & Co., of Boston, U. S.,

Bought of Samuel Snow,

45	yds. Broadcloth,	at	8s.	4d.	
50	" "	"	10s.	6d.	
56	" "	"	3s.	7½d.	
63	" "	"	12s.	11¾d.	
72	" "	"	19s.	11d.	
81	" "	"	9s.	3d.	
35	" "	"	19s.	7½d.	
99	" "	"	16s.	0½d.	
66	" "	"	8s.	11d.	
33	" "	"	16s.	11½d.	
					376£. 7s. 0¾d.

Received payment,

Samuel Snow.

2. Quebec, Jan. 8, 1835.

Mr. John Vose, Bought of Vans & Conant,

46 Ivory Combs,	at	3s.	5¼d.
47 lbs. Colored Thread,	"	6s.	9¼d.
51 yds. Durant,	"	1s.	8d.
52 Silk Vests,	"	6s.	7d.
53 Leghorns,	"	11s.	9½d.
57 ps. Nankin,	"	8s.	3¼d.
58 lbs. White Thread,	"	9s.	11¼d.
			128£. 16s. 5½d.

Received payment, Vans & Conant.

3. Montreal, July 4, 1835.

Mr. James Savage, Bought of Joseph Dowe,

83 gals. Lisbon Wine,	at	6s.	7d.
85 " Port do.	"	3s.	9½d.
86 " Madeira do.	"	4s.	11½d.
87 " Temperance do.	"	3s.	6½d.
89 " Oil,	"	5s.	3d.
91 Leghorns,	"	19s.	10¼d.
92 lbs. Green Tea,	"	3s.	1½d.
93 pair Thread Hose,	"	4s.	4½d.
94 " Silk Gloves,	"	3s.	3½d.
95 " Silk Hose,	"	6s.	6½d.
97 yds. Linen,	"	5s.	5½d.
98 gals. Winter Strained Oil,		7s.	7½d.
			338£. 19s. 2¼d.

Received payment, Joseph Dowe.

4. Montreal, June 17, 1835.

Mr. Samuel Simpson, Bought of Lackington, Grey, & Co.

19 yds. Cloth,	at	1s.	6d.
23 " Worsted,	"	7s.	8¼d.
26 " Baize,	"	3s.	11½d.
29 " Camlet,	"	6s.	10½d.
31 " Bombazine,	"	1s.	5¼d.
34 " Linen,	"	3s.	7d.
37 " Cotton,	"	11s.	9d.
38 " Flannel,	"	6s.	11d.
39 " Calico,	"	3s.	10¼d.
41 " Broadcloth,	"	6s.	9½d.
43 " Nankin,	"	7s.	5¾d.
			106£. 1s. 11½d.

Received payment, Lackington, Grey, & Co.

5. Liverpool, June 2, 1835.

John Jones, of Philadelphia, U. S.,

Bought of Thomas Hasseltine,

297 yds.	Black Broadcloth,	at	17s.	3½d.	
473 "	Blue do.	"	9s.	11¼d.	
512 "	Red do.	"	15s.	10d.	
624 "	Green do.	"	12s.	8d.	
765 "	White do.	"	19s.	9½d.	
169 "	Black Velvet,	"	13s.	5¼d.	
698 "	Green do.	"	15s.	6¾d.	
315 "	Red do.	"	14s.	3½d.	
713 "	White do.	"	11s.	7½d.	
519 "	Carpet,	"	13s.	6¼d.	
147 "	Black Kerseymere,	"	16s.	7¾d.	
386 "	Blue do.	"	14s.	3¼d.	
137 "	Green do.	"	19s.	9d.	
999 "	Black Silk,	"	15s.	8d.	
					5012£. 0s. 11½d.

Received payment,

Thomas Hasseltine.

6. London, May 11, 1846.

Messrs. Kimball, Jewett, & Co., of Boston, U. S.,

Bought of Benjamin Fowler,

							£.	s.	d.
2345 yds.	Red Broadcloth,	at	1£.	17s.	9¼d.	=	4428	12	7¼
7186 "	Green do.	"	3£.	15s.	8½d.	=	27202	0	1
8011 "	Black do.	"	2£.	18s.	10¼d.	=	23574	0	8¾
6789 "	Blue do.	"	1£.	6s.	9d.	=	9080	5	9
3178 "	White do.	"	2£.	1s.	7¾d.	=	6617	10	5½
2365 "	Pongee Silk,	"	1£.	2s.	8½d.	=	2685	5	2½
5107 "	Black do.	"	1£.	7s.	5¼d.	=	7006	3	3¾
4444 bales	Cotton Cloth,	"	3£.	16s.	8¼d.	=	17039	19	3
7777 "	Irish Linen,	"	17£.	19s.	9d.	=	139888	15	9
1234 "	Nankin,	"	7£.	15s.	11d.	=	9620	1	2
4567 "	Flannel,	"	8£.	16s.	7½d.	=	40332	6	4½
9876 "	Bombazine,	"	3£.	5s.	5¾d.	=	32333	12	3
7658 "	Calico,	"	9£.	17s.	6½d.	=	75638	14	1
8107 "	Camlet,	"	8£.	6s.	0¾d.	=	67313	8	8¼
4725 "	Baize,	"	3£.	7s.	9¾d.	=	16020	14	0¾
5670 "	Durant,	"	4£.	18s.	5½d.	=	27907	0	7½
							506688£.	10s.	4¾d

Received payment,

Benjamin Fowler.

SECTION XV.

COMPOUND DIVISION.

COMPOUND DIVISION is when the dividend consists of several denominations.

EXAMPLES.

1. Divide 598£. 8s. 9d. equally among 5 persons.

```
  £.    s.  d.
5)598   8   9
 119.  13   9
```

Having divided the pounds by 5, we find 3£. remaining, which are 60s.; to these we add the 8s. in the question, and again divide by 5 and find 3s. remaining, which are 36d.; to these we add the 9d. in the question, and divide their sum by 5. The several quotients we write under their respective denominations.

2. Divide 168£. 15s. 0d. equally among 36 men.

```
    £.    s.  d.
36)168   15   0(4£.
   144
    24
    20
 36)495(13s.
    36
    135
    108
     27
     12
 36)324(9d.
    324
```

When the divisor is more than 12, we usually perform the operation by *Long Division.* In the present example, we first divide the pounds by 36, and obtain 4£. for the quotient and 24£. remaining, which we reduce to shillings and annex the 15s., and again divide by 36, and obtain 13s. for the quotient. The remainder we reduce to pence, and again divide, and obtain 9d. for the quotient.

From the above examples and illustrations we deduce the following

RULE.

Divide the highest denomination of the dividend by the divisor, and, if there be a remainder, reduce it to the next lower denomination, adding to the number thus found the number in the dividend of the same denomination. Divide the result thus obtained by the divisor; and, if there be a remainder, proceed as before, till all the denominations of the dividend are taken, or till the work is finished. The successive quotients will be of the same denominations with the successive numbers divided, or will correspond with the several denominations of the dividend.

NOTE.—When the operation is performed by Short Division, the several quotients must be placed under their respective denominations.

2. When 10£. 13s. 0d. are paid for 9 barrels of flour, what is the cost of one barrel?

3. If 7bbls. of flour cost 10£. 0s. 4½d., what cost one barrel?

4. Paid 3£. 2s. 4d. for 8lbs. of Cayenne pepper; what was the cost of one pound?

5. Divide 54yd. 2qr. 3na. equally among 5 persons.

6. Divide 29cwt. 3qr. 16lb. equally among 9 persons.

7. If 5 pair of oxen in one year consume 37 tons 17cwt. 0qr. 16lb. of hay, what quantity would be sufficient for one pair?

8. If one man could perform a piece of labor in 76 days 11h. 53m., how long would it take 10 men to perform the same labor?

9. Divide 151£. 19s. 11¼d. by 9. Ans. 16£. 17s. 9¼d.
10. Divide 350£. 17s. 3¾d. by 519. Ans. 13s. 6¼d.
11. Divide 225£. 1s. 10½d. by 63. Ans. 3£. 11s. 5½d.
12. Divide 159£. 4s. 9d. by 12. Ans. 13£. 5s. 4¾d.
13. Divide 75£. 6s. 6d. by 4. Ans. 18£. 16s. 7½d.
14. Divide 111£. 8s. 11d. by 7.
15. Divide 159£. 4s. 9d. by 12.
16. Divide 158lb. 9oz. 1dwt. 21gr. by 9.
17. Divide 111lb. 11oz. 17dwt. 9gr. by 7.
18. Divide 183 tons 1cwt. 2qr. 11lb. 4oz. by 11.
19. Divide 105 tons 7cwt. 0qr. 6lb. 8oz. by 8.
20. Divide 19℔ 9℥ 4ʒ 2℈ 9gr. by 8.
21. Divide 335yd. 2qr. 0na. 0½in. by 7.
22. Divide 215m. 7fur. 1rd. 14ft. 6in. by 12.
23. Divide 149deg. 8m. 0fur. 0rd. 1yd. 1ft. 6in. by 9.
24. Divide 97deg. 55m. 7fur. 35rd. 4ft. 2in. 1bar. by 6.
25. Divide 181A. 3R. 11p. 6yd. 4ft. 41in. by 11.
26. Divide 31 cords 83ft. 1332in. by 4.
27. Divide 209hhd. 55gal. 3qt. 0pt. 1gi. by 7.
28. Divide 35 tuns 3hhd. 4gal. 2qt. by 9.
29. Divide 56hhd. 45gal. by 8.
30. Divide 118bu. 1pk. 5qt. by 6.
31. Divide 255ch. 24bu. 3pk. 1qt. by 7.
32. Divide 110y. 343d. 3h. 41m. 12sec. by 8.

33. A man divides his farm of 214A. 3R. 12p. equally among his 9 sons; how much does each receive?

34. If one man perform a certain piece of labor in 3da. 16h. 54m., how long would it take 12 men to perform the same work?

35. A farmer has 29 bushels of rye, which he wishes to put into 8 bags; how much must each bag contain?

CASE II.

When the divisor is a composite number, proceed as in the following

EXAMPLE.

36. If 42 yards of cloth cost 14£. 3s. 6d., what is the value of 1 yard? Ans. 6s. 9d.

	£	s.	d.
7)	14	3	6
6)	2	0	6
	0	6	9

In this question, we find the component parts of 42 are 6 and 7; we therefore first divide the price by 7, and then divide the quotient by 6.

From the above, we deduce the following

RULE.

Divide the dividend by one of the component parts, and the quotient thence arising by the other, and the last quotient will be the answer.

NOTE. — To find the true remainder; multiply the last remainder by the first divisor, and to the product add the first remainder.

37. If 16 yards of velvet cost 2£. 18s. 8d., what will 1 yard cost?

38. If 72 yards of broadcloth cost 71£. 14s. 0d., what is the value of 1 yard?

39. If 84 yards of cotton cost 8£. 1s. 0d., what will 1 yard cost?

40. If 90 hogsheads of sugar weigh 56T. 13cwt. 3qr. 10lb., what is the weight of 1 hogshead?

41. What will be the price of 1 sheep, if 18 cost 5£. 4s. 3d.?

42. If 21 yards of cloth cost 10£. 8s. 3d., what is the price of 1 yard?

43. What is the value of 1 hat, when 22 cost 12£. 13s. 0d.?

44. When 96 shares of a certain stock are valued at 1290£. 4s. 0d., what would be the cost of 1 share?

45. If 120 spoons weigh 32lb. 9oz. 15dwt., what does 1 weigh?

46. If a man in 1 month travel 746m. 5fur. 0rd., how far does he go in 1 day?

47. If the earth revolve 15° on its axis in 1 hour, how far does it revolve in 1 minute?

48. Divide 1275A. 2R. 16p. 22yd. 8ft. 32in. equally among 32 men.

49. If a man walk round the earth in 2y. 68d. 19h. 54m., how long would it take him to walk 1 degree, allowing 365$\frac{1}{4}$ days to a year?

The following questions are to be performed as the second example of this section.

50. If 53 tons of iron cost 1001£. 9s. 7d., what is the value of 1 ton?

51. If 57 gallons of wine cost 23£. 11s. 5$\frac{1}{4}$d., what cost 1 gallon?

52. Divide 3419A. 2R. 23p. by 29.

53. If 89 pieces of cloth contain 3375yds. 3qr. 1na. 0$\frac{1}{4}$in., how much does 1 piece contain?

54. If 59 casks contain 44hhd. 52gal. 2qt. 1pt. of wine, what are the contents of 1 cask?

55. If a man travel in 1 year (365 days) 6357m. 5fur. 14rd. 11$\frac{1}{2}$ft., how far is that per day?

56. When 175gal. 2qt. of beer are drunk in 52 weeks, how much is consumed in 1 week?

57. When 17 sticks of timber measure 15T. 38ft. 1074in., how many feet does 1 contain?

58. Divide 132 cords 2ft. by 17.

59. Divide 89hhd. 52gal. 3qt. 1pt. by 39.

60. Divide 179bu. 3pk. 5qt. 0pt. 1gi. by 53.

61. Divide 275ch. 19bu. 2pk. equally among 17 men.

62. Divide 796£. 19s. 8d. by 386. Ans. 2£. 1s. 3$\frac{206}{386}$d.

63. Divide 618£. 16s. 7d. by 571. Ans. 1£. 1s. 8$\frac{59}{571}$d.

64. Divide 1678£. 14s. 3d. by 97. Ans. 17£. 6s. 1$\frac{50}{97}$d.

65. Divide 697T. 18cwt. 3qr. 14lb. by 146.
Ans. 4T. 15cwt. 2qr. 12$\frac{26}{146}$lb.

66. Divide 916m. 3fur. 30rd. 10ft. 6in. by 47.
Ans. 19m. 3fur. 39rd. 13ft. 2$\frac{26}{47}$in.

67. Divide 718A. 3R. 37p. by 29. Ans. 24A. 3R. 6$\frac{23}{29}$p.

68. Divide 815A. 1R. 17p. 200ft. by 87.
Ans. 9A. 1R. 19p. 139$\frac{86}{87}$ft.

69. Divide 144A. 3R. 18p. 3yd. 1ft. 36in. by 11.
Ans. 13A. 0R. 27p. 3yd. 0ft. 45$\frac{9}{11}$in.

70. Divide 6718£. 19s. 11d. by 47.
Ans. 142£. 19s. 1$\frac{36}{47}$d.

71. Divide 1237£. 17s. 4d. by 86. Ans. 14£. 7s. 10$\frac{44}{86}$d.

72. Purchased 18T. 17cwt. 3qr. 20lb. of copperas, at 4 cents per pound. I sold 4T. 6cwt. 1qr. 14lb. at 5 cents per pound,

and 7T. 1cwt. 3qr. 10lb. at 6 cents per pound. Moses Atwood purchased one fourth of the remainder at 6 cents per pound. One half of what then remained I sold to J. Gale at 10 cents per pound. The remaining quantity I sold to J. Smith at 12 cents per pound; but he has become a bankrupt, and I lose half my debt. What have I gained by my purchase?

Ans. $1001.34.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

1. If 7 pair of shoes cost $8.75, what will one pair cost? what will 20 pairs cost? Ans. $25.00.

2. If 5 tons of hay cost $85, what will 1 ton cost? what will 17 tons cost? Ans. $289.00.

3. When $0.75 are paid for 3gal. of molasses, what is the value of 1gal.? What cost 37gal.? Ans. $9.25.

4. Gave $1.92 for 4lbs. of tea; what cost 1lb.? what cost 37lbs.? Ans. $17.76.

5. For 12lbs. of rice I paid $1.08; what was paid for 1lb.; and what must I give for 25lbs.? Ans. $2.25.

6. Gave S. Smith $63.00 for 9 tubs of butter; what was the cost of 1 tub? What cost 27 tubs? Ans. $189.00.

7. T. Swan can walk 20 miles in 5 hours; how far can he walk in 1 hour? How long would it take him to walk from Bradford to Boston, the distance being in a straight line 28 miles? Ans. 7 hours.

8. If a hungry boy would eat 49 crackers in 1 week, how many would he eat in 1 day? how many would be sufficient to last him 19 days? Ans. 133 crackers.

9. Gave $20 for 5 barrels of flour; what cost 1 barrel? what cost 40 barrels? Ans. $160.00.

10. For 3lbs. of lard there were paid 36 cents; what was the cost of 37lbs.? Ans. $4.44.

11. Paid F. Johnson 72 cents for 9 nutmegs; how many cents were paid for 1 nutmeg; and what should be charged for 37 nutmegs? Ans. $2.96.

12. Paid 2£. 17s. 5d. for 52lbs. of sugar; what cost 1lb.? what cost 76lbs.? Ans.

13. Paid 4£. 3s. 11d. for 76 pounds of sugar; what cost 52lbs.? Ans.

14. If 52lbs. of sugar cost 2£. 17s. 5d., how many pounds can be purchased for 4£. 3s. 11d.? Ans.

15. When 4£. 3s. 11d. are paid for 76lbs. of sugar, how many pounds can be obtained for 2£. 17s. 5d.? Ans.

16. Bought 20 bushels of wheat for 8£. 3s. 11d.; what cost 1 bushel? what cost 200 bushels? Ans.

17. Paid E. Bradley 81£. 19s. 2d. for 200 bushels of wheat; what cost 20 bushels? Ans.

18. Mr. Day paid 3£. 4s. 2d. for 10yds. of cloth; what should he have paid for 97yds.? Ans. 31£. 2s. 5d.

19. If 8 barrels of flour cost 2£. 12s., what cost 29 barrels? Ans. 9£. 8s. 6d.

20. If 17 bushels of wheat cost 6£. 13s. 2d., what cost 101 bushels? Ans. 39£. 11s. 2d.

21. Gave 10£. 4s. 3d. for 19 yards of cloth; what cost 97 yards? Ans. 52£. 2s. 9d.

SECTION XVI.

VULGAR FRACTIONS.

FRACTIONS are parts of an integer, or whole number.

An integer is any whole number or quantity, as 1, 7, 11, &c., or a pound, a yard.

VULGAR FRACTIONS are expressed by two numbers, called the Numerator and Denominator; the former *above*, and the latter *below* a line.

Thus, $\left\{ \begin{array}{l} \text{Numerator} \\ \text{Denominator} \end{array} \right. \frac{7}{11}$

The Denominator shows into how many parts the integer, or whole number, is divided.

The numerator shows how many of these parts are taken, or expressed by the fraction.

1. A proper fraction is one whose numerator is less than the denominator; as $\frac{5}{7}$.
2. An improper fraction is one whose numerator exceeds or is equal to the denominator; as $\frac{17}{12}$ or $\frac{8}{8}$.
3. A single or simple fraction consists of but *one* numerator and one denominator; as $\frac{7}{8}$.
4. A compound fraction is a fraction of a fraction, connected by the word *of*; as $\frac{7}{8}$ of $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{3}{4}$.
5. A mixed number is an integer with a fraction; as $7\frac{6}{11}$, $5\frac{3}{8}$.
6. A complex fraction is a fraction having a fraction or a mixed number for its numerator or denominator, or both; as, $\frac{7\frac{1}{4}}{9\frac{1}{11}}$, $\frac{\frac{3}{4}}{\frac{7}{9}}$, $\frac{\frac{1}{4}}{7\frac{1}{6}}$, $\frac{8\frac{1}{4}}{11}$, or $\frac{7}{9\frac{7}{8}}$.

7. The *terms* of a fraction are the numerator and denominator; the numerator being the upper term, and the denominator the lower.
8. The greatest common measure of two or more numbers is the largest number that will divide them without a remainder.
9. The least common multiple of two or more numbers is the least number that may be divided by them without a remainder.
10. A fraction is in its lowest terms, when no number but a unit will measure both its terms.
11. A prime number is that which can be measured only by itself or a unit; as 7, 11, and 19.
12. Numbers are said to be *prime* to each other, when only a unit measures or divides them both without a remainder; thus, 7 and 11 are prime to each other.
13. Prime factors of numbers are those factors which can be divided by no number but by themselves or a unit; thus the prime factors of 21 are 7 and 3.
14. An even number is that which can be divided into two equal whole numbers.
15. An odd number is that which cannot be divided into two equal whole numbers.
16. A *square number* is the product of a number multiplied by itself.
17. A *cube number* is the product of a number multiplied by its square.
18. A composite number is that produced by multiplying two or more numbers together.
19. The factors of a number are those whose continued product will exactly produce the number.
20. An *aliquot part* is that which is contained a *precise* number of times in another.
21. An *aliquant part* is such a number as is contained in another a certain number of times with some part or parts *over*.
22. A perfect number is that which is equal to the sum of all its aliquot parts, or is equal to the sum of all the numbers that will divide it without a remainder; thus 6 is a perfect number, because it can be divided by 3, 2, and 1; and the sum of these numbers is 6. But 12 is *not* a perfect number, because its *aliquot* parts are *more* than 12; thus $6+4+3+1=14$. 8 is not a perfect number, because its aliquot parts are *less* than 8; thus $4+2+1=7$. But 28, 496, and 8128

are perfect numbers. The chief use of a knowledge of these numbers is in the higher branches of mathematics.

23. A fraction is equal to the number of times the numerator will contain the denominator.

24. The value of a fraction depends on the proportion which the numerator bears to the denominator.

25. Ratio is the relation which two numbers or quantities of the same kind bear to each other, and may be found by dividing one number by the other. For example, the ratio of 12 to 4 is 3, because $12 \div 4 = 3$; and the ratio of 4 to 8 is $\frac{1}{2}$, because $4 \div$ by $8 = \frac{1}{2}$.

CASE I.

To find the greatest common measure of two or more numbers, or to find the greatest number that will divide two or more numbers, without a remainder.

RULE. — *Divide the greater number by the less, and, if there be a remainder, divide the last divisor by it, and so continue dividing the last divisor by the last remainder until nothing remains, and the last divisor is the greatest common measure.*

If there be more than two numbers, find the greatest common measure of two of them, and then of that common measure and the other numbers. If it should happen that 1 is the common measure, the numbers are prime to each other, and are incommeasurable.

The above rule may be illustrated and demonstrated by the following example.

Let it be required to find the greatest common measure or divisor of 24 and 88.

According to the *rule*, we first divide 88, the *greater* number, by 24, the *less;* for it is evident that no number greater than the *less* of two numbers can measure or divide those numbers. As therefore 24 will exactly measure or divide itself, if it will also divide 88, it will be the *greatest common divisor* sought.

OPERATION.

```
24)88(3
   72
   16)24(1
      16
       8)16(2
         16
         --
```

Now we find that 24 will not exactly measure or divide 88, but there is a remainder, 16. 24, therefore, is not the common divisor of the two numbers. Now as 72, the number which we subtracted from 88, is an exact multiple of 24, we know that any number which will exactly measure or divide 24 will also divide 72; and as 16, the *remainder* of the division of 88 by 24, is that part of 88

which 24 will not measure or divide, it is a number which *must be divided* by the common divisor of 24 and 88. Now, since no number can divide 16 greater than 16 itself, and since, if it will divide 24, we know that it will also divide 88, because 88 is a multiple of 24, + 16, we proceed according to the rule to try whether 16 will measure or divide 24, and therefore place 24, the last divisor, at the right of 16, the last remainder. We know, also, that if 16 will divide 24 it is the *greatest* common divisor of 24 and 88; because we have before shown that any number which will divide 24 and 88 *must also divide* 16.

On dividing 24 by 16 we again find a remainder, 8. Now 8 being the remainder after the division of 24 by 16, we know, according to the reasoning before adopted, that no number *greater* than 8 can measure or divide 16 and 24, and that if it will measure 16, it will also measure 24, because 24 is a multiple of 16, + 8, and that for the same reason it will divide 88, for 88 is a multiple of 24, + 16. Making 8, therefore, the divisor, and 16 the dividend, according to the rule, we find that 8 will exactly divide 16, and hence know that 8 is the greatest common divisor of 24 and 88. Q. E. D.

2. What is the greatest common measure of 56 and 168? Ans. 56.
3. What is the greatest common measure of 96 and 128? Ans. 32.
4. What is the greatest common measure of 57 and 285? Ans. 57.
5. What is the greatest common measure of 169 and 175? Ans. 1.
6. What is the greatest common measure of 175 and 455? Ans. 35.
7. What is the greatest common measure of 169 and 866? Ans. 1.
8. What is the greatest common measure of 47 and 478? Ans. 1.
9. What is the greatest common measure of 84 and 1068? Ans. 12.
10. What is the greatest common measure of 75 and 165? Ans. 15.
11. What is the greatest common measure of 78, 234, and 468? Ans. 78.

12. What is the greatest common measure of 144, 485, and 25? Ans. 1.

13. What is the greatest common measure of 671, 2013, and 4026? Ans. 671.

14. What is the greatest common measure of 16, 20, and 24? Ans. 4.

15. What is the greatest common measure of 21, 27, and 81? Ans. 3.

CASE II.

To reduce fractions to their lowest terms.

1. Reduce $\frac{16}{48}$ to its lowest terms.

OPERATION.

$$4)\tfrac{16}{48} = 4)\tfrac{4}{12} = \tfrac{1}{3} \text{ Ans.}$$

NOTE.—That $\frac{1}{3}$ is equal to $\frac{16}{48}$ may be demonstrated as follows:—16 is the same multiple of 1, that 48 is of 3, therefore 16 has the same ratio to 48, that 1 has to 3; and as the value of a fraction depends on the ratio which the numerator has to the denominator, it is evident when their ratios are the same that their values are equal; therefore, $\frac{1}{3}$ is equal to $\frac{16}{48}$. Q. E. D.

RULE.—*Divide the numerator and denominator by any number, that will divide them both without a remainder; and so continue, until no number will divide them but a unit. Or divide the numerator and denominator by their greatest common measure.*

2. Reduce $\frac{16}{84}$ to its lowest terms. Ans. $\frac{4}{21}$.
3. Reduce $\frac{24}{32}$ to its lowest terms. Ans. $\frac{3}{4}$.
4. Reduce $\frac{36}{84}$ to its lowest terms. Ans. $\frac{3}{7}$.
5. Reduce $\frac{14}{108}$ to its lowest terms. Ans. $\frac{7}{54}$.
6. Reduce $\frac{15}{175}$ to its lowest terms. Ans. $\frac{3}{35}$.
7. Reduce $\frac{162}{844}$ to its lowest terms. Ans. $\frac{81}{422}$.
8. Reduce $\frac{617}{4319}$ to its lowest terms. Ans. $\frac{1}{7}$.
9. Reduce $\frac{416}{875}$ to its lowest terms. Ans. $\frac{416}{875}$.
10. Reduce $\frac{811}{1116}$ to its lowest terms. Ans. $\frac{811}{1116}$.
11. Reduce $\frac{615}{915}$ to its lowest terms. Ans. $\frac{41}{61}$.
12. Reduce $\frac{816}{1018}$ to its lowest terms. Ans. $\frac{408}{509}$.
13. Reduce $\frac{123}{456}$ to its lowest terms. Ans. $\frac{41}{152}$.

CASE III.

To reduce mixed numbers to improper fractions.

1. How many fifths of a gallon in $17\frac{3}{5}$ gallons?

OPERATION.

$17\frac{3}{5}$
5
$\overline{\frac{88}{5}} =$ Ans.

We analyze this question by saying, that, as there are 5 fifths in 1 gallon, there will be 5 times as many fifths as gallons. Therefore in 17 gallons and 3 fifths of a gallon there will be 88 fifths, which should be expressed thus, $\frac{88}{5}$. And this fraction by definition 2d, page 89, is an improper fraction.

RULE. — *Multiply the whole number by the denominator of the fraction, and to the product add the numerator, and place their sum over the denominator of the fraction.*

2. Reduce $16\frac{3}{11}$ to an improper fraction. Ans. $\frac{179}{11}$.
3. Reduce $14\frac{3}{7}$ to an improper fraction. Ans. $\frac{101}{7}$.
4. Reduce $126\frac{16}{37}$ to an improper fraction. Ans. $\frac{4678}{37}$.
5. Reduce $149\frac{116}{137}$ to an improper fraction. Ans. $\frac{20529}{137}$.
6. Reduce $161\frac{1}{16}$ to an improper fraction. Ans. $\frac{2577}{16}$.
7. Reduce $171\frac{57}{59}$ to an improper fraction. Ans. $\frac{10146}{59}$.
8. Reduce $98\frac{48}{97}$ to an improper fraction. Ans. $\frac{9554}{97}$.
9. Reduce $116\frac{31}{61}$ to an improper fraction. Ans. $\frac{7107}{61}$.
10. Reduce $718\frac{45}{97}$ to an improper fraction. Ans. $\frac{69691}{97}$.
11. Reduce $100\frac{100}{199}$ to an improper fraction. Ans. $\frac{20000}{199}$.
12. Reduce $478\frac{5}{116}$ to an improper fraction. Ans. $\frac{55453}{116}$.
13. Reduce $871\frac{3}{116}$ to an improper fraction. Ans. $\frac{101039}{116}$.
14. Reduce $167\frac{100}{117}$ to an improper fraction. Ans. $\frac{19639}{117}$.
15. Reduce $613\frac{101}{307}$ to an improper fraction. Ans. $\frac{188292}{307}$.
16. Reduce $159\frac{100}{201}$ to an improper fraction. Ans. $\frac{32059}{201}$.
17. Reduce $999\frac{999}{1000}$ to an improper fraction. Ans. $\frac{999999}{1000}$.
18. Reduce 7 to an improper fraction. Ans. $\frac{7}{1}$.
19. Change 11 to an improper fraction. Ans. $\frac{11}{1}$.
20. Change 1 to an improper fraction. Ans. $\frac{1}{1}$.
21. Change 100 to an improper fraction. Ans. $\frac{100}{1}$.

NOTE. — To reduce a whole number to an equivalent fraction having a given denominator, we multiply the whole number by the given denominator, and we then place the product over the given denominator.

22. Reduce 11 to a fraction whose denominator shall be 7.

OPERATION.

$11 \times 7 = 77$; $\frac{77}{7}$ Ans.

23. Reduce 5 to a fraction whose denominator shall be 17. Ans. $\frac{85}{17}$.
24. Reduce 19 to a fraction whose denominator shall be 13. Ans. $\frac{247}{13}$.

CASE IV.

To reduce improper fractions to integers or mixed numbers.

1. How many yards in $\frac{117}{19}$ of a yard?

OPERATION.

$$\begin{array}{l} 19)117(6\frac{3}{19} \text{ Ans.} \\ \underline{\;114\;} \\ \quad\;\; 3 \end{array}$$

This question may be analyzed by saying, as 19 nineteenths make one yard, there will be as many yards as 117 contains 19, which is 6 times and 3 nineteenths times, which is written thus, $6\frac{3}{19}$; and this expression, by definition 5th, page 89, is a mixed number.

RULE. — *Divide the numerator by the denominator, and if there be a remainder, place it over the denominator at the right hand of the integer.*

2. Reduce $\frac{167}{15}$ to a mixed number. Ans. $11\frac{2}{15}$.
3. Reduce $\frac{1631}{116}$ to a mixed number. Ans. $14\frac{7}{116}$.
4. Reduce $\frac{131}{17}$ to a mixed number. Ans. $7\frac{12}{17}$.
5. Reduce $\frac{617}{181}$ to a mixed number. Ans. $3\frac{74}{181}$.
6. Change $\frac{1000}{9}$ to a mixed number. Ans. $111\frac{1}{9}$.
7. Change $\frac{4122}{45}$ to a mixed number. Ans. $91\frac{27}{45}$.
8. Change $\frac{1125}{9}$ to a whole number. Ans. 125.
9. Change $\frac{37}{37}$ to a whole number. Ans. 1.

CASE V.

To reduce complex fractions to simple ones.

1. Reduce $\frac{\frac{2}{9}}{\frac{5}{8}}$ to a simple fraction.

OPERATION.

$$\frac{\frac{2}{9}}{\frac{5}{8}} = \frac{2}{9} \times \frac{8}{5} = \frac{16}{45} \text{ Ans.}$$

In performing this question, we divide the numerator by the denominator; because all fractions are equal to the number of times the numerator contains the denominator.

RULE. — *If the numerator or denominator be a whole or a mixed number, let it be reduced to an improper fraction. Then multiply the denominator of the lower fraction into the numerator of the upper fraction for a new numerator, and the denominator of the upper fraction into the numerator of the lower fraction for a new denominator; or, we may invert the denominator of the complex fraction, when reduced, and place it in a line with the numerator, then multiply the two numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE. — Every fraction denotes a division of the numerator by the denominator, and its value is equal to the quotient obtained by such division. Hence the necessity of inverting the denominator of the complex fraction.

2. Reduce $\frac{7}{\frac{2}{3}}$ to a simple fraction.

OPERATION.

$$\frac{7}{\frac{2}{3}} = \frac{7}{1} \times \frac{3}{2} = \frac{21}{2} = 10\tfrac{1}{2} \text{ Ans.}$$

3. Reduce $\frac{\frac{3}{5}}{8}$ to a simple fraction.

OPERATION.

$$\frac{\frac{3}{5}}{8} = \frac{3}{5} \times \frac{1}{8} = \frac{3}{40} \text{ Ans.}$$

4. Reduce $\frac{4\frac{3}{7}}{\frac{2}{3}}$ to a simple fraction.

OPERATION.

$$\frac{4\frac{3}{7}}{\frac{2}{3}} = \frac{\frac{31}{7}}{\frac{2}{3}} = \frac{31}{7} \times \frac{3}{2} = \frac{93}{14} = 6\tfrac{9}{14} \text{ Ans.}$$

5. Reduce $\frac{\frac{3}{4}}{5\frac{2}{3}}$ to a simple fraction.

OPERATION.

$$\frac{\frac{3}{4}}{5\frac{2}{3}} = \frac{\frac{3}{4}}{\frac{17}{3}} = \frac{3}{4} \times \frac{3}{17} = \frac{9}{68} \text{ Ans.}$$

6. Reduce $\frac{7}{4\frac{2}{3}}$ to a simple fraction.

OPERATION.

$$\frac{7}{4\frac{2}{3}} = \frac{\frac{7}{1}}{\frac{14}{3}} = \frac{7}{1} \times \frac{3}{14} = \frac{21}{14} = 1\tfrac{1}{2} \text{ Ans.}$$

7. Reduce $\frac{7\frac{4}{11}}{8}$ to a simple fraction.

OPERATION.

$$\frac{7\frac{4}{11}}{8} = \frac{\frac{81}{11}}{\frac{8}{1}} = \frac{81}{11} \times \frac{1}{8} = \frac{81}{88} \text{ Ans.}$$

8. Reduce $\frac{6\frac{2}{9}}{8\frac{2}{3}}$ to a simple fraction.

OPERATION.

$$\frac{6\frac{2}{9}}{8\frac{2}{3}} = \frac{\frac{56}{9}}{\frac{26}{3}} = \frac{56}{9} \times \frac{3}{26} = \frac{168}{234} = \frac{28}{39} \text{ Ans.}$$

9. Reduce $\frac{\frac{2}{7}}{\frac{3}{5}}$ to a simple fraction. Ans. $\frac{10}{21}$.

10. Reduce $\frac{8}{\frac{1}{3}}$ to a whole number. Ans. 24.

11. Reduce $\frac{\frac{4}{7}}{2}$ to a simple fraction. Ans. $\frac{2}{7}$.

12. Reduce $\frac{5\frac{1}{3}}{\frac{3}{7}}$ to a mixed number. Ans. $12\frac{4}{9}$.

13. Reduce $\frac{\frac{1}{4}}{6\frac{1}{2}}$ to a simple fraction. Ans. $\frac{1}{26}$.

14. Reduce $\frac{3}{2\frac{1}{2}}$ to a mixed number. Ans. $1\frac{1}{5}$.

15. Reduce $\frac{3\frac{1}{4}}{9}$ to a simple fraction. Ans. $\frac{13}{36}$.

16. Reduce $\frac{11\frac{2}{3}}{12\frac{3}{5}}$ to a simple fraction. Ans. $\frac{25}{27}$.

17. If 7 were to be the denominator to the following quantity, $\frac{7\frac{7}{8}}{11\frac{4}{5}}$, what would be its value? Ans. $\frac{50}{581}$.

CASE VI.

To reduce compound fractions to simple fractions.

1. What is $\frac{3}{4}$ of $\frac{7}{8}$?

OPERATION.
$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$ Ans.

This question may be analyzed by saying, If $\frac{1}{8}$ of an orange be divided into 4 equal parts, one of those parts is $\frac{1}{32}$ of the orange; and, if $\frac{1}{4}$ of $\frac{1}{8}$ be $\frac{1}{32}$, it is evident that $\frac{1}{4}$ of $\frac{7}{8}$ will be seven times as much. And 7 times $\frac{1}{32}$ is $\frac{7}{32}$. If, therefore, $\frac{1}{4}$ of $\frac{7}{8}$ be $\frac{7}{32}$, $\frac{3}{4}$ of $\frac{7}{8}$ will be 3 times as much; and 3 times $\frac{7}{32}$ is $\frac{21}{32}$.

RULE. — *Change mixed numbers and whole numbers, if there be any, to improper fractions; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator; the fraction should then be reduced to its lowest terms. If there be numbers in the numerator similar to those in the denominator, they may be cancelled in the operation.*

2. What is $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{11}{12}$? Ans. $\frac{770}{1728} = \frac{385}{864}$.
3. What is $\frac{6}{7}$ of $\frac{11}{15}$ of $\frac{3}{5}$ of $\frac{1}{18}$? Ans. $\frac{198}{9450} = \frac{11}{525}$.

4. What is $\frac{4}{7}$ of $\frac{5}{8}$ of $\frac{3}{8}$ of $\frac{11}{13}$? Ans. $\frac{660}{5824}=\frac{165}{1456}$.
5. What is $\frac{6}{11}$ of $\frac{6}{7}$ of $\frac{1}{4}$ of 21? Ans. $\frac{756}{308}=2\frac{5}{11}$.
6. What is $\frac{7}{11}$ of $15\frac{7}{8}$ of $5\frac{7}{10}$ of 100? Ans. $\frac{5067300}{880}=5758\frac{13}{44}$.
7. What is $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{7}{11}$ of $\frac{11}{24}$?

STATEMENT. CANCELLED.

$$\frac{3}{4}\times\frac{4}{7}\times\frac{7}{11}\times\frac{11}{24}=\frac{3}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{11}}\times\frac{\cancel{11}}{24}=\frac{3}{24}=\frac{1}{8}\text{ Ans.}$$

8. What is $\frac{3}{7}$ of $\frac{7}{15}$ of $\frac{15}{29}$ of $\frac{29}{33}$?

STATEMENT. CANCELLED.

$$\frac{3}{7}\times\frac{7}{15}\times\frac{15}{29}\times\frac{29}{33}=\frac{3}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{15}}\times\frac{\cancel{15}}{\cancel{29}}\times\frac{\cancel{29}}{33}=\frac{3}{33}=\frac{1}{11}\text{ Ans.}$$

9. What is $\frac{7}{11}$ of $\frac{11}{17}$ of $\frac{17}{23}$ of $\frac{23}{28}$?

STATEMENT. CANCELLED.

$$\frac{7}{11}\times\frac{11}{17}\times\frac{17}{23}\times\frac{23}{28}=\frac{7}{\cancel{11}}\times\frac{\cancel{11}}{\cancel{17}}\times\frac{\cancel{17}}{\cancel{23}}\times\frac{\cancel{23}}{28}=\frac{7}{28}=\frac{1}{4}\text{ Ans.}$$

10. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{12}$ of 24?

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{12}\times\frac{24}{1}=\frac{24}{12}=2\text{ Ans.}$$

11. What is the value of $\frac{7}{11}$ of $\frac{11}{25}$ of $\frac{25}{31}$ of \$$7\frac{3}{4}$?

$$\frac{7}{\cancel{11}}\times\frac{\cancel{11}}{\cancel{25}}\times\frac{\cancel{25}}{\cancel{31}}\times\frac{\cancel{31}}{4}=\frac{7}{4}=\$1.75\text{ Ans.}$$

12. What is $\frac{4}{9}$ of $\frac{9}{17}$ of $\frac{17}{18}$ of $3\frac{3}{5}$ gallons?

$$\frac{4}{\cancel{9}}\times\frac{\cancel{9}}{\cancel{17}}\times\frac{\cancel{17}}{\cancel{18}}\times\frac{\cancel{18}}{5}=\frac{4}{5}\text{ gal. Ans.}$$

RULE 2. — *When there are any two numbers, one in the numerators, and the other in the denominators, which may be divided by a number without a remainder, the quotients arising from such division may be used in the operation of the question, instead of the original numbers. The quotients also may be cancelled, as other numbers.*

1. Reduce $\frac{4}{7}$ of $\frac{14}{27}$ of $\frac{21}{25}$ of $\frac{5}{11}$ to its lowest terms.

OPERATION.

$$\frac{4\times\overset{2}{\cancel{14}}\times\overset{7}{\cancel{21}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{7}}\times\underset{9}{\cancel{27}}\times\underset{5}{\cancel{25}}\times 11}=\frac{56}{495}\text{ Ans.}$$

In performing this question, we find that 14 among the numerators, and 7 among the denominators, may be divided by 7, and that their quotients wil

be 2 and 1. We write the 2 *above* the 14, and 1 *below* the 7. We also find a 21 among the numerators, and a 27 among the denominators, which may be divided by 3, and that their quotients will be 7 and 9. We write the 7 *above* the 21, and 9 *below* the 27. We again find a 5 among the numerators, and a 25 among the denominators, which may be divided by 5, and that their quotients will be 1 and 5. We write the 1 *over* the 5, and the 5 *below* the 25. We then multiply the 4, 2, 7, and 1 together for a numerator = 56, and the 1, 9, 5, and 11 for a denominator = 495. The answer will therefore be $\frac{56}{495}$.

2. Reduce $\frac{14}{15}$ of $\frac{18}{25}$ of $\frac{10}{11}$ of $\frac{3}{21}$ to a simple fraction.

$$\frac{\overset{2}{\cancel{14}}\times\overset{6}{\cancel{18}}\times\overset{2}{\cancel{10}}\times\overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}}\times\underset{5}{\cancel{25}}\times 11\times\underset{\cancel{3}}{\cancel{21}}}=\frac{24}{275}\text{ Ans.}$$

3. What is the value of $\frac{4}{7}$ of $\frac{9}{20}$ of $\frac{15}{16}$ of $\frac{14}{17}$ of \$34?

$$\frac{\overset{1}{\cancel{4}}\times 9\times\overset{3}{\cancel{15}}\times\overset{\cancel{2}}{\cancel{14}}\times\overset{\cancel{2}}{\cancel{34}}}{\underset{\cancel{1}}{\cancel{7}}\times\underset{\cancel{4}}{\cancel{20}}\times\underset{4}{\cancel{16}}\times\underset{1}{\cancel{17}}\times 1}=\frac{27}{4}=\$6.75\text{ Ans.}$$

Note. — The above rule will apply, when the product of several numbers is to be divided by the product of other numbers.

4. What is the continued product of 8, 4, 9, 2, 12, 16, and 5, divided by the continued product of 40, 6, 6, 3, 8, 4, and 20?

$$\frac{\overset{1}{\cancel{8}}\times\cancel{4}\times\cancel{9}\times\cancel{2}\times\cancel{12}\times\cancel{16}\times\cancel{5}}{\underset{5}{\cancel{40}}\times\cancel{6}\times\cancel{6}\times\cancel{3}\times\cancel{8}\times\cancel{4}\times\cancel{20}}=\frac{1}{5}\text{ Ans.}$$

The product of 4 and 9 in the *upper* line is equal to the product of 6 and 6 in the *lower*, therefore they are cancelled; and the product of 2 and 12 in the *upper* line is equal to the product of 3 and 8 in the *lower* line; also the product of 16 and 5 in the *upper* line is equal to the product of 4 and 20 in the *lower* line; these are all cancelled. We also find, that the 8 in the upper line and the 40 in the lower line may be divided by 8, and their quotients will be 1 and 5. We write the 1 *above* the 8, and the 5 *below* the 40. By the usual process, we now find our answer is $\frac{1}{5}$.

5. What is the continued product of 12, 13, 14, 15, 16, 18, 20, 21, and 24, divided by the continued product of 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11?

$$\frac{\overset{3}{\cancel{12}}\times13\times\overset{2}{\cancel{14}}\times\overset{3}{\cancel{15}}\times\overset{2}{\cancel{16}}\times\overset{2}{\cancel{18}}\times\overset{2}{\cancel{20}}\times\overset{7}{\cancel{21}}\times\overset{2}{\cancel{24}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{5}}\times\underset{1}{\cancel{6}}\times\underset{1}{\cancel{7}}\times\underset{1}{\cancel{8}}\times\underset{1}{\cancel{9}}\times\underset{1}{\cancel{10}}\times11}=\frac{26208}{11}=2382\tfrac{6}{11}$$ [Ans.

CASE VII.

To find the least common multiple of two or more numbers.

1. What is the least common multiple of 4, 6, 8, 16, and 20?

OPERATION.

$$\begin{array}{r|rrrrr}
2) & 4 & 6 & 8 & 16 & 20 \\ \hline
2) & 2 & 3 & 4 & 8 & 10 \\ \hline
2) & 1 & 3 & 2 & 4 & 5 \\ \hline
 & 1 & 3 & 1 & 2 & 5
\end{array}$$

$2\times2\times2\times3\times2\times5 = 240$ Ans.

In this operation, we have divided the given numbers by that number which would *divide* most of the given numbers without a remainder, that is, by one of the prime factors, and have continued the division until no number would divide two of them. We have then multiplied all the divisors and the *last* quotients together and found their product to be 240, which *is a* common multiple of the given numbers, 4, 6, 8, 16, and 20.

It is not only a common multiple, but it is the *least* common multiple.

To prove this, we assume the following admitted propositions, which are self-evident: —

1. Every number not prime itself is the product of two or more primes or factors, and is resolvable into its original primes by division.

2. The least number which contains all the prime factors of two or more numbers is the least common multiple of those quantities.

The factors of each number in the question are as follows: —

Factors of 4	are	2 and 2	$=2\times2=4$
6	"	2 and 3	$=2\times3=6$
8	"	2, 2, and 2	$=2\times2\times2=8$
16	"	2, 2, 2, and 2	$=2\times2\times2\times2=16$
20	"	2, 2, and 5	$=2\times2\times5=20$

Now 240 is the least number which contains *all* the factors common to each of these numbers, $2\times2\times2\times2\times3\times5=$

240; therefore it is the least common multiple of *all* these numbers.

Or the above principle may be illustrated as in the following question.

2. What is the least common multiple of 4, 5, 6, 7, 8, and 9?

$$\begin{array}{r|rrrrrr}
2) & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline
2) & 2 & 5 & 3 & 7 & 4 & 9 \\ \hline
2) & 1 & 5 & 3 & 7 & 2 & 9 \\ \hline
3) & 1 & 5 & 3 & 7 & 1 & 9 \\ \hline
3) & 1 & 5 & 1 & 7 & 1 & 3 \\ \hline
5) & 1 & 5 & 1 & 7 & 1 & 1 \\ \hline
7) & 1 & 1 & 1 & 7 & 1 & 1 \\ \hline
 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}$$

In this question, we divide by the prime factors, 2, 3, 5, 7, &c., successively, until the last quotients terminate in units; and the product of all the prime factors is the least common multiple.

Thus $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$ Ans.

That 2520 is the *least* common multiple is evident from the fact, that the divisors whose product produces this number are the prime factors, and the *only* prime factors of the given numbers, as may be seen below.

Factors of	4	are	2 and $2 = 2 \times 2 = 4$
	5	"	5 and $1 = 5 \times 1 = 5$
	6	"	2 and $3 = 2 \times 3 = 6$
	7	"	7 and $1 = 7 \times 1 = 7$
	8	"	2, 2, and $2 = 2 \times 2 \times 2 = 8$
	9	"	3 and $3 = 3 \times 3 = 9$

RULE. — *Divide by such a number as will divide most of the given numbers without a remainder, and set the several quotients, with the several undivided numbers, in a line beneath, and so continue to divide, until no number greater than unity will divide two or more of them. Then multiply all the divisors, the last quotients, and undivided numbers together, and the product is the least common multiple.*

Or, divide the given numbers successively by their prime factors, until the last quotients terminate in units. The continued product of all the divisors will be the least common multiple.

3. What is the least common multiple of 6, 8, 10, 18, 20, and 24? Ans. 360.

4. What is the least common multiple of 14, 19, 38, and 57? Ans. 798.

5. What is the least common multiple of 20, 36, 48, and 50? Ans. 3600.

6. What is the least common multiple of 15, 25, 35, 45, and 100? Ans. 6300.

7. What is the least common multiple of 100, 200, 300, 400, and 575? Ans. 27600.

8. I have four different measures; the first contains 4 quarts, the second 6 quarts, the third 10 quarts, and the fourth 12 quarts. How large is a vessel, that may be filled by each one of these, taken any number of times full? Ans. 60 quarts.

NOTE. — In finding the common multiple of two or more *numbers*, any one number that can measure another may be cancelled.

9. What is the least common multiple of 4, 6, 8, *12*, *16*, *10*, and 20?

$$\begin{array}{r|ccccccc} 4 & \cancel{4} & \cancel{6} & \cancel{8} & 12 & 16 & \cancel{10} & 20 \\ \hline & & & & 3 & 4 & & 5 \end{array} \qquad 4\times3\times4\times5=240 \text{ Ans.}$$

By examining this question, we find that 8 may be divided by 4, 12 by 6, 16 by 8, and 20 by 10; therefore we cancel 4, 6, 8, and 10.

10. What is the least common multiple of 5, 15, 30, 7, 14, and 28?

$$\begin{array}{r|cccccc} 2 & \cancel{5} & \cancel{15} & 30 & \cancel{7} & \cancel{14} & 28 \\ \hline & & & 15 & & & 14 \end{array} \qquad 2\times15\times14=420 \text{ Ans.}$$

In this question, we find that 15 may be measured by 5, 30 by 15, 14 by 7, and 28 by 14; we therefore cancel 5, 15, 7, and 14.

11. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 8, and 9?

$$\begin{array}{r|ccccccccc} 2 & \cancel{1} & \cancel{2} & \cancel{3} & \cancel{4} & 5 & 6 & 7 & 8 & 9 \\ \hline 3 & & & & & 5 & 3 & 7 & 4 & 9 \\ \hline & & & & & 5 & 1 & 7 & 4 & 3 \end{array} \qquad 2\times3\times5\times7\times4\times3=2520 \text{ Ans.}$$

12. What is the least common multiple of 9, 8, 12, 18, 24, 36, and 72?

$$\cancel{9}\ \cancel{8}\ \cancel{12}\ \cancel{18}\ \cancel{24}\ \cancel{36}\ 72 \qquad 72 \text{ Ans.}$$

13. What is the least number that 18, 24, 36, 12, 6, 20, and 48 will measure?

$$\begin{array}{r|rrrrrrr} 4) & \cancel{16} & \cancel{24} & 36 & \cancel{12} & \cancel{6} & 20 & 48 \\ \hline & & 3) & 9 & & & 5 & 12 \\ \hline & & & 3 & & & 5 & 4 \end{array} \quad 4 \times 3 \times 3 \times 5 \times 4 = 720 \text{ Ans.}$$

CASE VIII.

To reduce fractions to a common denominator, that is, to change fractions to other fractions, all having their denominators *alike*, yet retaining the same value.

1. Reduce $\frac{7}{8}$, $\frac{5}{12}$, and $\frac{11}{16}$ to other fractions of equal value, having the same, or a common, denominator.

First Method.

OPERATION.

$$\begin{array}{r|rrr} 4) & 8 & 12 & 16 \\ \hline 2) & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array} \quad 4 \times 2 \times 3 \times 2 = 48 \text{ common denominator.}$$

$$\begin{array}{r|l} 8 & 6 \times 7 = 42 \text{ numerator for } \frac{7}{8} = \frac{42}{48}. \\ 12 & 4 \times 5 = 20 \text{ numerator for } \frac{5}{12} = \frac{20}{48}. \\ 16 & 3 \times 11 = 33 \text{ numerator for } \frac{11}{16} = \frac{33}{48}. \end{array}$$

Having first obtained the least common multiple of all the denominators of the given fractions by the last rule, we assume this as the common denominator required. This number (48) we divide by the denominators of the given fractions, 8, 12, and 16, and find their quotients to be 6, 4, and 3, which we place under the 48; these numbers we then multiply by the numerators 7, 5, and 11, and find their products to be 42, 20, and 33, and these numbers are the numerators of the fractions required.

Second Method.

OPERATION.

$$7 \times 12 \times 16 = 1344 \text{ numerator for } \frac{7}{8} = \frac{1344}{1536}.$$
$$5 \times 8 \times 16 = 640 \text{ numerator for } \frac{5}{12} = \frac{640}{1536}.$$
$$11 \times 8 \times 12 = 1056 \text{ numerator for } \frac{11}{16} = \frac{1056}{1536}.$$

The numerators are produced by multiplying the numerators of the given fraction by each of the other denominators, and the common denominator is obtained by multiplying all the denominators. By this process, we obtain the following Ans. $\frac{1344}{1536}$, $\frac{640}{1536}$, $\frac{1056}{1536}$.

The pupil will perceive, that this method does not express the fractions in so low terms as the other; although they both have the same value.

RULE. — *Find the least common multiple of all the denominators by Case VII., and it will be the denominator required. Divide the common multiple by each of the denominators, and multiply the quotients by the respective numerators of the fractions and their products will be the numerators required.*

Or, multiply each numerator into all the denominators except its own for a new numerator; and all the denominators into each other for a common denominator.

Questions to be performed by the first method.

2. Reduce $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{8}$, and $\frac{5}{12}$. Ans. $\frac{54}{72}$, $\frac{40}{72}$, $\frac{63}{72}$, $\frac{30}{72}$.
3. Reduce $\frac{6}{11}$, $\frac{8}{15}$, $\frac{19}{20}$, and $\frac{5}{24}$. Ans. $\frac{720}{1320}$, $\frac{704}{1320}$, $\frac{1254}{1320}$, $\frac{275}{1320}$.
4. Reduce $\frac{1}{7}$, $\frac{3}{14}$, $\frac{8}{21}$, and $\frac{5}{28}$. Ans. $\frac{12}{84}$, $\frac{18}{84}$, $\frac{32}{84}$, $\frac{15}{84}$.
5. Reduce $\frac{16}{21}$, $\frac{4}{7}$, $\frac{11}{14}$, and $\frac{1}{2}$. Ans. $\frac{32}{42}$, $\frac{24}{42}$, $\frac{33}{42}$, $\frac{21}{42}$.
6. Reduce $\frac{15}{22}$, $\frac{3}{11}$, $\frac{4}{7}$, and $\frac{3}{4}$. Ans. $\frac{210}{308}$, $\frac{84}{308}$, $\frac{176}{308}$, $\frac{231}{308}$.
7. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.
8. Reduce $\frac{6}{7}$, $\frac{4}{9}$, $\frac{5}{11}$, and $\frac{7}{11}$. Ans. $\frac{594}{693}$, $\frac{308}{693}$, $\frac{315}{693}$, $\frac{441}{693}$.
9. Reduce $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{6}$, and $3\frac{4}{5}$. Ans. $\frac{90}{120}$, $\frac{105}{120}$, $\frac{20}{120}$, $\frac{456}{120}$.
10. Reduce $\frac{5}{12}$, $\frac{3}{8}$, $\frac{4}{7}$, and $4\frac{3}{4}$. Ans. $\frac{70}{168}$, $\frac{63}{168}$, $\frac{96}{168}$, $\frac{798}{168}$.
11. Reduce $\frac{4}{7}$, $\frac{5}{14}$, $\frac{11}{21}$, and $\frac{1}{2}$. Ans. $\frac{24}{42}$, $\frac{15}{42}$, $\frac{22}{42}$, $\frac{21}{42}$.
12. Reduce $\frac{8}{9}$, $\frac{7}{12}$, $\frac{17}{18}$, and $\frac{2}{21}$. Ans. $\frac{224}{252}$, $\frac{147}{252}$, $\frac{238}{252}$, $\frac{24}{252}$.
13. Reduce $\frac{19}{20}$, $\frac{7}{30}$, $\frac{11}{40}$, and $\frac{1}{50}$. Ans. $\frac{570}{600}$, $\frac{140}{600}$, $\frac{165}{600}$, $\frac{12}{600}$.
14. Reduce $\frac{7}{8}$, $\frac{7}{12}$, $\frac{7}{16}$, and $\frac{7}{20}$. Ans. $\frac{210}{240}$, $\frac{140}{240}$, $\frac{105}{240}$, $\frac{84}{240}$.
15. Reduce $\frac{2}{9}$, 7, 8, and $5\frac{1}{7}$. Ans. $\frac{14}{63}$, $\frac{441}{63}$, $\frac{504}{63}$, $\frac{324}{63}$.

Questions to be performed by the second method.

16. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$ to fractions having a common denominator. Ans. $\frac{80}{120}$, $\frac{96}{120}$, $\frac{105}{120}$.
17. Reduce $\frac{4}{7}$, $\frac{8}{9}$, and $\frac{3}{10}$. Ans. $\frac{360}{630}$, $\frac{560}{630}$, $\frac{189}{630}$.
18. Reduce $\frac{6}{11}$, $\frac{4}{7}$, and $\frac{8}{13}$. Ans. $\frac{546}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$.
19. Reduce $\frac{7}{12}$, $\frac{4}{13}$, and $7\frac{3}{4}$. Ans. $\frac{364}{624}$, $\frac{192}{624}$, $\frac{4836}{624}$.
20. Reduce $\frac{11}{17}$, $\frac{4}{9}$, and $\frac{4}{15}$. Ans. $\frac{1485}{2295}$, $\frac{1020}{2295}$, $\frac{612}{2295}$.
21. Reduce $\frac{8}{9}$, $\frac{4}{17}$, and $11\frac{7}{15}$. Ans. $\frac{2040}{2295}$, $\frac{540}{2295}$, $\frac{26316}{2295}$.
22. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{7}$, and 8. Ans. $\frac{21}{42}$, $\frac{28}{42}$, $\frac{24}{42}$, $\frac{336}{42}$.
23. Reduce $\frac{4}{9}$, $\frac{7}{11}$, and $\frac{2}{3}$ of $7\frac{5}{8}$. Ans. $\frac{528}{1188}$, $\frac{756}{1188}$, $\frac{6039}{1188}$.
24. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{6}$, and 17. Ans. $\frac{24}{48}$, $\frac{36}{48}$, $\frac{8}{48}$, $\frac{816}{48}$.
25. Reduce $\frac{11}{12}$, $\frac{4}{5}$ of 6, and $21\frac{1}{2}$. Ans. $\frac{110}{120}$, $\frac{576}{120}$, $\frac{2580}{120}$.
26. Reduce $\frac{6}{7}$, $\frac{4}{11}$, $\frac{5}{13}$, $\frac{4}{7}$, and $\frac{1}{2}$.
Ans. $\frac{12012}{14014}$, $\frac{5096}{14014}$, $\frac{5390}{14014}$, $\frac{8008}{14014}$, $\frac{7007}{14014}$.
27. Reduce $\frac{27}{169}$, $\frac{127}{611}$, and $\frac{7}{1728}$.
Ans. $\frac{28506816}{178431552}$, $\frac{37088064}{178431552}$, $\frac{722813}{178431552}$.

NOTE. 1. — If there be complex fractions, they must first be changed to simple fractions, and then they may be reduced by the foregoing Rule.

28. Reduce $\frac{4}{6\frac{2}{3}}$, $\frac{7\frac{1}{2}}{9}$, and $\frac{3\frac{3}{4}}{11\frac{1}{4}}$ to a common denominator.

Ans. $\frac{54}{90}$, $\frac{75}{90}$, $\frac{30}{90}$.

29. Reduce $\frac{\frac{3}{8}}{11\frac{1}{4}}$, $\frac{6\frac{1}{3}}{31\frac{2}{3}}$, and $\frac{29\frac{1}{7}}{65\frac{4}{7}}$ to a common denominator.

Ans. $\frac{3}{90}$, $\frac{18}{90}$, $\frac{40}{90}$.

30. Reduce 16, $\frac{2}{7}$ of $\frac{8\frac{4}{5}}{11\frac{7}{10}}$, and $\frac{3}{5}$ of $8\frac{2}{9}$ to a common denominator. Ans. $\frac{65520}{4095}$, $\frac{880}{4095}$, $\frac{20202}{4095}$.

NOTE 2. — To reduce complex fractions to equivalent fractions, having a given fraction for a common denominator, we first change the complex fractions to simple fractions, and multiply each of their numerators by the numerator of the required common denominator, and their denominators by the denominator of the required denominator; under these fractions we write the required denominator.

31. Change $\frac{4\frac{1}{2}}{6\frac{1}{4}}$ and $\frac{\frac{3}{5}}{\frac{7}{8}}$ to other fractions of equal value, having $\frac{3}{5}$ for a common denominator.

OPERATION.

$\frac{4\frac{1}{2}}{6\frac{3}{4}} = \frac{2}{3}$; $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$, first numerator.

$\frac{\frac{3}{5}}{\frac{7}{8}} = \frac{24}{35}$; $\frac{24}{35} \times \frac{3}{5} = \frac{72}{175}$, second numerator. $\frac{\frac{2}{5}}{\frac{3}{5}}$ and $\frac{\frac{72}{175}}{\frac{3}{5}}$ Ans.

CASE IX.

To reduce fractions of a lower denomination to a higher.

1. What part of a pound is $\frac{3}{8}$ of a penny?

OPERATION.

$\frac{1}{12} \times \frac{3}{8}$d. $= \frac{3}{96} = \frac{1}{32}$s.

$\frac{1}{20} \times \frac{1}{32}$s. $= \frac{1}{640}$£.

This question may be thus analyzed. Since 12 pence make a shilling, there will be $\frac{1}{12}$ as many shillings as pence; therefore $\frac{1}{12}$ of $\frac{3}{8}$ of a penny is $\frac{3}{96} = \frac{1}{32}$ of a shilling. Again, as 20 shillings make a pound, there will be $\frac{1}{20}$ as many pounds as shillings; therefore $\frac{1}{20}$ of $\frac{1}{32}$ of a shilling is $\frac{1}{640}$ of a pound. Q. E. D.

This question may be abridged.

Thus, $\frac{3}{8} \times \frac{1}{12} \times \frac{1}{20} = \frac{3}{1920} = \frac{1}{640}$ Ans.

RULE. — *Multiply the denominator of the given fraction by all the denominations between it and the one to which it is to be reduced, and over the product write the given numerator.*

2. Reduce $\frac{4}{5}$ of a farthing to the fraction of a pound.

3. Reduce $\frac{2}{5}$ of a grain troy to the fraction of a pound.

4. Reduce $\frac{5}{8}$ of a scruple to the fraction of a pound.
5. Reduce $\frac{5}{11}$ of an ounce to the fraction of a hundred weight.
6. Reduce $\frac{3}{4}$ of a pound to the fraction of a ton.
7. Reduce $\frac{1}{5}$ of an inch to the fraction of an ell English.
8. Reduce $\frac{4}{7}$ of an inch to the fraction of a mile.
9. Reduce $\frac{1}{2}$ of a barleycorn to the fraction of a league.
10. Reduce $\frac{3}{4}$ of an inch to the fraction of an acre.
11. Reduce $\frac{7}{8}$ of a quart to the fraction of a tun, wine measure.
12. Reduce $\frac{3}{5}$ of a pint to the fraction of a bushel.
13. Reduce $\frac{1}{8}$ of a minute to the fraction of a year (365$\frac{1}{4}$ days).

CASE X.

To reduce fractions of a higher denomination to a lower.

1. What part of a penny is $\frac{1}{640}$ of a pound?

OPERATION.

$$\frac{1}{640} \times \frac{20}{1} = \frac{20}{640} = \frac{1}{32}\text{s.}$$
$$\frac{1}{32} \times \frac{12}{1} = \frac{12}{32} = \frac{3}{8}\text{d. Ans.}$$

We explain this question in the following manner. As shillings are twentieths of a pound, there will be 20 times as many parts of a shilling in $\frac{1}{640}$ of a pound, as there are parts of a pound; therefore, $\frac{1}{640}$ of a pound is equal to $\frac{1}{640}$ of $\frac{20}{1} = \frac{20}{640} = \frac{1}{32}$ of a shilling. And as pence are twelfths of a shilling, there will be twelve times as many parts of a penny in $\frac{1}{32}$ of a shilling, as there are parts of a shilling; therefore, $\frac{1}{32}$ of a shilling is equal to $\frac{1}{32}$ of $\frac{12}{1} = \frac{12}{32} = \frac{3}{8}$ of a penny, Ans.

The operation of this question may be facilitated in the following manner.

OPERATION.

$$\frac{1}{640} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{640} = \frac{3}{8}\text{d. Ans.}$$

RULE. — *Let the given numerator be multiplied by all the denominations between it and the one to which it is to be reduced; then place the product over the given denominator and reduce the fraction to its lowest terms.*

2. Reduce $\frac{1}{1200}$ of a pound to the fraction of a farthing.
3. Reduce $\frac{1}{9600}$ of a pound troy to the fraction of a grain.
4. Reduce $\frac{5}{2592}$ of a pound, apothecaries' weight, to the fraction of a scruple.
5. Reduce $\frac{3}{9856}$ of a cwt. to the fraction of an ounce.
6. Reduce $\frac{3}{8960}$ of a ton to the fraction of a pound.

7. Reduce $\frac{1}{225}$ of an ell English to the fraction of an inch.
8. Reduce $\frac{1}{110880}$ of a mile to the fraction of an inch.
9. Reduce $\frac{1}{1140480}$ of a league to the fraction of a barley-corn.
10. Reduce $\frac{3}{250905 60}$ of an acre to the fraction of an inch.
11. Reduce $\frac{1}{1152}$ of a tun of wine measure to the fraction of a quart.
12. Reduce $\frac{3}{320}$ of a bushel to the fraction of a pint.
13. Reduce $\frac{1}{4207680}$ of a year to the fraction of a minute.

CASE XI.

To find the value of a fraction in numbers of a lower denomination.

1. What is the value of $\frac{3}{11}$ of a £.?

By Case IX. $\frac{3}{11}$£. $= \frac{3}{11} \times \frac{20}{1} = \frac{60}{11}$s. $= 5\frac{5}{11}$s.; and $\frac{5}{11}$s. $= \frac{5}{11} \times \frac{12}{1} = \frac{60}{11}$d. $= 5\frac{5}{11}$d.; and $\frac{5}{11}$d. $= \frac{5}{11} \times \frac{4}{1} = \frac{20}{11}$qr. $= 1\frac{9}{11}$qr. Ans. 5s. 5d. $1\frac{9}{11}$qr.

This question may be analyzed thus: — If 1£. is 20s., $\frac{3}{11}$ of a £. is $\frac{3}{11}$ of 20s. $= 5\frac{5}{11}$s.; and if 1s. is 12d., $\frac{5}{11}$ of a shilling is $\frac{5}{11}$ of 12d. $= 5\frac{5}{11}$d.; and if 1d. is 4qr., $\frac{5}{11}$ of a penny is $\frac{5}{11}$ of 4qr. $= 1\frac{9}{11}$qr., Answer, as before.

Or, it may be performed in the following manner.

OPERATION.

```
    3
   20
11)60(5s.
   55
    5
   12
11)60(5d.
   55
    5
    4
11)20(1 9/11qr.
   11
    9
```

In this operation, it will be perceived that we multiply the numerator of the fraction by the successive lower denominations, beginning with the highest, and divide each product by the denominator.

From these illustrations, we derive the following

RULE. — *Multiply the numerator by the next lower denomination of the integer, and divide the product by the denominator; if any thing remain, multiply it by the next less denomination, and divide as before, and so continue as far as may be required; and the several quotients will be the answer.*

2. What is the value of $\frac{7}{24}$ of a shilling? Ans. $3\frac{1}{2}$d.
3. What is the value of $\frac{7}{9}$ of a guinea, at 28 shillings?
Ans. 21s. 9d. $1\frac{1}{3}$qr.
4. What is the value of $\frac{7}{11}$ of a cwt.?
Ans. 2qr. 15lb. 4oz. $5\frac{9}{11}$dr.
5. What is the value of $\frac{4}{9}$ of a lb. avoirdupois?
Ans. 7oz. $1\frac{7}{9}$dr.
6. What is the value of $\frac{8}{9}$ of a lb. troy?
Ans. 10oz. 13dwt. 8gr.
7. What is the value of $\frac{4}{13}$ of a ℔. apothecaries' weight.
Ans. 3℥ 5ʒ 1℈ $12\frac{4}{13}$gr.
8. What is the value of $\frac{7}{13}$ of a yard?
Ans. 2qr. 0na. $1\frac{5}{13}$in.
9. What is the value of $\frac{5}{9}$ of an ell English?
Ans. 2qr. 3na. $0\frac{1}{4}$in.
10. What is the value of $\frac{11}{13}$ of a mile?
Ans. 6fur. 30rd. 12ft. $8\frac{4}{13}$in.
11. What is the value of $\frac{8}{9}$ of a furlong?
Ans. 35rd. 9ft. 2in.
12. What is the value of $\frac{7}{13}$ of an acre?
Ans. 2R. 6rd. 4yd. 5ft. $127\frac{5}{13}$in.
13. What is the value of $\frac{9}{17}$ of a rod?
Ans. 144ft. $19\frac{1}{17}$in.
14. What is the value of $\frac{1}{13}$ of a cord?
Ans. 9ft. $1462\frac{2}{13}$in.
15. What is the value of $\frac{2}{19}$ of a hhd. of wine?
Ans. 6gal. 2qt. 1pt. $0\frac{4}{19}$gi.
16. What is the value of $\frac{7}{9}$ of a hhd. of beer?
Ans. 42gal.
17. What is the value of $\frac{11}{23}$ of a year ($365\frac{1}{4}$ days)?
Ans. 174d. 16h. 26m. $5\frac{5}{23}$sec.
18. What is the value of $7\frac{3\frac{3}{11}}{4\frac{2}{5}}$ of a dollar?
Ans. \$$7.74\frac{46}{121}$.

CASE XII.

To reduce any mixed quantity of weights, measures, &c., to the fraction of the integer.

1. What part of a shilling is 1d.? is 2d.? is 3d.? is 4d.?
2. What part of a pound is 2s.? 3s.? 4s.? 5s.? 6s.? 9s.?
3. What part of a furlong is 3rd.? 4rd.? 5rd.? 8rd.? 9rd.?
4. What part of a hogshead is 5gal.? 8gal.? 10gal.?
5. What part of a foot is 2 inches? 3in.? 4in.? 5in.? 8in.?

6. What part of a £. is 5s. 5d. $1\frac{9}{11}$qr. ?

	s.	d.	qr.
20	5	5	$1\frac{9}{11}$
12	12		
240	65		
4	4		
960	261		
11	11		
10560	2880		

$\frac{2880}{10560} = \frac{3}{11}$ Ans.

In this question, the shillings, pence, farthings, &c. are reduced to elevenths of farthings for the numerator of a fraction. A pound is also reduced to the same denomination, for a denominator. The fraction is then reduced to its lowest terms.

Rule. — *Reduce the given number to the lowest denomination it contains for a numerator; and then reduce the integer to the same denomination, for the denominator of the fraction required.*

7. Reduce $3\frac{1}{2}$d. to the fraction of a shilling. Ans. $\frac{7}{24}$.
8. Reduce 21s. 9d. $1\frac{1}{3}$qr. to the fraction of a guinea. Ans. $\frac{7}{8}$.
9. Reduce 2qr. 15lb. 4oz. $5\frac{9}{11}$dr. to the fraction of a cwt. Ans. $\frac{7}{11}$.
10. What part of a pound are 7oz. $1\frac{7}{9}$dr. ? Ans. $\frac{4}{9}$.
11. What part of a pound troy are 10oz. 13dwt. 8gr. ? Ans. $\frac{8}{9}$.
12. What part of a pound apothecaries' weight are 3℥ 5ʒ 1℈ $12\frac{4}{13}$gr. ? Ans. $\frac{4}{13}$.
13. What part of a yard are 2qr. 0na. $1\frac{5}{13}$in. ? Ans. $\frac{7}{13}$.
14. What part of an ell English are 2qr. 3na. $0\frac{1}{4}$in. ? Ans. $\frac{5}{9}$.
15. What part of a mile are 6fur. 30rd. 12ft. 8in. $0\frac{12}{13}$br. ? Ans. $\frac{11}{13}$.
16. Reduce 35rd. 9ft. 2in. to the fraction of a furlong. Ans. $\frac{8}{9}$.
17. What part of an acre are 2R. 6rd. 4yd. 5ft. $127\frac{5}{13}$in. ? Ans. $\frac{7}{13}$.
18. What part of a square rod are 144ft. $19\frac{1}{17}$in. ? Ans. $\frac{9}{17}$.
19. What part of a cord are 9ft. $1462\frac{2}{13}$in. ? Ans. $\frac{1}{13}$.
20. What part of a hogshead of wine are 6gal. 2qt. 1pt. $0\frac{4}{19}$gi. ? Ans. $\frac{2}{19}$.
21. What part of a hhd. of beer are 42gal. ? Ans. $\frac{7}{9}$.
22. What part of a year ($365\frac{1}{4}$ da.) are 174d. 16h. 26m. $5\frac{5}{23}$ sec. ? Ans. $\frac{11}{23}$.

SECTION XVII.

ADDITION OF VULGAR FRACTIONS.

CASE I.

To add fractions that have a common denominator.

1. What part of an apple is $\frac{1}{4}$ and $\frac{2}{4}$? $\frac{1}{3}$ and $\frac{1}{3}$? $\frac{2}{5}$ and $\frac{1}{5}$?
2. What part of a dollar is $\frac{1}{8}$ and $\frac{2}{8}$? $\frac{2}{10}$ and $\frac{3}{10}$? $\frac{4}{10}$ and $\frac{6}{10}$?
3. What part of a shilling is $\frac{3}{12}$ and $\frac{4}{12}$? $\frac{5}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$?
4. What part of an orange is $\frac{1}{3}$ and $\frac{2}{3}$? $\frac{1}{2}$ and $\frac{1}{2}$? $\frac{6}{9}$ and $\frac{8}{9}$?
5. Add $\frac{3}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, and $\frac{11}{12}$ together.

OPERATION.

$$3+5+7+11=\frac{26}{12}=\frac{13}{6}=2\frac{1}{6} \text{ Ans.}$$

In this question, we add the numerators, and divide their sum by the denominator.

RULE. — *Write the sum of the numerators over the common denominator, and reduce the fraction if necessary.*

6. Add $\frac{5}{17}$, $\frac{6}{17}$, $\frac{9}{17}$, $\frac{11}{17}$, $\frac{14}{17}$, and $\frac{16}{17}$ together. Ans. $3\frac{10}{17}$.
7. Add $\frac{4}{23}$, $\frac{9}{23}$, $\frac{11}{23}$, $\frac{13}{23}$, and $\frac{19}{23}$ together. Ans. $2\frac{10}{23}$.
8. Add $\frac{4}{87}$, $\frac{17}{87}$, $\frac{29}{87}$, $\frac{49}{87}$, and $\frac{77}{87}$ together. Ans. $2\frac{2}{87}$.
9. Add $\frac{11}{144}$, $\frac{19}{144}$, $\frac{29}{144}$, and $\frac{117}{144}$ together. Ans. $1\frac{2}{9}$.
10. Add $\frac{619}{631}$, $\frac{491}{631}$, $\frac{567}{631}$, and $\frac{11}{631}$ together. Ans. $2\frac{426}{631}$.
11. Add $\frac{491}{781}$, $\frac{741}{781}$, $\frac{698}{781}$, and $\frac{780}{781}$ together. Ans. $3\frac{367}{781}$.
12. Add $\frac{1671}{9110}$, $\frac{8747}{9110}$, and $\frac{7126}{9110}$ together. Ans. $1\frac{4217}{4555}$.
13. Add $\frac{3361}{3370}$, $\frac{3333}{3370}$, and $\frac{3000}{3370}$ together. Ans. $2\frac{1477}{1685}$.
14. Add $\frac{999}{1000}$, $\frac{888}{1000}$, and $\frac{777}{1000}$ together. Ans. $2\frac{83}{125}$.

CASE II.

To add fractions that have not a common denominator.

1. What is the sum of $\frac{7}{8}$, $\frac{5}{12}$, $\frac{11}{16}$, and $\frac{13}{20}$?

First Method.

OPERATION.

$$\begin{array}{r|rrrr} 4) & 8 & 12 & 16 & 20 \\ \hline 2) & 2 & 3 & 4 & 5 \\ \hline & 1 & 3 & 2 & 5 \end{array} \qquad 4\times2\times3\times2\times5=240 \text{ common denominator.}$$

$$\begin{array}{r|l} 8 & 30\times\ 7=210 \\ 12 & 20\times\ 5=100 \\ 16 & 15\times11=165 \\ 20 & 12\times13=156 \\ \hline & \frac{631}{240}=2\frac{151}{240} \text{ Ans.} \end{array}$$

Having found a common denominator by Case VIII., we proceed as in the last Case.

Second Method.

OPERATION.

$$\begin{array}{r}
7 \times 12 \times 16 \times 20 = 26880 \\
5 \times 8 \times 16 \times 20 = 12800 \\
11 \times 8 \times 12 \times 20 = 21120 \\
13 \times 8 \times 12 \times 16 = \underline{19968} \\
\dfrac{80768}{8 \times 12 \times 16 \times 20 = 30720} = 2\tfrac{151}{240}
\end{array}$$

[Ans.

Let the pupil examine the second method of reducing fractions to a common denominator in Case VIII., Sec. XVI.

RULE. — *Reduce mixed numbers to improper fractions, and compound fractions to simple fractions; then reduce all the fractions to a common denominator, and the sum of their numerators written over the common denominator will be the answer required.*

2. Add $\frac{1}{4}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{5}{9}$ together. Ans. $2\frac{43}{72}$.
3. Add $\frac{7}{11}$, $\frac{9}{22}$, $\frac{5}{8}$, and $\frac{1}{6}$ together. Ans. $1\frac{221}{264}$.
4. Add $\frac{3}{10}$, $\frac{5}{18}$, $\frac{11}{12}$, and $\frac{7}{8}$ together. Ans. $2\frac{133}{360}$.
5. Add $\frac{5}{17}$, $\frac{3}{34}$, $\frac{16}{136}$, and $\frac{1}{2}$ together. Ans. 1.
6. Add $\frac{14}{15}$, $\frac{17}{20}$, $\frac{19}{30}$, and $\frac{9}{10}$ together. Ans. $3\frac{19}{60}$.
7. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ together. Ans. $1\frac{83}{140}$.
8. Add $\frac{8}{9}$, $\frac{3}{7}$, and $5\frac{3}{8}$ together. Ans. $6\frac{349}{504}$.
9. Add $\frac{4}{11}$, $\frac{7}{22}$, and $9\frac{3}{11}$ together. Ans. $9\frac{21}{22}$.
10. Add $\frac{3}{4}$, $\frac{5}{6}$, and $4\frac{3}{4}$ together. Ans. $6\frac{1}{3}$.
11. Add $\frac{4}{5}$, $7\frac{1}{2}$, and $8\frac{3}{4}$ together. Ans. $17\frac{1}{20}$.
12. Add $\frac{7}{9}$, $3\frac{1}{4}$, and $5\frac{3}{7}$ together. Ans. $9\frac{115}{252}$.
13. Add $6\frac{3}{5}$, $7\frac{8}{9}$, and $4\frac{3}{8}$ together. Ans. $18\frac{311}{360}$.

NOTE. — If the quantity be a mixed number, the better way is to add their fractional parts separately, as in the following example.

14. What is the sum of $11\frac{3}{4}$, $15\frac{7}{8}$, $12\frac{5}{12}$, and $17\frac{5}{6}$?

OPERATION.

$$\begin{array}{r|rrrr}
4 & 4 & 8 & 12 & 6 \\
3 & 1 & 2 & 3 & 6 \\
2 & 1 & 2 & 1 & 2 \\
\hline
 & 1 & 1 & 1 & 1
\end{array}
\qquad
4 \times 3 \times 2 = 24$$

$$\begin{array}{r|l}
11\frac{3}{4} & 6 \times 3 = 18 \\
15\frac{7}{8} & 3 \times 7 = 21 \\
12\frac{5}{12} & 2 \times 5 = 10 \\
17\frac{5}{6} & 4 \times 5 = \underline{20} \\
\text{Ans.} = 57\frac{7}{8} & \dfrac{69}{24} = 2\frac{7}{8}
\end{array}$$

15. What is the sum of $11\frac{6}{7}$, $19\frac{5}{12}$, and $23\frac{5}{9}$? Ans. $54\frac{209}{252}$.
16. What is the sum of $18\frac{4}{5}$, $27\frac{7}{10}$, and $49\frac{1}{2}$? Ans. 96.
17. What is the sum of $21\frac{4}{9}$, $18\frac{2}{3}$, and $26\frac{5}{6}$? Ans. $66\frac{17}{18}$.

18. What is the sum of $17\frac{2}{9}$, $14\frac{1}{2}$, and $13\frac{4}{7}$? Ans. $45\frac{1}{126}$.

19. What is the sum of $16\frac{2}{3}$, $8\frac{7}{8}$, $9\frac{3}{5}$, $3\frac{1}{4}$, and $1\frac{7}{8}$?
Ans. $40\frac{4}{15}$.

20. What is the sum of $371\frac{11}{12}$, $614\frac{19}{20}$, and $81\frac{3}{4}$?
Ans. $1068\frac{37}{60}$.

21. Add $\frac{4}{7}$ of $18\frac{3}{11}$, and $\frac{11}{12}$ of $\frac{4}{9}$ of $6\frac{3}{11}$ together.
Ans. $12\frac{691}{693}$.

22. Add $\frac{5}{8}$ of 18, and $\frac{4}{11}$ of $\frac{11}{12}$ of $7\frac{4}{11}$ together.
Ans. $13\frac{31}{44}$.

23. Add $\frac{6}{7}$ of $15\frac{1}{4}$, and $\frac{3}{4}$ of $107\frac{1}{7}$ together. Ans. $93\frac{3}{7}$.

24. Add $\frac{1}{3}$ of $\frac{2}{7}$ of $28\frac{3\frac{6}{7}}{4\frac{2}{7}}$ to $3\frac{39\frac{1}{2}}{105}$. Ans. $6\frac{9}{70}$.

25. Add $\frac{1}{8}$, $2\frac{5}{8}$, $\frac{45}{94\frac{7}{11}}$, and $\frac{47\frac{5}{8}}{314\frac{3}{5}}$ together. Ans. $3\frac{142857991}{852999832}$.

CASE III.

To add any two fractions, whose numerators are a unit.

RULE. — *Place the sum of the denominators over their product.*

EXAMPLE.

1. Add $\frac{1}{4}$ to $\frac{1}{5}$. $\dfrac{4+5=9}{4\times 5=20}$ Answer.

2. Add $\frac{1}{6}$ to $\frac{1}{4}$, $\frac{1}{2}$ to $\frac{1}{3}$, $\frac{1}{7}$ to $\frac{1}{5}$, $\frac{1}{3}$ to $\frac{1}{5}$, $\frac{1}{2}$ to $\frac{1}{8}$, $\frac{1}{2}$ to $\frac{1}{9}$, $\frac{1}{2}$ to $\frac{1}{10}$.
3. Add $\frac{1}{12}$ to $\frac{1}{4}$, $\frac{1}{3}$ to $\frac{1}{2}$, $\frac{1}{3}$ to $\frac{1}{3}$, $\frac{1}{3}$ to $\frac{1}{4}$, $\frac{1}{3}$ to $\frac{1}{5}$, $\frac{1}{3}$ to $\frac{1}{6}$, $\frac{1}{3}$ to $\frac{1}{7}$.
4. Add $\frac{1}{11}$ to $\frac{1}{2}$, $\frac{1}{11}$ to $\frac{1}{3}$, $\frac{1}{11}$ to $\frac{1}{4}$, $\frac{1}{11}$ to $\frac{1}{5}$, $\frac{1}{11}$ to $\frac{1}{6}$, $\frac{1}{11}$ to $\frac{1}{7}$.
5. Add $\frac{1}{10}$ to $\frac{1}{2}$, $\frac{1}{10}$ to $\frac{1}{3}$, $\frac{1}{10}$ to $\frac{1}{4}$, $\frac{1}{10}$ to $\frac{1}{5}$, $\frac{1}{10}$ to $\frac{1}{6}$, $\frac{1}{10}$ to $\frac{1}{7}$.
6. Add $\frac{1}{4}$ to $\frac{1}{2}$, $\frac{1}{4}$ to $\frac{1}{3}$, $\frac{1}{4}$ to $\frac{1}{4}$, $\frac{1}{4}$ to $\frac{1}{5}$, $\frac{1}{4}$ to $\frac{1}{6}$, $\frac{1}{4}$ to $\frac{1}{7}$, $\frac{1}{4}$ to $\frac{1}{8}$.
7. Add $\frac{1}{5}$ to $\frac{1}{6}$, $\frac{1}{5}$ to $\frac{1}{7}$, $\frac{1}{5}$ to $\frac{1}{8}$, $\frac{1}{5}$ to $\frac{1}{9}$, $\frac{1}{5}$ to $\frac{1}{2}$, $\frac{1}{5}$ to $\frac{1}{3}$, $\frac{1}{5}$ to $\frac{1}{4}$.
8. Add $\frac{1}{7}$ to $\frac{1}{2}$, $\frac{1}{7}$ to $\frac{1}{3}$, $\frac{1}{7}$ to $\frac{1}{4}$, $\frac{1}{7}$ to $\frac{1}{5}$, $\frac{1}{7}$ to $\frac{1}{6}$, $\frac{1}{7}$ to $\frac{1}{8}$, $\frac{1}{7}$ to $\frac{1}{9}$.
9. Add $\frac{1}{9}$ to $\frac{1}{2}$, $\frac{1}{9}$ to $\frac{1}{3}$, $\frac{1}{9}$ to $\frac{1}{4}$, $\frac{1}{9}$ to $\frac{1}{5}$, $\frac{1}{9}$ to $\frac{1}{6}$, $\frac{1}{9}$ to $\frac{1}{7}$, $\frac{1}{9}$ to $\frac{1}{8}$.

NOTE. — The truth of this rule is evident from the fact, that this process reduces the fractions to a common denominator, and then adds the numerators.

If the numerators of the given fractions be alike, and more than a unit, multiply the sum of the denominators by one of the numerators for a new numerator, then multiply the denominators together for a new denominator.

10. Add $\frac{3}{4}$ to $\frac{3}{5}$. $\qquad \dfrac{4+5=9\times 3=27}{4\times 5=20}=1\frac{7}{20}$ Ans.

11. Add $\frac{4}{5}$ to $\frac{4}{7}$, $\frac{4}{5}$ to $\frac{4}{9}$, $\frac{4}{5}$ to $\frac{4}{11}$, $\frac{2}{5}$ to $\frac{2}{7}$, $\frac{2}{5}$ to $\frac{2}{9}$, $\frac{2}{5}$ to $\frac{2}{11}$, $\frac{3}{5}$ to $\frac{3}{7}$.
12. Add $\frac{3}{5}$ to $\frac{3}{11}$, $\frac{3}{5}$ to $\frac{3}{5}$, $\frac{5}{6}$ to $\frac{5}{7}$, $\frac{5}{6}$ to $\frac{5}{8}$, $\frac{5}{6}$ to $\frac{5}{9}$, $\frac{5}{6}$ to $\frac{5}{11}$.
13. Add $\frac{6}{7}$ to $\frac{6}{8}$, $\frac{6}{7}$ to $\frac{6}{11}$, $\frac{6}{7}$ to $\frac{6}{13}$, $\frac{6}{11}$ to $\frac{6}{13}$, $\frac{6}{11}$ to $\frac{6}{17}$, $\frac{6}{11}$ to $\frac{6}{19}$.
14. Add $\frac{8}{9}$ to $\frac{8}{11}$, $\frac{8}{11}$ to $\frac{8}{11}$, $\frac{8}{11}$ to $\frac{8}{13}$, $\frac{8}{13}$ to $\frac{8}{15}$, $\frac{8}{13}$ to $\frac{8}{17}$, $\frac{8}{13}$ to $\frac{8}{19}$.
15. Add $\frac{9}{10}$ to $\frac{9}{11}$, $\frac{9}{10}$ to $\frac{9}{14}$, $\frac{9}{10}$ to $\frac{9}{16}$, $\frac{9}{10}$ to $\frac{9}{17}$, $\frac{9}{10}$ to $\frac{9}{19}$.
16. Add $\frac{7}{8}$ to $\frac{7}{9}$, $\frac{7}{8}$ to $\frac{7}{10}$, $\frac{7}{8}$ to $\frac{7}{11}$, $\frac{7}{8}$ to $\frac{7}{12}$, $\frac{7}{8}$ to $\frac{7}{13}$, $\frac{7}{8}$ to $\frac{7}{14}$.
17. Add $\frac{8}{9}$ to $\frac{8}{11}$, $\frac{8}{9}$ to $\frac{8}{13}$, $\frac{8}{9}$ to $\frac{8}{15}$, $\frac{8}{9}$ to $\frac{8}{17}$, $\frac{8}{9}$ to $\frac{8}{19}$, $\frac{8}{9}$ to $\frac{8}{21}$.
18. Add $\frac{10}{11}$ to $\frac{10}{13}$, $\frac{10}{11}$ to $\frac{10}{17}$, $\frac{10}{11}$ to $\frac{10}{19}$, $\frac{10}{13}$ to $\frac{10}{13}$, $\frac{10}{13}$ to $\frac{10}{17}$.
19. Add $\frac{11}{12}$ to $\frac{11}{13}$, $\frac{11}{12}$ to $\frac{11}{14}$, $\frac{11}{12}$ to $\frac{11}{15}$, $\frac{11}{12}$ to $\frac{11}{16}$, $\frac{11}{12}$ to $\frac{11}{17}$.

NOTE.—The preceding rule may be found very useful, because all similar questions may be readily performed *mentally*.

CASE IV.

To add compound numbers.

1. Add $\frac{7}{13}$ of a £. to $\frac{9}{11}$ of a £.

Value of $\frac{7}{13}$ of a £. =	10s.	9d.	$0\frac{12}{13}$qr.	
Value of $\frac{9}{11}$ of a £. =	16s.	4d.	$1\frac{5}{11}$qr.	
1£.	7s.	1d.	$2\frac{54}{143}$qr.	

This question is performed by finding the values of $\frac{7}{13}$ of a £. and $\frac{9}{11}$ of a £. by Case XI., Sec. XVI. The fractions $\frac{12}{13}$ and $\frac{5}{11}$ are added by Case II. of Addition of Fractions. The following questions are performed in the same manner.

The above question may be performed by first adding the fractions of the pounds together, and then finding their value by Case XI.; thus:

OPERATION.

$\frac{7}{13}$£. + $\frac{9}{11}$£. = $\frac{194}{143}$£. = 1£. 7s. 1d. $2\frac{54}{143}$qr. Ans.

2. Add together $\frac{4}{9}$ of a £., $\frac{3}{7}$ of a £., and $\frac{2}{5}$ of a shilling.

OPERATION.

	£.	s.	d.	qr.
$\frac{4}{9}$ of a £. =	0	8	10	$2\frac{2}{3}$
$\frac{3}{7}$ of a £. =	0	8	6	$3\frac{3}{7}$
$\frac{2}{5}$ of a s. =	0	0	4	$3\frac{1}{5}$
	0	17	10	$1\frac{31}{105}$

The above question may be solved by first reducing $\frac{2}{5}$ of a shilling to the fraction of a pound by Case IX., Sec. XVI.

and then adding it to the other numbers, and finding their value by Case XI., Sec. XVI. Thus:

$$\tfrac{2}{5} \text{ of a shilling} = \tfrac{2}{5} \times \tfrac{1}{20} = \tfrac{2}{100} = \tfrac{1}{50}\text{£}.$$

$$\tfrac{4}{9}\text{£} + \tfrac{3}{7}\text{£} + \tfrac{1}{50}\text{£} = \tfrac{2813}{3150}\text{£} = 0\text{£}.\ 17\text{s}.\ 10\text{d}.\ 1\tfrac{31}{105}qr.\ \textit{Ans.}$$

NOTE.—The pupil should solve the following questions by both processes.

3. Add together $\frac{7}{11}$ of a ton, and $\frac{17}{22}$ of a cwt.
Ans. *13cwt.* 2qr.

4. Add together $\frac{3}{5}$ of a yard, $\frac{1}{7}$ of an ell English, and $\frac{6}{7}$ of a qr. Ans. 3qr. 3na. $1\frac{139}{140}$*in.*

5. Add together $\frac{7}{11}$ of a mile, $\frac{4}{13}$ of a furlong, and $\frac{9}{22}$ of a yard. Ans. 5fur. 16rd. 0ft. 3in. $1\frac{136}{143}$bar.

6. A. has three house-lots; the first contains $\frac{5}{7}$ of an acre, the second $\frac{2}{3}$ of an acre, and the third $\frac{13}{14}$ of an acre. How many acres do they all contain?
Ans. 2A. 1R. 9p. 142ft. $87\frac{3}{7}$in.

7. A man travelled $18\frac{3}{7}$ miles the first day, $23\frac{11}{14}$ miles the second day, and $19\frac{2}{21}$ miles the third day. How far did he travel in the three days? Ans. 61m. 2fur. 3rd. 13ft. $4\frac{2}{7}$in.

8. Add $\frac{11}{12}$ of a gallon of wine to $\frac{1}{12}$ of a hhd.
Ans. 6gal. 0qt. 1pt. $1\frac{1}{3}$gi.

9. Add $\frac{5}{16}$ of a week to $\frac{1}{5}$ of a day. Ans. 2d. 9h. 18m.

10. Add $\frac{3}{4}$ of a square foot to $\frac{1}{2}$ a foot square. Ans. 1 foot.

11. Add 6 inches to 11rd. 16ft. 5in. Ans. 12rd. 0ft. 5in.

SECTION XVIII.

SUBTRACTION OF VULGAR FRACTIONS.

CASE I.

To subtract fractions that have a common denominator.

1. If $\frac{1}{4}$ be taken from $\frac{3}{4}$ what will be left?
2. If $\frac{3}{8}$ be taken from $\frac{5}{8}$ what will be left?
3. If $\frac{2}{10}$ be taken from $\frac{7}{10}$ what will be left?
4. What portion of a dollar will be left, if $\frac{1}{4}$ be taken from $\frac{1}{2}$?
5. Subtract $\frac{5}{12}$ from $\frac{11}{12}$.

$$11 - 5 = 6. \quad \tfrac{6}{12} = \tfrac{1}{2} \text{ Ans.}$$

RULE. — *Subtract the less numerator from the greater, and under the remainder write the common denominator, and reduce the fraction if necessary.*

6. Subtract $\frac{6}{17}$ from $\frac{13}{17}$. Ans. $\frac{7}{17}$.
7. Subtract $\frac{4}{19}$ from $\frac{13}{19}$. Ans. $\frac{9}{19}$.
8. Subtract $\frac{8}{37}$ from $\frac{23}{37}$. Ans. $\frac{15}{37}$.
9. Subtract $\frac{17}{39}$ from $\frac{19}{39}$. Ans. $\frac{2}{39}$.
10. Subtract $\frac{18}{29}$ from $\frac{23}{29}$. Ans. $\frac{5}{29}$.
11. Subtract $\frac{14}{37}$ from $\frac{29}{37}$. Ans. $\frac{15}{37}$.
12. Subtract $\frac{16}{31}$ from $\frac{30}{31}$. Ans. $\frac{14}{31}$.
13. Subtract $\frac{116}{137}$ from $\frac{119}{137}$. Ans. $\frac{3}{137}$.
14. Subtract $\frac{1}{100}$ from $\frac{26}{100}$. Ans. $\frac{1}{4}$.
15. Subtract $\frac{14}{24}$ from $\frac{20}{24}$. Ans. $\frac{1}{4}$.
16. Subtract $\frac{16}{144}$ from $\frac{36}{144}$. Ans. $\frac{5}{36}$.
17. Subtract $\frac{19}{160}$ from $\frac{47}{160}$. Ans. $\frac{7}{40}$.
18. Subtract $\frac{327}{1728}$ from $\frac{596}{1728}$. Ans. $\frac{269}{1728}$.
19. Subtract $\frac{48}{1000}$ from $\frac{100}{1000}$. Ans. $\frac{13}{250}$.

CASE II.

To subtract fractions whose denominators are unlike.

1. Subtract $\frac{4}{7}$ from $\frac{10}{11}$.

OPERATION.

Common denominator 77

$$\begin{array}{r|l}
11 & 7 \times 10 = 70 \\
7 & 11 \times \ \ 4 = 44 \\
 & \qquad\qquad \overline{26} \\
 & \qquad\qquad \frac{26}{77} = \text{Ans.}
\end{array}$$

In this question, we find the common denominator, 77, by multiplying the two denominators, 7 and 11; and then obtain the numerators, as in Case VIII., Sec. XVI.; the difference of which we write over the common denominator.

2. From $\frac{11}{16}$ take $\frac{5}{12}$. Ans. $\frac{13}{48}$.

OPERATION.

$$4\underline{)16 \quad 12} \qquad 4 \times 4 \times 3 = 48 \text{ common denominator.}$$
$$\ \ 4 \quad\ 3$$

$$\begin{array}{r|l}
16 & 3 \times 11 = 33 \\
12 & 4 \times \ \ 5 = 20 \\
 & \qquad\qquad \overline{13} \\
 & \qquad\qquad \frac{13}{48} = \text{Ans.}
\end{array}$$

3. From $9\frac{7}{8}$ take $5\frac{11}{16}$. Ans. $4\frac{3}{16}$.

OPERATION.

$$9\tfrac{7}{8}=\tfrac{79}{8};\ 5\tfrac{11}{16}=\tfrac{91}{16}.$$

$$\begin{array}{r} 8)\underline{8\ \ 16} \\ 1\ \ \ 2 \end{array}\quad 8\times 2=16 \text{ common denominator.}$$

$$\begin{array}{r|l} 8 & 2\times 79=158 \\ 16 & 1\times 91=\ \ \underline{91} \\ & \qquad\quad \dfrac{67}{16}=4\tfrac{3}{16}\text{ Ans.} \end{array}$$

4. From $\frac{3}{8}$ of $16\frac{5}{6}$ take $\frac{5}{12}$ of $9\frac{1}{5}$. Ans. $2\frac{23}{48}$.

OPERATION.

$$16\tfrac{5}{6}=\tfrac{101}{6};\ 9\tfrac{1}{5}=\tfrac{46}{5}.$$

$$\tfrac{3}{8}\times\tfrac{101}{6}=\tfrac{303}{48};\ \tfrac{5}{12}\times\tfrac{46}{5}=\tfrac{230}{60}=\tfrac{23}{6}.$$

$$\tfrac{303}{48}-\tfrac{23}{6}\qquad \begin{array}{r} 6)\underline{48\ \ 6} \\ 8\ \ 1 \end{array}$$

$$6\times 8\times 1=48$$

$$\begin{array}{r|l} 48 & 1\times 303=303 \\ 6 & 8\times\ 23=\underline{184} \\ & \qquad\quad \dfrac{119}{48}=2\tfrac{23}{48}\text{ Ans.} \end{array}$$

RULE. — *Reduce compound fractions to simple ones, and mixed ones to improper fractions; then, having found a common denominator, divide this by each of the denominators of the fraction, and multiply their quotients by their respective numerators. The difference of these products, placed over the common denominator, will give the answer required.*

5. From $\frac{7}{12}$ take $\frac{2}{9}$. Ans. $\frac{13}{36}$.
6. From $\frac{8}{9}$ take $\frac{3}{8}$. Ans. $\frac{37}{72}$.
7. From $\frac{5}{6}$ take $\frac{1}{3}$. Ans. $\frac{1}{2}$.
8. From $\frac{12}{17}$ take $\frac{5}{34}$. Ans. $\frac{19}{34}$.
9. From $\frac{7}{10}$ take $\frac{5}{12}$. Ans. $\frac{17}{60}$.
10. From $\frac{11}{17}$ take $\frac{5}{19}$. Ans. $\frac{124}{323}$.
11. From $\frac{18}{19}$ take $\frac{3}{38}$. Ans. $\frac{627}{722}$.
12. From $\frac{16}{21}$ take $\frac{4}{11}$. Ans. $\frac{92}{231}$.
13. From $\frac{31}{50}$ take $\frac{3}{10}$. Ans. $\frac{8}{25}$.
14. From $\frac{17}{25}$ take $\frac{7}{20}$. Ans. $\frac{33}{100}$.
15. From $\frac{10}{27}$ take $\frac{5}{18}$. Ans. $\frac{5}{54}$.
16. From $\frac{3}{28}$ take $\frac{3}{56}$. Ans. $\frac{3}{56}$.
17. From $7\frac{3}{4}$ take $\frac{5}{7}$ of 9. Ans. $1\frac{9}{28}$.
18. From $\frac{3}{5}$ of $8\frac{3}{4}$ take $\frac{2}{3}$ of 5. Ans. $1\frac{11}{12}$.
19. From $\frac{1}{4}$ of 3 take $\frac{1}{3}$ of 2. Ans. $\frac{1}{12}$.

CASE III.

To subtract a proper fraction or a mixed one from a whole number.

1. From \$ 7 take \$ $3\frac{5}{8}$.

OPERATION.
7
$3\frac{5}{8}$
\$ $3\frac{3}{8}$ Ans.

To subtract $\frac{5}{8}$ in this example we must borrow 1 from the 7 in the minuend, and reduce it to eighths ($\frac{8}{8}$) and the $\frac{5}{8}$ must be taken from them; $\frac{5}{8}$ from $\frac{8}{8}$ leaves $\frac{3}{8}$. To pay for the 1 which was borrowed, 1 must be added to the 3 in the subtrahend, $1 + 3 = 4$, and 4 taken from 7 leaves 3, and the $\frac{3}{8}$ placed at the right hand of it gives the answer \$ $3\frac{3}{8}$.

By adopting the following rule, the same result will be obtained.

RULE. — *Subtract the numerator from the denominator of the fraction, and under the remainder write the denominator, and carry one to the whole number of the subtrahend to be subtracted from the minuend.*

OPERATION.

2.	3.	4.	5.	6.	7.
32	16	671	385	16	18
$5\frac{3}{8}$	$4\frac{3}{4}$	$0\frac{41}{50}$	$16\frac{17}{39}$	$0\frac{117}{237}$	$1\frac{3}{4}$
$26\frac{5}{8}$	$11\frac{1}{4}$	$670\frac{9}{50}$	$368\frac{22}{39}$	$15\frac{120}{237}$	$16\frac{1}{4}$

8.	9.	10.	11.	12.	13.
19	27	169	711	46	81
$13\frac{3}{7}$	$8\frac{3}{7}$	$91\frac{14}{23}$	$30\frac{15}{79}$	$15\frac{5}{8}$	$49\frac{11}{19}$

If it be required to subtract one mixed number from another mixed number, the following method may be adopted.

14. From $8\frac{3}{7}$ take $4\frac{4}{5}$. Ans. $3\frac{22}{35}$.

OPERATION.
$8\frac{3}{7} = 8\frac{15}{35}$
$4\frac{4}{5} = 4\frac{28}{35}$
$3\frac{22}{35}$

In this question, we multiply the 3 and the 7, the numerator and the denominator of the fraction in the minuend, by 5, the denominator of the fraction in the subtrahend, and we have a new fraction $\frac{15}{35}$, which we write at the right hand of the 8, thus, $8\frac{15}{35}$. We then multiply the numerator and denominator of the subtrahend by 7, the denominator of the minuend; and we have another new fraction, $\frac{28}{35}$, which we place at the right hand of the 4, thus, $4\frac{28}{35}$. It will now be perceived, that we have changed the fractions $8\frac{3}{7}$ and $4\frac{4}{5}$ to other fractions of the same value, having a *common denominator*. We now subtract as in question 1, by adding 1 ($=\frac{35}{35}$) to $\frac{15}{35}$, which makes $\frac{50}{35}$, and

from this we subtract $\frac{28}{35}$; thus, $\frac{50}{35} - \frac{28}{35} = \frac{22}{35}$. We then carry the 1 we borrowed to the 4, $1 + 4 = 5$, which we take from 8, and find 3 remaining. The answer, then, is $3\frac{22}{35}$.

If the fraction in the subtrahend be less than the fraction in the minuend, we proceed as in the following problem.

15. From $9\frac{4}{7}$ take $3\frac{5}{12}$. Ans. $6\frac{13}{84}$.

OPERATION.

$$9\frac{4}{7} = 9\frac{48}{84}$$
$$3\frac{5}{12} = 3\frac{35}{84}$$
$$\overline{\quad 6\frac{13}{84}}$$

Having reduced the fractions to a common denominator, as in the last problem, we subtract 35, the numerator of the subtrahend, from 48, the numerator of the minuend, and the remainder 13 we write over the common denominator $= \frac{13}{84}$, which we annex to the difference between 9 and 3, $= 6$; thus, $6\frac{13}{84}$.

	16.	17.	18.	19.	20.	21.
	cwt.	Tuns.	\$	lb.	oz.	Miles.
From	$18\frac{3}{8}$	$73\frac{4}{5}$	$67\frac{1}{3}$	$29\frac{5}{18}$	$144\frac{3}{4}$	$171\frac{15}{22}$
Take	$9\frac{5}{8}$	$16\frac{11}{12}$	$16\frac{2}{3}$	$15\frac{4}{9}$	$99\frac{11}{13}$	$91\frac{9}{10}$

	22.	23.	24.	25.	26.	27.
	Furlongs.	Rods.	Inches.	Feet.	Bushels.	Pecks.
From	$101\frac{7}{8}$	$165\frac{1}{2}$	$77\frac{11}{12}$	$84\frac{23}{24}$	$671\frac{111}{721}$	$17\frac{123}{457}$
Take	$93\frac{4}{5}$	$98\frac{7}{8}$	$19\frac{19}{20}$	$15\frac{11}{12}$	$183\frac{69}{83}$	$8\frac{234}{711}$

28. From a hhd. of wine there leaked out $7\frac{9}{11}$ gallons; what quantity remained? Ans. $55\frac{2}{11}$gal.

29. A man engaged to labor 30 days, but was absent $5\frac{7}{12}$ days; how many days did he work? Ans. $24\frac{5}{12}$da.

30. From 144 pounds of sugar there were taken at one time $17\frac{5}{8}$ pounds, and at another $28\frac{7}{12}$ pounds; what quantity remains? Ans. $97\frac{19}{24}$lb.

31. A man sells $9\frac{7}{8}$ yards from a piece of cloth containing 34 yards; how many yards remain? Ans. $24\frac{1}{8}$yd.

32. The distance from Boston to Providence is 40 *miles*. A. having set out from Boston, has travelled $\frac{2}{17}$ of the distance; and B. having set out at the same time from Providence, has gone $\frac{2}{11}$ of the distance; how far is A. from B.?

Ans. $28\frac{4}{187}$m.

33. From $\frac{1}{3}$ of a square yard take $\frac{1}{3}$ of a yard squared.

Ans. 2 square feet.

34. What is the difference between $\dfrac{49\frac{5}{8}}{97}$ and $\dfrac{34\frac{3}{5}}{145\frac{3}{11}}$?

Ans. $\frac{847651}{3100020}$.

CASE IV.

To subtract one fraction from another, when both fractions have a unit for a numerator.

OPERATION.

1. Take $\frac{1}{7}$ from $\frac{1}{3}$. $\frac{7-3}{7 \times 3} = \frac{4}{21}$ Ans.

The student will perceive, that this operation reduces the fractions to a common denominator.

RULE. — *Write the difference of the denominators over their product*

2. Take $\frac{1}{9}$ from $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$; $\frac{1}{20}$ from $\frac{1}{10}$, $\frac{1}{11}$, $\frac{1}{12}$, $\frac{1}{15}$.
3. Take $\frac{1}{8}$ from $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$; $\frac{1}{19}$ from $\frac{1}{18}$, $\frac{1}{16}$, $\frac{1}{15}$.
4. Take $\frac{1}{6}$ from $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{13}$ from $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{8}$.
5. Take $\frac{1}{5}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; $\frac{1}{7}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
6. Take $\frac{1}{11}$ from $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.
7. Take $\frac{1}{3}$ from $\frac{1}{2}$; $\frac{1}{4}$ from $\frac{1}{2}$, $\frac{1}{3}$; $\frac{1}{9}$ from $\frac{1}{8}$.
8. Take $\frac{1}{12}$ from $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.
9. Take $\frac{1}{16}$ from $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.
10. Take $\frac{1}{10}$ from $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.

NOTE. — If the numerators of the given fractions be alike, and more than a unit, multiply the difference of the denominators by one of the numerators for a new numerator, then multiply the denominators together for a new denominator.

OPERATION.

11. Take $\frac{2}{7}$ from $\frac{2}{3}$. $7-3=4$; $\frac{4 \times 2}{3 \times 7} = \frac{8}{21}$ Ans.

12. Take $\frac{3}{5}$ from $\frac{3}{4}$; $\frac{3}{8}$ from $\frac{3}{7}$; $\frac{3}{10}$ from $\frac{3}{4}$; $\frac{3}{11}$ from $\frac{3}{5}$.
13. Take $\frac{4}{9}$ from $\frac{4}{7}$; $\frac{4}{11}$ from $\frac{4}{5}$; $\frac{4}{11}$ from $\frac{4}{7}$; $\frac{4}{11}$ from $\frac{4}{10}$.
14. Take $\frac{5}{7}$ from $\frac{5}{6}$; $\frac{5}{11}$ from $\frac{5}{6}$; $\frac{5}{11}$ from $\frac{5}{7}$; $\frac{5}{11}$ from $\frac{5}{10}$.
15. Take $\frac{6}{11}$ from $\frac{6}{7}$; $\frac{6}{11}$ from $\frac{6}{8}$; $\frac{6}{13}$ from $\frac{6}{7}$; $\frac{6}{13}$ from $\frac{6}{11}$.
16. Take $\frac{4}{15}$ from $\frac{4}{5}$; $\frac{4}{15}$ from $\frac{4}{7}$; $\frac{4}{15}$ from $\frac{4}{9}$; $\frac{4}{15}$ from $\frac{4}{11}$.
17. Take $\frac{8}{11}$ from $\frac{8}{9}$; $\frac{8}{11}$ from $\frac{8}{7}$; $\frac{8}{11}$ from $\frac{8}{5}$; $\frac{8}{11}$ from $\frac{8}{3}$.
18. Take $\frac{2}{11}$ from $\frac{2}{5}$; $\frac{2}{11}$ from $\frac{2}{9}$; $\frac{2}{11}$ from $\frac{2}{3}$; $\frac{2}{11}$ from $\frac{2}{7}$.
19. Take $\frac{10}{13}$ from $\frac{10}{7}$; $\frac{10}{19}$ from $\frac{10}{17}$; $\frac{10}{29}$ from $\frac{10}{19}$; $\frac{10}{17}$ from $\frac{10}{11}$.
20. Take $\frac{12}{7}$ from $\frac{12}{5}$; $\frac{12}{7}$ from $\frac{12}{1}$; $\frac{12}{5}$ from $\frac{12}{3}$; $\frac{12}{13}$ from $\frac{12}{7}$.

NOTE. — The above questions, and those of a similar kind, may readily be performed *mentally*.

CASE V.

To subtract compound numbers.

1. From $\frac{7}{11}$ of a £. take $\frac{2}{9}$ of a £.

OPERATION.

Value of $\frac{7}{11}$ £. = 12s. $8\frac{8}{11}$d.	33	common denominator
Value of $\frac{2}{9}$ £. = 4s. $5\frac{1}{3}$d.	$\overline{24}$	
Ans. 8s. $3\frac{13}{33}$d.	11	
	$\overline{13}$	
	$\overline{33}$	

To perform this question, we find by Case XI., Sect. XVI., the value of $\frac{7}{11}$£. = 12s. $8\frac{8}{11}$d.; and also of $\frac{2}{9}$ £. = 4s. $5\frac{1}{3}$d.; we then find a common denominator of the fractional part, by multiplying together their denominators, $11 \times 3 = 33$. We then proceed as in Case II., Sect XVIII.

This question can be performed by first subtracting the fraction $\frac{2}{9}$ of a £. from $\frac{7}{11}$ of a £., and then reducing the remainder by Case XI., Sect. XVI.; thus:

$$\frac{7}{11} £. - \frac{2}{9} £. = \frac{41}{99} £. = 0 £.\ 8s.\ 3d.\ 1\frac{19}{33}qr.\ \text{Ans.}$$

2. From $\frac{2}{31}$ of a ton take $\frac{4}{5}$ of a cwt.

OPERATION.

	cwt.	qr.	lb.	oz.	dr.
$\frac{2}{31}$ of a ton =	1	1	4	8	$4\frac{4}{31}$
$\frac{4}{5}$ of a cwt. =		3	5	9	$9\frac{3}{5}$
Ans.	0	1	26	14	$10\frac{82}{155}$

This question may also be performed by first reducing $\frac{4}{5}$ of a cwt. to the fraction of a ton by Case IX., Sect. XVI., and subtracting it from $\frac{2}{31}$ of a ton, and then reducing the remainder to its proper terms by Case XI., Sect. XVI. Thus:

$$\frac{4}{5} \times \frac{1}{20} = \frac{4}{100} = \frac{1}{25} \text{ of a ton.}$$

$$\frac{2}{31} - \frac{1}{25} = \frac{19}{775} \text{ of a ton} = 1qr.\ 26lb.\ 14oz.\ 10\frac{82}{155}dr.\ \text{Ans.}$$

RULE. — *Find the value of the fractions in integers; then subtract as in the foregoing rules.*

3. From $\frac{7}{8}$ of an ell English take $\frac{2}{7}$ of a yard.
Ans. 3qr. 0na. $2\frac{5}{56}$in.

4. From $\frac{2}{9}$ of a mile take $\frac{7}{11}$ of a furlong.
Ans. 1fur. 5rd. 10ft. 10in.

5. From $\frac{5}{7}$ of a degree take $\frac{3}{5}$ of a mile.
Ans. 49m. 0fur. 13rd. 11ft. 9in. $1\frac{2}{7}$bar.

6. From $\frac{4}{11}$ of an acre take $\frac{4}{9}$ of a rod.
Ans. 1R. 17p. 22yd. 2ft. 108in.

7. From $\frac{9}{10}$ of a cord take $\frac{2}{11}$ of a cord.
Ans. 91ft. 1602$\frac{18}{55}$in.

8. From $\frac{7}{13}$ of a hhd. of wine there leaked out $\frac{4}{5}$ of it; what remained?
Ans. 6gal. 3qt. 0pt. 1$\frac{7}{65}$gi.

9. From Boston to Concord, N. H., the distance is 72 miles; having travelled $\frac{4}{7}$ of this distance, how much remains?
Ans. 30m. 6fur. 34rd. 4ft. 8in. 1$\frac{4}{7}$bar.

10. From $\frac{2}{7}$ of a year take $\frac{4}{9}$ of a week.
Ans. 101da. 5h. 54m. 17$\frac{1}{7}$sec.

11. From $\frac{4}{11}$ of an acre take $\frac{1}{2}$ of a foot.
Ans. 1R. 18p. 5yd. 4ft. 0in.

SECTION XIX.

MULTIPLICATION OF VULGAR FRACTIONS.

CASE I.

To multiply a simple fraction by a simple fraction.

1. Multiply $\frac{7}{8}$ by $\frac{3}{5}$.

OPERATION.

$\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$ Ans.

This process may be understood by supposing a man to have found $\frac{7}{8}$ of a dollar, and that he gave $\frac{3}{5}$ of it to his wife, and that he wished to ascertain what part of a dollar his wife received. If $\frac{1}{8}$ of a dollar be divided into 5 equal parts, one of these parts will be $\frac{1}{40}$ of a dollar. And, if $\frac{1}{8}$ of $\frac{1}{5}$ of a dollar be $\frac{1}{40}$ of a dollar, $\frac{7}{8}$ of $\frac{1}{5}$ will be 7 times as much, and 7 times $\frac{1}{40}$ are $\frac{7}{40}$. If then, $\frac{7}{8}$ of $\frac{1}{5}$ be $\frac{7}{40}$, $\frac{7}{8}$ of $\frac{3}{5}$ will be 3 times as much, and 3 times $\frac{7}{40}$ are $\frac{21}{40}$. The wife will therefore receive $\frac{21}{40}$ of a dollar.

RULE. — *Multiply the numerators together for a new numerator, and the denominators for a new denominator. The fraction should then be reduced to its lowest terms.*

2. Multiply $\frac{4}{11}$ by $\frac{2}{3}$. Ans. $\frac{8}{33}$.
3. Multiply $\frac{3}{7}$ by $\frac{19}{20}$. Ans. $\frac{57}{140}$.
4. Multiply $\frac{4}{5}$ by $\frac{6}{11}$. Ans. $\frac{24}{55}$.
5. Multiply $\frac{7}{12}$ by $\frac{11}{16}$. Ans. $\frac{77}{192}$.
6. Multiply $\frac{8}{11}$ by $\frac{4}{7}$. Ans. $\frac{32}{77}$.
7. Multiply $\frac{3}{4}$ by $\frac{15}{16}$. Ans. $\frac{45}{64}$.
8. Multiply $\frac{16}{17}$ by $\frac{11}{12}$. Ans. $\frac{44}{51}$.

9. Multiply $\frac{7}{11}$ by $\frac{11}{21}$. Ans. $\frac{1}{3}$.

OPERATION.

$$\frac{7}{\cancel{11}} \times \frac{\cancel{11}}{21} = \frac{7}{21} = \frac{1}{3}$$

The operation of the following questions may be abridged by cancelling.

10. Multiply $\frac{14}{17}$ by $\frac{17}{42}$. Ans. $\frac{1}{3}$.
11. Multiply $\frac{9}{11}$ by $\frac{22}{27}$. Ans. $\frac{2}{3}$.
12. Multiply $\frac{4}{9}$ by $\frac{18}{19}$. Ans. $\frac{8}{19}$.
13. Multiply $\frac{3}{13}$ by $\frac{26}{39}$. Ans. $\frac{2}{13}$.
14. Multiply $\frac{7}{9}$ by $\frac{9}{11}$. Ans. $\frac{7}{11}$.
15. Multiply $\frac{1}{99}$ by $\frac{99}{100}$. Ans. $\frac{1}{100}$.

CASE II.

To multiply a whole number by a fraction, or a fraction by a whole number.

1. If a man earn $\frac{7}{8}$ of a dollar in one day, how much will he earn in 9 days?

OPERATION.

$\frac{7}{8} \times \frac{9}{1} = \frac{63}{8} = \$\,7\frac{7}{8}$ Ans.

To analyze this question, we say, if he earn 7 eighths of a dollar in 1 day, in 9 days he will earn 9 times as much, and 9 times 7 eighths are 63 eighths $= \frac{63}{8} = \$\,7\frac{7}{8}$ Ans.

RULE. — *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator, and the quotient is the answer required.*

2. Multiply 12 by $\frac{5}{7}$. Ans. $8\frac{4}{7}$.
3. Multiply 15 by $\frac{6}{11}$. Ans. $8\frac{2}{11}$.
4. Multiply $\frac{4}{9}$ by 11. Ans. $4\frac{8}{9}$.
5. Multiply $\frac{2}{7}$ by 12. Ans. $3\frac{3}{7}$.
6. Multiply $\frac{8}{9}$ by 19. Ans. $16\frac{8}{9}$.
7. Multiply $\frac{9}{10}$ by 14. Ans. $12\frac{3}{5}$.
8. Multiply 13 by $\frac{4}{7}$. Ans. $7\frac{3}{7}$.
9. Multiply 16 by $\frac{3}{19}$. Ans. $2\frac{10}{19}$.
10. Multiply 11 by $\frac{4}{7}$. Ans. $6\frac{2}{7}$.

NOTE. — If any of the fractions are compound, they must be reduced to simple fractions by Case VI., Sect. XVI.

11. Multiply $\frac{4}{9}$ of $\frac{7}{8}$ of $\frac{1}{2}$ by 12. Ans. $2\frac{1}{3}$.

OPERATION.

$$\frac{4}{9} \times \frac{7}{8} \times \frac{1}{2} = \frac{28}{144} = \frac{7}{36}.$$

$$\frac{7}{36} \times \frac{12}{1} = \frac{84}{36} = 2\frac{1}{3} \text{ Ans.}$$

12. Multiply $\frac{3}{7}$ of $\frac{7}{11}$ of $\frac{11}{24}$ by 100. Ans. $12\frac{1}{2}$.
13. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ by 11. Ans. $3\frac{5}{24}$.
14. What cost $\frac{7}{12}$ of a ton of hay, at $ 17 per ton?
Ans. $$9\frac{11}{12}$.
15. What cost $\frac{9}{20}$ of an acre of land, at $ 37 per acre?
Ans. $$16\frac{13}{20}$.
16. At $\frac{5}{8}$ of a dollar per foot, what cost 7 cords of wood?
Ans. $ 35.

CASE III.

To multiply a mixed number by a whole number, or a whole number by a mixed number.

1. Multiply $7\frac{5}{8}$ by 9.

OPERATION.

$$\begin{array}{r} 7\frac{5}{8} \\ 9 \\ \hline 68\frac{5}{8} \end{array}$$

In performing this question, we say 9 times 5 eighths are 45 eighths, and 45 eighths are equal to $5\frac{5}{8}$. We write down the $\frac{5}{8}$ and carry the 5 to the product of 9 times 7 = 68.

RULE. — *Multiply the numerator of the mixed number by the whole number, and divide the product by the denominator of the fraction, and as many times as it contains the denominator, so many units must be carried to the product of the integers. If, after division, any thing remains, let it be a numerator and the divisor a denominator to a fraction to be affixed to the product.*

2. Multiply $8\frac{3}{5}$ by 7. Ans. $60\frac{1}{5}$.
3. Multiply $9\frac{3}{7}$ by 8. Ans. $75\frac{3}{7}$.
4. Multiply $11\frac{7}{8}$ by 7. Ans. $82\frac{1}{8}$.
5. Multiply $15\frac{6}{11}$ by 7. Ans. $108\frac{9}{11}$.
6. Multiply $14\frac{3}{5}$ by 9. Ans. $131\frac{2}{5}$.
7. Multiply $23\frac{4}{9}$ by 11. Ans. $257\frac{8}{9}$.
8. Multiply $47\frac{3}{11}$ by 15. Ans. $709\frac{1}{11}$.
9. Multiply $37\frac{3}{4}$ by 18. Ans. $679\frac{1}{2}$.
10. Multiply $678\frac{11}{12}$ by 24. Ans. 16294.
11. What will $23\frac{7}{8}$ pounds of lead cost, at 8 cents a pound?
Ans. $1.91.
12. What will $15\frac{11}{16}$ pounds of sugar cost, at 12 cents a pound? Ans. $$1.88\frac{1}{4}$.
13. What will $29\frac{5}{12}$ cwt. of hay cost, at $ 1.12 per cwt.?
Ans. $$ 32.94\frac{2}{3}$.
14. What will $9\frac{7}{8}$ yards of broadcloth cost, at $ 8 per yard?
Ans. $ 79.00.
15. What will $17\frac{9}{16}$ tons of potash cost, at $ 97 per ton?
Ans. $$ 1703.56\frac{1}{4}$.

16. What will $3\frac{17}{112}$ tons of plaster of Paris cost, at \$12.75 per ton? Ans. \$ $40.18\frac{59}{112}$.

17. What will $17\frac{5}{12}$ dozen candles cost, at 16 cents a dozen? Ans. \$ $2.78\frac{2}{3}$.

18. What will $27\frac{3}{8}$ pounds of bar soap cost, at 7 cents a pound? Ans. \$ $1.91\frac{5}{8}$.

19. What will $29\frac{7}{12}$ dozen of axes cost, at *\$11.62 per* dozen? Ans. \$ $343.75\frac{5}{6}$.

20. Bought 28 bales of cotton cloth, each bale contain*ing* $31\frac{3}{8}$ yards; what will be the cost, at 16 cents per yard? Ans. \$ 140.56.

Note. — If both numbers be mixed, or either of them be a complex fraction, they must first be reduced to simple ones.

21. Multiply $8\frac{7}{8}$ by $11\frac{3}{8}$.

OPERATION.

$$8\frac{7}{8}=\frac{71}{8};\ 11\frac{3}{8}=\frac{91}{8}.$$
$$\frac{71}{8}\times\frac{91}{8}=\frac{6461}{64}=100\frac{61}{64}\ \text{Ans.}$$

22. Multiply $161\frac{11}{15}$ by $19\frac{11}{25}$. Ans. $3136\frac{33}{75}$.
23. Multiply $\frac{3}{7}$ by $8\frac{3}{4}$. Ans. $3\frac{3}{4}$.
24. Multiply $\frac{9}{10}$ by $17\frac{3}{11}$. Ans. $15\frac{6}{11}$.
25. Multiply $\frac{8}{9}$ by $71\frac{4}{5}$. Ans. $63\frac{37}{45}$.
26. Multiply $\frac{3}{4}$ of $9\frac{1}{4}$ by $\frac{2}{3}$ of 17. Ans. $78\frac{5}{8}$.
27. Multiply $\frac{9}{10}$ of 7 by $\frac{11}{15}$ of $87\frac{3}{11}$. Ans. $403\frac{1}{5}$.
28. Multiply 7 by $3\frac{3}{4}$. Ans. $26\frac{1}{4}$.
29. Multiply 8 by $\frac{7}{9}$. Ans. $6\frac{2}{9}$.
30. Multiply 12 by $\frac{5}{7}$. Ans. $8\frac{4}{7}$.
31. Multiply 15 by $\frac{6}{11}$. Ans. $8\frac{2}{11}$.
32. A merchant owns $\frac{7}{8}$ of a ship, he sells $\frac{4}{11}$ of his share to A. What part is that of the whole ship? Ans. $\frac{7}{22}$.
33. Multiply $3\frac{7}{9}$ by $10\frac{4}{7}$. Ans. $39\frac{59}{63}$.
34. Multiply $\frac{2}{3}$ of $7\frac{1}{4}$ by $\frac{7}{8}$ of $11\frac{3}{4}$. Ans. $49\frac{133}{192}$.
35. Multiply $\frac{2}{7}$ of 9 by $\frac{3}{5}$ of 17. Ans. $26\frac{8}{35}$.
36. Multiply $\frac{4}{7}$ of $8\frac{3}{10}$ by $\frac{4}{7}$ of $9\frac{1}{4}$. Ans. $25\frac{17}{245}$.
37. Multiply $18\frac{8}{9}$ by $15\frac{6}{7}$. Ans. *$299\frac{11}{21}$*.
38. Multiply $\frac{3}{4}$ of $\dfrac{7\frac{1}{2}}{11\frac{4}{7}}$ by $\frac{4}{7}$ of $\dfrac{5\frac{3}{7}}{8\frac{1}{4}}$. Ans. $\frac{380}{2079}$.
39. What cost $7\frac{3}{8}$ barrels of flour, at \$ $5\frac{3}{8}$ per barrel? Ans. \$ $39\frac{41}{64}$.
40. What cost $9\frac{4}{11}$ acres of land, at \$ $27\frac{3}{7}$ per acre? Ans. \$ $256\frac{64}{77}$.

41. What cost $8\frac{7}{8}$ bushels of apples, at $\$1\frac{1}{4}$ per bushel?

Ans. $\$11\frac{3}{32}$.

42. Bought $11\frac{7}{8}$ bushels of corn, at $\$1\frac{3}{8}$ per bushel, and sold $\frac{3}{8}$ of it for $\$1\frac{5}{8}$ per bushel, and the remainder at $\$1\frac{7}{8}$ per bushel; what is gained on the sale of the corn? Ans. $\$3\frac{2}{24}$.

43. What is the continued product of the following numbers: $\frac{27}{37\frac{4}{5}}$, $\frac{87\frac{2}{9}}{98\frac{1}{8}}$, $\frac{\frac{7}{8}}{2\frac{1}{3}}$, and $\frac{81\frac{5}{11}}{128}$? Ans. $\frac{5}{33}$.

SECTION XX.

DIVISION OF VULGAR FRACTIONS.

CASE I.

To divide a fraction by an integer or whole number.

1. How many times will $\frac{5}{7}$ contain 8?

OPERATION.

$\frac{5}{7} \times \frac{1}{8} = \frac{5}{56}$ Ans.

To illustrate this question we will suppose $\frac{5}{7}$ of an orange to be divided equally among 8 persons. Now, if we divide $\frac{1}{7}$ of an orange into 8 equal parts, one of these parts will be equal to $\frac{1}{56}$ of a whole orange, but as there are $\frac{5}{7}$ to be divided among 8 persons, each person will receive 5 times $\frac{1}{56} = \frac{5}{56}$ Ans.

RULE. — *Multiply the whole number by the denominator of the fraction, and write the product under the numerator. Or, divide the numerator of the fraction by the whole number, if it can be done without a remainder, and write the quotient over the denominator.*

2. Divide $\frac{7}{11}$ by 18. Ans. $\frac{7}{198}$.
3. Divide $\frac{15}{22}$ by 28. Ans. $\frac{15}{616}$.
4. Divide $\frac{1}{17}$ by 27. Ans. $\frac{1}{459}$.
5. Divide $\frac{2}{15}$ by 128. Ans. $\frac{1}{960}$.
6. Divide $\frac{11}{17}$ by 98. Ans. $\frac{11}{1666}$.
7. Divide $\frac{14}{23}$ by 19. Ans. $\frac{14}{437}$.
8. Divide $\frac{5}{6}$ by 167. Ans. $\frac{5}{1002}$.
9. Divide $\frac{16}{29}$ by 49. Ans. $\frac{16}{1421}$.
10. Divide $\frac{1}{15}$ by 15. Ans. $\frac{1}{225}$.

CASE II.

To divide a whole number by a fraction.

1. How many times will 17 contain $\frac{3}{4}$?

OPERATION.

$\frac{17}{1} \times \frac{4}{3} = \frac{68}{3} = 22\frac{2}{3}$ Ans.

It is evident that 17 will contain $\frac{1}{4}$ as many times as there are fourths in 17, which are $4 \times 17 = 68$ times. Again, if 17 contain 1 fourth 68 times, it will contain 3 fourths as many times as 68 will contain $3 = 22\frac{2}{3}$ Ans.

RULE. — *Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

2. Divide 18 by $\frac{7}{11}$. Ans. $28\frac{2}{7}$.
3. Divide 28 by $\frac{15}{22}$. Ans. $41\frac{1}{15}$.
4. Divide 27 by $\frac{1}{17}$. Ans. 459.
5. Divide 128 by $\frac{2}{15}$. Ans. 960.
6. Divide 98 by $\frac{11}{17}$. Ans. $151\frac{5}{11}$.
7. Divide 19 by $\frac{14}{23}$. Ans. $31\frac{3}{14}$.
8. Divide 167 by $\frac{15}{18}$. Ans. $200\frac{2}{5}$.
9. Divide 49 by $\frac{15}{27}$. Ans. $88\frac{3}{15}$.
10. Divide 15 by $\frac{1}{15}$. Ans. 225.

CASE III.

To divide a mixed number by an integer or whole number.

1. Divide $27\frac{3}{5}$ by 6.

OPERATION.

$6)27\frac{3}{5}$

$4\frac{18}{30} = 4\frac{3}{5}$ Ans.

We divide 27 by 6, and find that it is contained 4 times, which we write under the 27, and we have 3 remaining, which we multiply by 5, the denominator of the fraction, and to the product we add the numerator 3, and the amount is 18. This we write over the product of 6, the divisor, multiplied by the denominator, $5, = 6 \times 5 = 30$. We then reduce the fraction.

RULE. — *Divide the integers as in whole numbers, and if any thing remains multiply it by the denominator of the fraction, and to the product add the numerator of the fraction, and write it over the product of the divisor, multiplied by the denominator.*

2. Divide $29\frac{3}{8}$ by 9. Ans. $3\frac{19}{72}$.
3. Divide $14\frac{1}{2}$ by 7. Ans. $2\frac{1}{14}$.
4. Divide $13\frac{3}{8}$ by 8. Ans. $1\frac{43}{64}$.
5. Divide $14\frac{3}{5}$ by 6. Ans. $2\frac{13}{30}$.
6. Divide \$ $37\frac{3}{7}$ among 9 men. Ans. \$ $4\frac{10}{63}$.
7. Divide \$ $96\frac{2}{3}$ among 11 persons. Ans. \$ $8\frac{26}{33}$.
8. What is $\frac{1}{8}$ of $167\frac{7}{11}$ cwt. of iron? Ans. $20\frac{21}{22}$ cwt.

9. Divide $\frac{7}{8}$ of a prize, valued at \$ 1723, equally between 12 seamen. Ans. \$ 125.63$\frac{13}{24}$.

10. What will a barrel of flour cost, if 19 barrels can be purchased for \$ 107$\frac{3}{8}$? Ans. \$ 5.65$\frac{5}{38}$.

11. If 15 pounds of raisins can be obtained for \$ 3$\frac{2}{7}$, what will 1 pound cost? Ans. \$ 0.21$\frac{19}{21}$.

12. If 12 quarts of wine cost \$ 3.75$\frac{3}{4}$, what will a quart cost? Ans. \$ 0.31$\frac{5}{16}$.

13. If \$ 19 will buy 375$\frac{11}{16}$ acres of land, how much can be bought for \$ 1? Ans. 19$\frac{235}{304}$ acres.

CASE IV.

To divide one fraction by another, whether the fractions be simple, compound, mixed, or complex.

1. Divide $\frac{4}{7}$ by $\frac{3}{8}$. Ans. 1$\frac{11}{21}$.

OPERATION.

$\frac{4}{7} \times \frac{8}{3} = \frac{32}{21} = 1\frac{11}{21}$ Ans.

To understand the *rationale* of this process, we find the two factors of $\frac{3}{8}$, which are $\frac{3}{1}$ and $\frac{1}{8}$; $\frac{3}{1}$ multiplied by $\frac{1}{8}$ make $\frac{3}{8}$, as is evident from a preceding rule. We now divide $\frac{4}{7}$ by $\frac{3}{1}$, which by Case I of this section will be $\frac{4}{7} \times \frac{1}{3} = \frac{4}{21}$. Again, we wish to divide $\frac{4}{21}$ by $\frac{1}{8}$. It is evident that $\frac{4}{21}$ will contain $\frac{1}{8}$ eight times as often as it will a unit, and it contains an unit $\frac{4}{21}$ times, therefore it contains $\frac{1}{8}$ eight times $\frac{4}{21} = \frac{8}{1} \times \frac{4}{21} = \frac{32}{21} = 1\frac{11}{21}$ Ans. It will be perceived, that in performing this question the numerator of the dividend has been multiplied by the denominator of the divisor, and the denominator of the dividend by the numerator of the divisor.

RULE. — *Invert the divisor, and proceed as in multiplication. If, however, there be mixed numbers in the question, they must first be reduced to improper fractions, and compound and complex fractions must be reduced to simple fractions.*

2. Divide $\frac{5}{8}$ by $\frac{7}{9}$. Ans. $\frac{45}{56}$.

OPERATION.

$$\frac{5}{8} \times \frac{9}{7} = \frac{45}{56} \text{ Ans.}$$

3. Divide 12$\frac{4}{7}$ by 9$\frac{2}{5}$. Ans. 1$\frac{111}{329}$.

OPERATION.

$$12\frac{4}{7} = \frac{88}{7};\ 9\frac{2}{5} = \frac{47}{5}.$$

$$\frac{88}{7} \times \frac{5}{47} = \frac{440}{329} = 1\frac{111}{329} \text{ Ans.}$$

4. Divide $\frac{2}{3}$ of $\frac{4}{11}$ by $\frac{1}{5}$ of $\frac{7}{8}$. Ans. 1$\frac{89}{231}$.

OPERATION.

$$\tfrac{2}{3} \times \tfrac{4}{11} = \tfrac{8}{33};\ \tfrac{1}{5} \times \tfrac{7}{8} = \tfrac{7}{40}.$$
$$\tfrac{8}{33} \times \tfrac{40}{7} = \tfrac{320}{231} = 1\tfrac{89}{231} \text{ Ans.}$$

5. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{4}{5}$ of $11\frac{4}{11}$. **Ans.** $\frac{341}{600}$.

OPERATION.

$$\tfrac{2}{3} \text{ of } 7\tfrac{3}{4} = \tfrac{2}{3} \times \tfrac{31}{4} = \tfrac{62}{12} = \tfrac{31}{6}$$
$$\tfrac{4}{5} \text{ of } 11\tfrac{4}{11} = \tfrac{4}{5} \times \tfrac{125}{11} = \tfrac{500}{55} = \tfrac{100}{11}$$
$$\tfrac{31}{6} \times \tfrac{11}{100} = \tfrac{341}{600} \text{ Ans.}$$

6. Divide $\dfrac{7\frac{4}{9}}{11\frac{2}{3}}$ by $\dfrac{\frac{3}{7}}{4\frac{1}{8}}$. Ans. $4\frac{728}{840}$.

OPERATION.

$$\frac{7\frac{4}{9}}{11\frac{2}{3}} = \frac{\frac{67}{9}}{\frac{35}{3}} = \tfrac{67}{9} \times \tfrac{3}{35} = \tfrac{201}{315} = \tfrac{67}{105}$$
$$\frac{\frac{3}{7}}{4\frac{1}{8}} = \frac{\frac{3}{7}}{\frac{33}{8}} = \tfrac{3}{7} \times \tfrac{8}{33} = \tfrac{24}{231} = \tfrac{8}{77}.$$
$$\tfrac{67}{105} \times \tfrac{77}{8} = \tfrac{5159}{840} = 6\tfrac{119}{840} \text{ Ans.}$$

7. Divide $\frac{4}{9}$ of 91 by $\frac{9}{10}$ of 87. Ans. $\frac{3640}{7047}$.

OPERATION.

$$\tfrac{4}{9} \times \tfrac{91}{1} = \tfrac{364}{9};\ \tfrac{9}{10} \times \tfrac{87}{1} = \tfrac{783}{10}.$$
$$\tfrac{364}{9} \times \tfrac{10}{783} = \tfrac{3640}{7047} \text{ Ans.}$$

8. Divide $\frac{7}{15}$ by $\frac{8}{9}$. $\frac{7}{15} \times \frac{9}{8} = \frac{63}{120} = \frac{21}{40}$ Ans.
9. Divide $\frac{11}{12}$ by $\frac{3}{4}$. Ans. $1\frac{2}{9}$.
10. Divide $\frac{3}{4}$ by $\frac{8}{13}$. Ans. $1\frac{7}{32}$.
11. Divide $\frac{5}{6}$ by $\frac{8}{9}$. Ans. $\frac{15}{16}$.
12. Divide $\frac{11}{15}$ by $\frac{7}{11}$. Ans. $1\frac{16}{105}$.
13. Divide $\frac{8}{25}$ by $\frac{14}{17}$. Ans. $\frac{68}{175}$.
14. Divide $\frac{16}{21}$ by $\frac{1}{15}$. Ans. $11\frac{3}{7}$.
15. Divide $\frac{9}{25}$ by $7\frac{3}{4}$. Ans. $\frac{36}{775}$.
16. Divide $\frac{8}{11}$ by $16\frac{4}{9}$. Ans. $\frac{18}{407}$.
17. Divide $11\frac{3}{7}$ by $\frac{4}{7}$. Ans. 20.
18. Divide $21\frac{4}{5}$ by $18\frac{4}{7}$. Ans. $1\frac{113}{650}$.
19. Divide $17\frac{3}{11}$ by $28\frac{11}{26}$. Ans. $\frac{4940}{8129}$.
20. Divide $161\frac{3}{17}$ by $14\frac{3}{5}$. Ans. $11\frac{49}{1241}$.
21. Divide $\frac{7}{11}$ of $\frac{4}{5}$ by $\frac{3}{5}$ of $\frac{8}{11}$. Ans. $1\frac{1}{6}$.
22. Divide $\frac{5}{9}$ of $7\frac{3}{11}$ by $\frac{4}{11}$ of $17\frac{3}{7}$. Ans. $\frac{350}{549}$.
23. Divide $\frac{6}{17}$ of 15 by $\frac{7}{15}$ of 22. Ans. $\frac{675}{1309}$.
24. Bought $\frac{7}{8}$ of a coal-mine for \$3675, and having sold

$\frac{5}{7}$ of my share, I gave $\frac{2}{3}$ of the remainder to a charitable society, and divided the residue among 7 poor persons; what was the share of each? Ans. $ 50 for each poor person.

25. Of an estate valued at $ 5000, the widow receives $\frac{1}{3}$, the oldest son $\frac{2}{3}$ of the remainder; the residue is equally divided among 7 daughters; what is the share of each daughter?

Ans. 158\frac{46}{63}$.

SECTION XXI.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

1. If $\frac{7}{8}$ of a bushel of corn cost 63 cents, what cost a bushel? what cost 15 bushels? Ans. $10.80.

Illustration. If 7 eighths of a bushel cost 63 cents, 1 eighth will cost 1 seventh of 63 cents = 9 cents; and 8 eighths will cost 8 times 9 cents = 72 cents, and 15 bushels will cost 15 times 72 cents = $10.80.

2. If 4$\frac{7}{9}$lb. of pepper cost $ 2.15, what cost one pound? what cost 30lb.? Ans. $13.50.

Illustration. In 4$\frac{7}{9}$lb. there are $\frac{43}{9}$lb. Then, if 43 ninths lb. cost $ 2.15, 1 ninth will cost 1 forty-third of $ 2.15 = $ 0.05, and 9 ninths or 1lb. will cost 9 times $ 0.05 = $ 0.45, and 30lb. will cost 30 times $ 0.45 = $13.50.

3. When $1728 are paid for 30$\frac{4}{15}$ tons of iron, what cost 1 ton? what cost 7$\frac{17}{30}$ tons? Ans. $ 432.

4. When $ 432 are paid for 7$\frac{17}{30}$ tons of iron, what quantity should be received for $1728? Ans.

5. For 7$\frac{17}{30}$ tons of iron there were paid $432; what sum will it require to pay for 30$\frac{4}{15}$ tons? Ans.

6. For 30$\frac{4}{15}$ tons of iron $1728 were paid; what quantity should be received for $ 432? Ans.

7. Gave 7$\frac{7}{10}$ bushels of rye for a barrel of flour; how much rye will it then require to purchase 6$\frac{4}{9}$ barrels of flour?

Ans. 49$\frac{28}{45}$ bushels.

8. Divide $1728 among 17 boys and 15 girls, and give each boy $\frac{7}{11}$ as much as a girl; what sum does each receive?

Ans. each girl $ 66$\frac{66}{71}$; each boy $ 42$\frac{42}{71}$.

9. If $\frac{7}{9}$ of a ton of hay cost $14.49, what cost 4$\frac{3}{7}$ tons?

Ans.

10. If $4\frac{3}{7}$ tons of hay cost \$ $82.50\frac{3}{7}$, what part of a ton will \$14.49 buy? Ans.

11. If \$14.49 will buy $\frac{7}{8}$ of a ton of hay, how much hay can be obtained for \$ $82.50\frac{3}{7}$? Ans.

12. When \$ $82.50\frac{3}{7}$ are paid for $4\frac{3}{7}$ tons of hay, what will be the cost of $\frac{7}{8}$ of a ton? Ans.

13. When $14\frac{7}{8}$ tons of copperas are sold for \$ 500, what is the value of 1 ton? what is the value of $9\frac{11}{12}$ tons? Ans.

14. When $9\frac{11}{12}$ tons of copperas are sold for \$ $333.33\frac{1}{3}$, what is the value of $14\frac{7}{8}$ tons? Ans.

15. Gave \$ $333.33\frac{1}{3}$ for $9\frac{11}{12}$ tons of copperas; what quantity of copperas should be received for \$ 500? Ans.

16. For $14\frac{7}{8}$ tons of copperas \$ 500 were paid; how much might be purchased for \$ $333.33\frac{1}{3}$? Ans.

17. Purchased $97\frac{7}{8}$ gallons of molasses for \$ 31.32; what cost 1 gallon? what cost $763\frac{5}{8}$ gallons? Ans.

18. Sold $763\frac{5}{8}$ gallons of molasses for \$ 244.36; what should I receive for $97\frac{7}{8}$ gallons? Ans.

19. If \$ 244.36 will buy $763\frac{5}{8}$ gallons of molasses, what quantity can be obtained for \$ 31.32? Ans.

20. Gave 1975lb. of flax for 40 barrels of flour; how many pounds were given for 1 barrel? how many pounds would it require to buy 144 barrels? Ans. 7110lb.

21. If 17 bushels of rye cost \$15.75, what cost 1 bushel? what cost $9\frac{1}{4}$ bushels? Ans. \$ $8.56\frac{67}{68}$.

22. If 9 barrels of flour cost \$ $50\frac{4}{7}$, what cost 1 barrel? what cost $87\frac{3}{5}$ barrels? Ans. \$ $492\frac{8}{35}$.

23. If 13 boarders consume a barrel of pork in 78 days, how long would it last, if 7 more boarders were added to their number? Ans. $50\frac{7}{10}$ days.

24. If a man by laboring 10 hours a day can, in 9 days, perform a certain piece of work, how many days would it require to do the same work, were he to labor 15 hours a day?

25. If a man, by laboring 15 hours a day, in 6 days can perform a certain piece of work, how many days would it require to do the same work by laboring 10 hours a day? Ans.

26. If a man, by laboring 10 hours a day, can in 9 days perform a certain piece of work, how many hours must he labor each day to perform the same work in 6 days? Ans.

27. Sold $17\frac{3}{11}$ bushels of corn for \$ $5\frac{3}{7}$; what was received for 1 bushel? what should I have charged for $97\frac{5}{8}$ bushels?

Ans. \$ $30\frac{208}{315}$.

28. Bought $9\frac{3}{5}$ tons of hay at \$ $19\frac{7}{8}$ per ton; for what must it be sold per cwt. to gain \$ 7 on my bargain? Ans. \$ $1\frac{73}{2880}$.

29. If I sell hay at \$$1\frac{3}{4}$ per cwt., what should I give for $9\frac{3}{5}$ tons, that I may make \$7 on my bargain? Ans. \$329.

30. How many bushels of corn at \$0.75 per bushel will it require to purchase $47\frac{3}{11}$ bushels of wheat at \$$2\frac{2}{3}$ per bushel? Ans. $168\frac{8}{99}$bu.

31. If 15 cords of wood cost \$$57\frac{9}{11}$, what cost 1 cord? what cost $19\frac{7}{8}$ cords? Ans.

32. If $19\frac{7}{8}$ cords of wood cost \$$76\frac{67}{110}$, how many cords may be obtained for \$$57\frac{9}{11}$? Ans.

33. At $7\frac{3}{10}$ shillings per yard, what cost $47\frac{1}{3}$ yards? Ans. 17£. 5s. $6\frac{2}{5}$d.

34. When 172£. 15s. $0\frac{2}{5}$d. are paid for $47\frac{1}{3}$ yards of broadcloth, what is the value of 1 yard? Ans. 3£. 12s. $11\frac{328}{355}$d.

35. If 1lb. of sugar cost $\frac{7}{13}$ of a dollar, what is the value of $43\frac{6}{7}$lb.? Ans. \$$23.61\frac{7}{13}$.

36. If $17\frac{3}{8}$lb. of sugar cost \$$2\frac{7}{11}$, what cost 50lb.? Ans. \$$7.58\frac{1018}{1529}$.

37. Bought $87\frac{3}{7}$ yards of broadcloth for \$612; what was the value of $14\frac{7}{10}$ yards? Ans. \$102.90.

38. If $\frac{7}{8}$ of an acre of land cost \$43.75, what cost 10 acres?

39. When \$500 are paid for 10 acres of land, how much might be obtained for \$43.75? Ans.

40. If 9 hogsheads of sugar cost \$71.87, what cost $\frac{4}{7}$ of a hogshead? Ans.

41. Paid \$$4.56\frac{20}{63}$ for $\frac{4}{7}$ of a hogshead of sugar; what ought to be given for 9 hogsheads? Ans.

42. If 19 men can grade a certain road in 111 days, how long would it require 47 men to perform the same labor?

43. When 47 men can grade a certain road in $44\frac{41}{47}$ days, how long would it require 19 men to perform the same labor?

44. If $\frac{4}{11}$ of a ton of hay cost \$9.20, what cost 17 tons?

45. When \$430.10 are paid for 17 tons of hay, what cost $\frac{4}{11}$ of a ton? Ans.

46. If $\frac{7}{16}$ of a tub of butter cost \$7.15, what cost 7 tubs?

47. When \$114.40 are paid for 7 tubs of butter, what cost $\frac{7}{16}$ of a tub? Ans.

48. If a man can mow a field of $7\frac{1}{4}$ acres in $8\frac{1}{2}$ days, when they are $11\frac{1}{4}$ hours long, how many days will it require to perform the same labor, when the days are 10 hours long? Ans. $9\frac{9}{16}$ days.

49. If 4 yards of cloth cost \$19, what cost 25 yards? Ans. \$118.75.

50. When \$87.25 is paid for 15 barrels of flour, what cost 100 barrels? Ans. \$$581.66\frac{2}{3}$.

51. Gave \$81.75 for 20 cords of wood; required the price of 70 cords at the same rate. Ans. \$286.12$\frac{1}{2}$.

52. John Smith gave Thomas Ayer \$19.75 for 2 dozen of hats; what then should Smith give for 19$\frac{7}{12}$ dozen?

Ans. \$193.38$\frac{13}{24}$.

53. If 3cwt. of potash cost \$47.25, what must be paid for 11$\frac{5}{12}$ cwt. Ans. \$179.81$\frac{1}{4}$.

54. If 47 casks of lime cost \$50, what is the value of 5 casks? Ans. \$5.31$\frac{43}{47}$.

55. Peter Quimby bought of Silas Noyes 19 tons of hay, for which he paid \$300; what then was the cost of 13$\frac{2}{7}$ tons?

Ans. \$209.77$\frac{58}{133}$.

56. If a staff 4 feet long cast a shadow 6 feet, what is the height of that steeple whose shadow is 180 feet?

Ans. 120 feet.

57. If 8 men eat a barrel of flour in 20 days, how long would it last 12 men? Ans. 13$\frac{1}{3}$ days.

58. If 23 men build 27$\frac{5}{8}$ rods of wall, which is 5$\frac{3}{8}$ feet high, in 5 days, by laboring 10 hours each day, required the number of days it will take 46 men to perform the same labor by working 8 hours each day? Ans. 3$\frac{1}{8}$ days.

59. Henry Smith can reap a field in 10 days, by laboring 8 hours a day. His wife, Sarah, the same field in 9 days, by laboring 12 hours a day. How long would it take both to reap the field, provided they labored 8 hours a day?

Ans. 5$\frac{35}{47}$ days.

60. If 15 tons 7cwt. 2qr. 18lb. of cotton cost \$2067.48, what cost 1lb.? what 1cwt.? what cost 1 ton? what 78 tons?

Ans. \$10483.20.

61. If it cost \$9$\frac{1}{8}$ to carry 17cwt. 3qr. 7$\frac{3}{4}$lb. of sugar 87 miles, how far may it be carried for \$73.87?

Ans. 650$\frac{1588}{1975}$ miles.

62. Bought a bale of cloth for \$96$\frac{3}{8}$; I dispose of it for $\frac{5}{8}$ of the cost, and by so doing I lose \$2 on a yard; required the number of yards in the bale? Ans. 18$\frac{9}{128}$yd.

63. Bought a horse and chaise for \$250, and I paid for the harness $\frac{7}{11}$ of what I paid for the horse. The chaise cost $\frac{11}{12}$ the value of the horse. What was the price of each?

Ans. Horse, \$130$\frac{10}{23}$; chaise, \$119$\frac{13}{23}$; harness, \$83$\frac{1}{253}$.

64. At $\frac{1}{7}$ of a dollar a peck, how many bushels can be obtained for \$17$\frac{5}{8}$? Ans. 30$\frac{27}{32}$ bushels.

65. If it require 2$\frac{3}{7}$ bushels of wheat to sow 1 acre, how many acres will 17$\frac{4}{7}$ bushels sow? Ans. 7$\frac{4}{17}$ acres.

66. If $17\frac{4}{7}$ bushels of wheat sow $7\frac{4}{17}$ acres, how many bushels will it require to sow 1 acre? Ans. $2\frac{3}{7}$ bushels.

67. John Jones sold his horse for \$176.18. He received in part pay $47\frac{3}{4}$ bushels of rye at \$$1.37\frac{1}{2}$ per bushel; and for the remainder he received wheat at \$$2\frac{2}{7}$ per bushel. Required the quantity of wheat. Ans. $45\frac{6833}{13600}$ bushels.

68. Sold apples at $\frac{1}{7}$ of a dollar a bushel. What did I receive for $8\frac{1}{5}$ bushels? Ans. \$$1.17\frac{1}{7}$.

69. Sold $18\frac{2}{9}$ bushels of rye for \$21.47. What was received for 1 bushel? what for $7\frac{4}{5}$ bushels? Ans. \$$9.19\frac{17}{820}$.

70. Gave 17£. 8s. 11d. for $9\frac{3}{11}$ tons of coal. What cost 1 ton? what cost $19\frac{4}{7}$ tons? Ans. 36£. 16s. $5\frac{1}{7}\frac{9}{14}$d.

71. How many garments can be made of $756\frac{4}{11}$ yards of cloth, if each garment requires $7\frac{31}{55}$ yards?

Ans. 100 garments.

72. If $18\frac{4}{7}$ cords of wood cost \$90.00, what cost 1 cord? what cost $171\frac{3}{8}$ cords? Ans. \$$830.50\frac{25}{26}$.

73. If a man travel $147\frac{3}{11}$ miles in $36\frac{4}{9}$ hours, how far will he travel in 1 hour? how far in $97\frac{1}{4}$ hours? Ans. $392\frac{3168}{3608}$m.

74. If a man travel 500 miles in $97\frac{3}{4}$ hours, how far will he travel in $32\frac{7}{12}$ hours? Ans. $166\frac{2}{3}$ miles.

75. If a horse eat $19\frac{3}{7}$ bushels of oats in $87\frac{4}{7}$ days, how many will 7 horses eat in 60 days? Ans. $93\frac{1}{3}$ bushels.

MISCELLANEOUS QUESTIONS.

1. How far will a man walk in $17\frac{3}{11}$ hours, provided he goes at the rate of $4\frac{7}{9}$ miles an hour?

Ans. 82m. 4fur. 8rd. 1ft. 4in.

2. How much land is there in a field which is $29\frac{7}{13}$ rods square? Ans. 5A. 1R. 32p. 141ft. $109\frac{155}{169}$in.

3. How much wood in a pile which is $17\frac{3}{4}$ feet long, $7\frac{1}{11}$ feet high, and $4\frac{3}{5}$ feet wide? Ans. 4cd. $66\frac{107}{110}$ft.

4. What is the value of $19\frac{7}{8}$ barrels of flour, at \$$6\frac{3}{4}$ a barrel? Ans. \$$134.15\frac{5}{8}$.

5. What is the value of $376\frac{11}{18}$ acres of land, at \$$75\frac{3}{8}$ per acre? Ans. \$$28387.06\frac{1}{4}$.

6. What cost $17\frac{19}{112}$ quintals of fish, at \$4.75 per quintal?

Ans. \$$81.55\frac{65}{112}$.

7. What cost $1670\frac{7}{13}$ pounds of coffee, at $12\frac{3}{4}$ cents per pound? Ans. \$$212.99\frac{19}{52}$.

8. What cost $28\frac{4}{11}$ tons of Lackawana coal, at \$ $11\frac{3}{4}$ a ton? Ans. \$ $333.27\frac{3}{11}$.

9. Bought $37\frac{11}{63}$ hogsheads of molasses, at \$ $17.62\frac{1}{2}$ a hhd.; what was the whole cost? Ans. \$ $655.20\frac{5}{21}$.

10. What cost $\frac{7}{8}$ of a cord of wood, at \$ 5.75 a cord? Ans. \$ $5.03\frac{1}{8}$.

11. What are the contents of a field which is $139\frac{4}{7}$ *rods* long, and $38\frac{4}{5}$ rods wide? Ans. 33A. 3R. $15\frac{13}{35}$p.

12. Bought 15 loads of wood, each containing $11\frac{3}{5}$ feet, *cord* measure. I divide it equally between 9 persons; what does each receive? Ans. $19\frac{1}{3}$ft.

13. If the transportation of $18\frac{3}{8}$ tons of iron cost \$ *$48.15\frac{3}{4}$*, what is it per ton? Ans. \$ $2.62\frac{4}{49}$.

14. If a hhd. of wine cost \$ $98\frac{7}{8}$, what is the price of *one* gallon? Ans. \$ $1.56\frac{119}{126}$.

15. If 5 bushels of wheat cost \$ $8\frac{2}{9}$, what will a bushel be worth? Ans. \$ *$1.64\frac{4}{9}$*.

16. What will 11 hogsheads and $17\frac{1}{2}$ gallons of wine cost, at $19\frac{3}{4}$ cents a gallon? Ans. \$ $140.32\frac{3}{8}$.

17. How many bottles, each containing $1\frac{3}{4}$ pints, are sufficient for bottling a hhd. of cider? Ans. 288.

18. I have a shed which is $18\frac{7}{12}$ feet long, $10\frac{5}{12}$ *feet wide*, and $7\frac{11}{12}$ feet high; how many cords of wood will it contain? Ans. 11cd. $124\frac{829}{1728}$ft.

19. What will $6\frac{7}{8}$ pounds of tea cost, at $65\frac{3}{4}$ cents per lb.? Ans. \$ $4.52\frac{1}{32}$.

20. How many cubic feet does a box contain, that *is* $8\frac{3}{4}$ feet long, $5\frac{7}{12}$ feet wide, and 3 feet high? Ans. $146\frac{9}{16}$ft.

21. How many feet of boards will it take to cover a side of a house which is $46\frac{5}{12}$ feet long and $17\frac{1}{2}$ feet high? Ans. $812\frac{7}{24}$ft.

22. Required the number of square feet on the surface of 7 boxes, each of which is $5\frac{1}{2}$ feet long, $2\frac{5}{12}$ feet high, and $3\frac{1}{12}$ feet wide; required also the number of cubic feet they *would* occupy. Ans. $527\frac{59}{72}$ft.; $286\frac{253}{288}$ cubic feet.

23. A certain room is 12 feet long, $11\frac{1}{2}$ feet wide, and $7\frac{1}{2}$ feet high; how much will it cost to plaster it, at $2\frac{3}{4}$ cents per square foot? Ans. \$ $13.48\frac{7}{8}$.

24. A man has a garden that is $14\frac{1}{2}$ rods long, and *$10\frac{1}{4}$ rods* wide; he wishes to have a ditch dug around it, that shall be 3 feet wide and $4\frac{1}{2}$ feet deep; what will be the expense, if he gives 2 cents per cubic foot? Ans. \$ *$223.76\frac{1}{4}$*.

25. How many bushels of grain will a box contain which is

$14\frac{7}{12}$ feet long, $5\frac{11}{12}$ feet deep, and $4\frac{1}{4}$ feet wide, there being $2150\frac{2}{5}$ cubic inches in a bushel? Ans. $294\frac{2429}{3584}$bu.

26. Which will contain the most, and by how much, a box that is 10 feet long, 8 feet wide, and 6 feet deep, or a cubical one, whose each side measures 8 feet?

Ans. The last contains 32 cubic feet the most.

27. Divide \$ $1112\frac{5}{8}$ equally among 129 men. Ans. \$ $8\frac{5}{8}$.

28. Bought 68 barrels of flour, at \$ $7\frac{11}{12}$ per barrel; what was the amount of the whole? Ans. \$ $538\frac{1}{3}$.

29. What cost $8\frac{2}{3}$ acres of land, at \$ $42\frac{3}{5}$ per acre?

Ans. \$ 369.20.

30. How shall four 3's be arranged, that their value shall be nothing?

31. I have a room 20 feet long, 15 feet wide, and $8\frac{1}{2}$ feet high. This room contains 4 windows, each of which is $5\frac{1}{2}$ feet in height and $3\frac{1}{4}$ feet in width. There are two doors 7 feet high and 3 feet wide. The mop-boards are $\frac{2}{3}$ of a foot wide. A mason has agreed to plaster this room at $6\frac{1}{4}$ cents per square yard; a painter is to lay on the paper at 9 cents per square yard; the paper which I wish to have laid on is $2\frac{3}{5}$ feet wide, for which I pay 5 cents per yard. What is the amount of my bill for plastering, for papering, and for paper?

Ans. For plastering, \$ $5.11\frac{29}{36}$; for papering, \$ 4.37; for paper, \$ $2.80\frac{5}{36}$.

SECTION XXII.

DECIMAL FRACTIONS.

A DECIMAL FRACTION is a fraction whose denominator is a unit with as many ciphers annexed as there are places in the numerator. Thus, $\frac{5}{10}$, $\frac{15}{100}$, $\frac{157}{1000}$, &c.

The denominator being in all cases formed in this obvious manner, there is no necessity of expressing it, and therefore it is not written; but a point is placed before the first figure of the numerator, to indicate that the figure or figures on the right of the point denote a decimal, and not a whole number. Thus, for $\frac{5}{10}$ we write .5, for $\frac{15}{100}$, .15, &c. This point is called the *separatrix*.

Ciphers annexed to the right hand of decimals do not increase their value; for .4, and .40, and .400 are decimals having

the same value, each being equal to $\frac{4}{10}$ or $\frac{2}{5}$; but when ciphers are placed on the left hand of a decimal, they decrease the value in a tenfold proportion. Thus, .4 is $\frac{4}{10}$, or four tenths; but .04 is $\frac{4}{100}$, or four hundredths; and .004 is $\frac{4}{1000}$, or four thousandths. The figure next the separatrix is reckoned so many tenths; the next at the right, so many hundredths; the third is so many thousandths; and so on, as may be seen by the following

TABLE.

Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.
7	6	5	4	3	2	1.	2	3	4	5	6	7

From this table it is evident, that in decimals, as well as in whole numbers, each figure takes its value by its distance from the place of units.

NOTE. — If there be one figure in the decimal, it is so many tenths; if there be two figures, they express so many hundredths; if there be three figures, they are so many thousandths, &c.

NUMERATION OF DECIMAL FRACTIONS.

Let the pupil write the following numbers.

1. Three hundred twenty-five, and seven tenths.
2. Four hundred sixty-five, and fourteen hundredths.
3. Ninety-three, and seven hundredths.
4. Twenty-four, and nine millionths.
5. Two hundred twenty-one, and nine hundred thousandths.
6. Forty-nine thousand, and forty-nine thousandths.
7. Seventy-nine million two thousand, and one hundred five thousandths.
8. Sixty-nine thousand fifteen, and fifteen hundred thousandths.
9. Eighty thousand, and eighty-three ten thousandths.

10. Nine billion nineteen thousand nineteen, and nineteen hundredths.

11. Twenty-seven, and nine hundred twenty-seven thousandths.

12. Forty-nine trillion, and one trillionth.

13. Twenty-one, and one ten thousandth.

14. Eighty-seven thousand, and eighty-seven millionths.

15. Ninety-nine thousand ninety-nine, and nine thousand nine billionths.

16. Seventeen, and one hundred seventeen ten thousandths.

17. Thirty-three, and thirty-three hundredths.

18. Forty-seven thousand, and twenty-nine ten millionths.

19. Fifteen, and four thousand seven hundred thousandths.

20. Eleven thousand, and eleven hundredths.

21. Seventeen, and eighty-one quatrillionths.

22. Nine, and fifty-seven trillionths.

23. Sixty-nine thousand, and three hundred forty-nine thousandths.

Let the following numbers be written in words.

27.86	86.0007	1.000007	16.300000007
48.07	5.6001	5.101016	1.315
15.716	34.1063	6.716678	0.0000001
161.3	15.0016	1.631	10.10101
87.006	16.1004	3.760701	1.000327

SECTION XXIII.

ADDITION OF DECIMALS.

1. Add 23.61 and 161.5 and 2.6789 and 61.111 and 27.0076 and 116.71 and 6151.7671 together.

OPERATION.

```
  23.61
 161.5
   2.6789
  61.111
  27.0076
 116.71
6151.7671
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6544.3846
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In this question, it will be perceived that tenths are written under tenths, hundredths under hundredths, &c.; and that the operation of addition is performed as in addition of whole numbers.

RULE. — *Write the numbers under each other according to their value, add as in whole numbers, and point off from the right hand as many places for decimals as there are in that number which contains the greatest number of decimals.*

2. Add together the following numbers: 81.61356, 6716.31, 413.1678956, 35.14671, 3.1671, 314.6. Ans. 7564.0052656.

3. What is the sum of the following numbers: 1121.6116, 61.87, 46.67, 165.13, 676.167895? Ans. 2071.449495.

4. Add 7.61, 637.1, 6516.14, 67.1234, 6.1234, together. Ans. 7234.0968.

5. Add 21.611, 6888.32, 3.6167, together. Ans. 6913.5477.

6. Add seventy-three and twenty-nine hundredths, eighty-seven and forty-seven thousandths, three thousand and five and one hundred six ten thousandths, twenty-eight and three hundredths, twenty-nine thousand and five thousandths, together. Ans. 32193.3826.

7. Add two hundred nine thousand and forty-six millionths, ninety-eight thousand two hundred seven and fifteen ten thousandths, fifteen and eight hundredths, and forty-nine ten thousandths, together. Ans. 307222.086446.

8. What is the sum of twenty-three million ten, one thousand and five hundred thousandths, twenty-seven and nineteen millionths, seven and five tenths? Ans. 23001044.500069.

9. Add the following numbers: fifty-nine and fifty-nine thousandths, twenty-five thousand and twenty-five ten thousandths, five and five millionths, two hundred five and five hundredths. Ans. 25269.111505.

10. What is the sum of the following numbers: twenty-five and seven millionths, one hundred forty-five and six hundred forty-three thousandths, one hundred seventy-five and eighty-nine hundredths, seventeen and three hundred forty-eight hundred thousandths? Ans. 363.536487.

SECTION XXIV.

SUBTRACTION OF DECIMALS.

RULE. — *Let the numbers be so written, that the separatrix of the subtrahend be directly under that of the minuend, subtract as in whole*

numbers, and point off as many places for decimals as there are in that number which contains the greatest number of decimals.

OPERATIONS.

1.	2	3.	4.
61.9634	39.3	5.	6.1
9.182	1.6789	1.678	1.99999
52.7814	37.6211	3.322	4.10001

5. From 41.7 take 21.9767. Ans. 19.7233.
6. From 29.167 take 19.66711. Ans. 9.49989.
7. From 91.61 take 2.6671. Ans. 88.9429.
8. From 96.71 take 96.709. Ans. .001.

9. Take twenty-seven and twenty-eight thousandths from ninety-seven and seven tenths. Ans. 70.672.

10. Take one hundred fifteen and seven hundredths from three hundred fifteen and twenty-seven ten thousandths. Ans. 199.9327.

11. From twenty-nine million four thousand and five take twenty-nine thousand and three hundred forty-nine thousand two hundred, and twenty-four hundred thousandths. Ans. 28625804.99976.

12. From one million take one millionth. Ans. 999999.999999.

SECTION XXV.

MULTIPLICATION OF DECIMALS.

EXAMPLES.	OPERATIONS.	VULG. FRAC.	DECIMALS.
1. Multiply $\frac{6}{10}$ by $\frac{3}{10}$.	$\frac{6}{10}\times\frac{3}{10}$	$=\frac{18}{100}$	$=.18$
2. Multiply $\frac{7}{10}$ by $\frac{8}{10}$.	$\frac{7}{10}\times\frac{8}{10}$	$=\frac{56}{100}$	$=.56$
3. Multiply $\frac{9}{10}$ by $\frac{9}{10}$.	$\frac{9}{10}\times\frac{9}{10}$	$=\frac{81}{100}$	$=.81$
4. Multiply $\frac{7}{10}$ by $\frac{7}{10}$.	$\frac{7}{10}\times\frac{7}{10}$	$=\frac{49}{100}$	$=.49$
5. Multiply $\frac{9}{100}$ by $\frac{4}{10}$.	$\frac{9}{100}\times\frac{4}{10}$	$=\frac{36}{1000}$	$=.036$
6. Multiply $\frac{2}{100}$ by $\frac{4}{100}$.	$\frac{2}{100}\times\frac{4}{100}$	$=\frac{8}{10000}$	$=.0008$
7. Multiply $\frac{7}{1000}$ by $\frac{9}{100}$.	$\frac{7}{1000}\times\frac{9}{100}$	$=\frac{63}{100000}$	$=.00063$
8. Multiply $\frac{769}{1000}$ by $\frac{436}{1000}$.	$\frac{769}{1000}\times\frac{436}{1000}$	$=\frac{335284}{1000000}$	$=.335284$
9. Multiply $\frac{671}{1000}$ by 32.	$\frac{671}{1000}\times\frac{32}{1}$	$=\frac{21472}{1000}$	$=21.472$
10. Multiply $\frac{7}{100}$ by 46.	$\frac{7}{100}\times\frac{46}{1}$	$=\frac{322}{100}$	$=3.22$

11. Multiply 76.81 by 3.2.

OPERATION.

```
  76.81
    3.2
  -----
  15362
 23043
 -------
 245.792
```

12. Multiply .1234 by .0046.

OPERATION.

```
    .1234
    .0046
    -----
     7404
    4936
 ---------
 .00056764
```

RULE. — *Multiply as in whole numbers, and point off as many figures for decimals in the product as there are decimals in the multiplicand and multiplier; but, if there should not be so many figures in the product as there are decimals in the multiplicand and multiplier, supply the defect by prefixing ciphers. See Example 12th.*

13. Multiply 61.76 by .0071. Ans. .438496.
14. Multiply .0716 by 1.326. Ans. .0949416.
15. Multiply .61001 by .061. Ans. .03721061.
16. Multiply 71.61 by 365. Ans. 26137.65.
17. Multiply .1234 by 1234. Ans. 152.2756.
18. Multiply 6.711 by 6543. Ans. 43910.073.
19. Multiply .0009 by .0009. Ans. .00000081.
20. Multiply forty-nine thousand by forty-nine thousandths. Ans. 2401.

21. What is the product of one thousand and twenty-five, multiplied by three hundred and twenty-seven ten thousandths? Ans. 33.5175.

22. What is the product of seventy-eight million two hundred five thousand and two, multiplied by fifty-three hundredths? Ans. 41448651.06.

23. Multiply one hundred and fifty-three thousandths by one hundred twenty-nine millionths. Ans. .000019737.

24. What is the product of fifteen thousand, multiplied by fifteen thousandths? Ans. 225.

25. What will 26.7 yards of cloth cost, at $ 5.75 a yard? Ans. $153.52,5.

26. What will 14.75 bushels of wheat cost, at $1.25 a bushel? Ans. $18.43,75.

27. What will 375.6 pounds of sugar cost, at $0.125 per pound? Ans. $ 46.95.

28. What will 26.58 cords of wood cost, at $ 5.625 a cord? Ans. $149.51,2½.

29. What will 28.75 tons of potash cost, at $125.78 per ton? Ans. $3616.17,5.

30. What will 369 gallons of molasses cost, at \$0.375 a gallon? Ans. \$138.37,5.

31. What will 97.48cwt. of hay cost, at \$1.125 per cwt.? Ans. \$109.66,5.

32. What will 63.5 bushels of corn be worth, at \$0.78 per bushel? Ans. \$49.53.

SECTION XXVI.

DIVISION OF DECIMALS.

EXAMPLES.	OPERATIONS.	VULG. FRAC.	DECIMALS.
1. Divide $\frac{8}{10}$ by $\frac{2}{10}$.	$\frac{8}{10}\times\frac{10}{2}$	$=\frac{80}{20}=\frac{40}{10}$	$=$ 4.
2. Divide $\frac{3}{10}$ by $\frac{6}{10}$.	$\frac{3}{10}\times\frac{10}{6}$	$=\frac{30}{60}=\frac{5}{10}$	$=$ .5
3. Divide $\frac{36}{100}$ by $\frac{4}{10}$.	$\frac{36}{100}\times\frac{10}{4}$	$=\frac{360}{400}=\frac{9}{10}$	$=$ .9
4. Divide $\frac{48}{10}$ by 4.	$\frac{48}{10}\times\frac{1}{4}$	$=\frac{48}{40}=\frac{12}{10}$	$=$ 1.2
5. Divide $\frac{96}{100}$ by $\frac{12}{100}$.	$\frac{96}{100}\times\frac{100}{12}$	$=\frac{9600}{1200}=\frac{80}{10}$	$=$ 8.
6. Divide $\frac{9}{100}$ by $\frac{9}{1000}$.	$\frac{9}{100}\times\frac{1000}{9}$	$=\frac{9000}{900}=\frac{10}{1}$	$=$ 10.

7. Divide 1.728 by 1.2. 8. Divide $\frac{1728}{1000}$ by $\frac{12}{10}$.

OPERATION BY DECIMALS.

```
1.2)1.728(1.44 Ans.
    12
    --
     52
     48
     --
      48
      48
      --
```

OPERATION BY VULGAR FRACTIONS.

$\frac{1728}{1000}\times\frac{10}{12}=\frac{17280}{12000}=\frac{1728}{1200}=\frac{144}{100}=1\frac{44}{100}$ [Ans.

RULE. — *Divide as in whole numbers, and point off as many decimals in the quotient as the number of decimals in the dividend exceed those of the divisor; but if the number of those in the divisor exceed that of the dividend, supply the defect by annexing ciphers to the dividend. And if the number of decimals in the quotient and divisor together are not equal to the number in the dividend, supply the defect by prefixing ciphers to the quotient.*

9. Divide 780.516 by 2.43. Ans. 321.2.
10. Divide 7.25406 by 9.57. Ans. .758.
11. Divide .21318 by .38. Ans. .561.
12. Divide 7.2091365 by .5201. Ans. 13.861+.
13. Divide 56.8554756 by .0759. Ans. 749.084.

14. Divide 30614.4 by .9567. Ans. 32000.
15. Divide .306144 by 9567. Ans. .000032.
16. Divide four thousand three hundred twenty-two and four thousand five hundred seventy-three ten thousandths by eight thousand and nine thousandths. Ans. .5403+.
17. Divide thirty-six and six thousand nine hundred forty-seven ten thousandths by five hundred and eighty-nine. Ans. .0623.
18. Divide three hundred twenty-three thousand seven hundred sixty-five by five millionths. Ans. 64753000000.
19. Divide 119109094.835 by 38123.45. Ans. 3124.3.
20. Divide 1191090.94835 by 3812345. Ans.
21. Divide 11910909483.5 by 38.12345. Ans.
22. Divide 11.9109094835 by 381234.5. Ans.
23. Divide 1191.09094835 by 3.812345. Ans.
24. Divide 11910909483.5 by .3812345. Ans.
25. Divide 1.19109094835 by 3.812345. Ans.
26. Divide .119109094835 by .3812345. Ans.

Section XXVII.

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its decimal.

1. Reduce $\frac{3}{8}$ to its decimal.

OPERATION.

$$\begin{array}{r} 8)\underline{3.000} \\ .375 \text{ Ans.} \end{array}$$

That the decimal .375 is equal to $\frac{3}{8}$ may be shown by writing it in a vulgar fraction and reducing it; thus, $\frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$ Ans.

RULE. — *Divide the numerator by the denominator, annexing one or more ciphers to the numerator, and the quotient will be the decimal required.*

NOTE. — It is not usually necessary that decimals should be carried to more than six places.

2. Reduce $\frac{5}{8}$ to a decimal. Ans. .625.
3. Reduce $\frac{1}{2}$ to a decimal. Ans. .5.
4. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{11}{12}$, $\frac{3}{16}$, $\frac{1}{25}$, and $\frac{1}{8}$ to decimals.
Ans. .666+, .75, .833+, .91666+, .1875, .04, .125.

5. Reduce $\frac{1}{17}$, $\frac{2}{27}$, $\frac{5}{37}$, $\frac{3}{431}$, $\frac{8}{111}$, and $\frac{1}{1234}$ to decimals.
Ans. .05882+, .07407+, .1351+, .00696+, .07207+, .0008103+.

CASE II.

To reduce denominate numbers to decimals.

1. Reduce 15s. 9¾d. to the decimal of a £. Ans. .790625.

OPERATION.

4	3.00
12	9.75000
20	15.81250
	.790625 Ans.

The 3 farthings are ¾ of a penny, and these reduced to a decimal are .75 of a penny, which we annex to the pence, and proceed in the same manner with the other terms.

RULE. — *Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest; opposite to each dividend, on the left hand, place such a number for a divisor, as will bring it to the next superior denomination, and draw a line between them. Begin at the highest, and write the quotient of each division, as decimal parts, on the right of the dividend next below it, and so on, till they are all divided; and the last quotient will be the decimal required.*

2. Reduce 9s. to the fraction of a pound. Ans. .45.
3. Reduce 15cwt. 3qr. 14lb. to the decimal of a ton. Ans. .79375.
4. Reduce 2qr. 21lb. 8oz. 12dr. to the decimal of a cwt. Ans. .6923828125.
5. Reduce 1qr. 3na. to the decimal of a yard. Ans. .4375.
6. Reduce 5fur. 35rd. 2yd. 2ft. 9in. to the decimal of a mile. Ans. .73603219+.
7. Reduce 3gal. 2qt. 1pt. of wine to the decimal of a hogs head. Ans. .0575396+.
8. Reduce 1pt. to the decimal of a bushel. Ans. .015625.
9. Reduce 2R. 16p. to the decimal of an acre. Ans. .6.

CASE III.

To find the decimal of any number of shillings, pence, and farthings, by *inspection.*

RULE. — *Write half the greatest even number of shillings for the first decimal figure, and if the number of shillings be odd, annex to the decimal the figure 5. Then write underneath the number of farthings contained in the given pence and farthings, setting the left-hand figure in the second place, if there be more than one figure, and the single figure*

in the third place, if there be but one, and increasing the number by 1 *when it exceeds* 12, *and by* 2 *when it exceeds* 36. *The sum of the whole will be the decimal required.*

EXAMPLES.

1. Find the decimal of 15s. $9\frac{3}{4}$d. by inspection.

.7	$= \frac{1}{2}$ of 14s.
.05	= for odd shilling.
39	= farthings in $9\frac{3}{4}$d.
2	= for excess of 36.
.791	

2. Find the value of 13s. $6\frac{3}{4}$d. by inspection. Ans. .678.
3. Find the value of 19s. $8\frac{1}{4}$d. by inspection. Ans. .984.
4. Value the following sums by inspection, and *find their total*: 19s. $11\frac{3}{4}$d., 16s. $9\frac{1}{4}$d., 1s. 11d., 3s. $0\frac{3}{4}$d., 17s. $5\frac{1}{2}$d., 13s. $4\frac{1}{2}$d., 18s. $8\frac{1}{2}$d., 19s. $11\frac{3}{4}$d., 13s. $3\frac{1}{4}$d., 16s. $0\frac{1}{4}$d., 17s. $7\frac{3}{4}$d.
Ans. 7.91£.

NOTE. — As shillings are so many twentieths of a pound, it *is evident*, that by taking one half of their number, we obtain their value in tenths or decimals of a pound. Thus, 16s. $= \frac{8}{10}$£.

In like manner, any number of farthings are so many nine *hundred sixtieths* of a pound. So that, in order to obtain their value in the denomination of pounds, we write the number of farthings for the numerator and 960 for a denominator, as 17 farthings $= \frac{17}{960}$£. But, in order *to treat this* fraction decimally, we must raise the denominator to 1000, which in the fraction $\frac{24}{960}$£. is done by adding 40 to the denominator and 1 to the numerator, and in the fraction $\frac{48}{960}$ by adding 2 to the numerator and *40 to the* denominator. Q. E. D.

CASE IV.

To find the value of a decimal in integral or whole numbers.

1. What is the value of .790625£. ?

OPERATION.

$$\begin{array}{r} .790625 \\ 20 \\ \hline 15.812500 \\ 12 \\ \hline 9.750000 \\ 4 \\ \hline 3.000000 \end{array}$$

Now it is evident, that .790625£. expressed in terms of a shilling must be the product of .790625£. multiplied by 20, and that *to continue* the reduction to the lowest terms we must multiply by the same number as in common reduction.

RULE. — *Multiply the given decimal by the number which will bring it to the next lower denomination, and cut off for a remainder as many places on the right as there are places in the given decimal.*

Multiply this remainder by the number which will bring it to the next lower denomination, cutting off for a remainder as before, and thus proceed till the reduction is carried to the denomination required. The several integral numbers, standing at the left hand, will be the answer sought in the different lower denominations.

2. What is the value of .625 of a shilling? Ans. $7\frac{1}{2}$d.
3. What is the value of .6725 of a cwt.? Ans. 2qr. 19lb. $5\frac{3}{25}$oz.
4. What is the value of .9375 of a yard? Ans. 3qr. 3na.
5. What is the value of .7895 of a mile? Ans. 6fur. 12rd. 10ft. $6\frac{18}{25}$in.
6. What is the value of .9378 of an acre? Ans. 3R. 30p. 13ft. $9\frac{99}{125}$in.
7. Reduce .5615 of a hogshead of wine to its value in gallons, &c. Ans. 35gal. 1qt. 0pt. $3\frac{123}{125}$gi.
8. Reduce .367 of a year to its value in days, &c. Ans. 134da. 1h. 7m. $19\frac{4}{5}$sec.
9. What is the value of .6923828125 of a cwt.? Ans. 2qr. 21lb. 8oz. 12dr.
10. What is the value of .015625 of a bushel? Ans. 1 pint.
11. What is the value of .55 of an ell English? Ans. 2qr. 3na.
12. What is the value of .6 of an acre? Ans. 2R. 16p.

Section XXVIII.

MISCELLANEOUS EXAMPLES.

1. What is the value of 7cwt. 2qr. 18lb. of sugar, at $ 11.75 per cwt.? Ans. $ 90.01,$3\frac{11}{28}$.
2. What cost 19cwt. 3qr. 14lb. of iron, at $ 9.25 per cwt.? Ans. $ 183.84,$3\frac{3}{4}$.
3. What cost 39A. 2R. 15p. of land, at $ 87.37,5 per acre? Ans. $ 3459.50,$3\frac{29}{32}$.
4. What would be the expense of making a turnpike 87m. 3fur. 15rd., at $ 578.75 per mile? Ans. $ 50595.41$\frac{1}{64}$.
5. What is the cost of a board 18ft. 9in. long, and 2ft. $3\frac{1}{2}$in. wide, at $ .05,3 per foot? Ans. $ 2.27,$7\frac{11}{32}$.
6. Goliath of Gath was $6\frac{1}{2}$ cubits high; what was his height in feet, the cubit being 1ft. 7.168in.? Ans. 10ft. 4.592in.

7. If a man travel 4.316 miles in an hour, how long would he be in travelling from Bradford to Boston, the distance being $29\frac{1}{2}$ miles? Ans. 6h. 50m. 6sec.+

8. What is the cost of 5yd. 1qr. 2na. of broadcloth, at \$ $5.62\frac{1}{2}$ per yard? Ans. \$ $30.23,4\frac{3}{8}$.

9. Bought 17 bags of hops, each weighing 4cwt. 3qr. 7lb., at \$ $5.87\frac{1}{2}$ per cwt.; what was the cost? Ans. \$ $480.64,8\frac{7}{16}$.

10. Purchased a farm, containing 176A. 3R. 25rd., at \$ $75.37\frac{1}{2}$ per acre; what did it cost? Ans. \$ $13334.30,8\frac{11}{16}$.

11. What cost 17625 feet of boards, at \$ 12.75 per thousand? Ans. \$ $224.71,8\frac{3}{4}$.

12. How many square feet in a floor 19ft. 3in. long, and 15ft. 9in. wide? Ans. 303ft. 27in.

13. How many square yards of paper will it take to cover a room 14ft. 6in. long, 12ft. 6in. wide, and 8ft. 9in. high? Ans. $52\frac{1}{2}$yd.

14. How many solid feet in a pile of wood 10ft. 7in. long, 4ft. wide, and 5ft. 10in. high? Ans. $246\frac{7}{18}$ft.

15. How many garments, each containing 4yd. 2qr. 3na., can be made from 112yd. 2qr. of cloth? Ans. 24.

16. Bought 1gal. 2qt. 1pt. of wine for \$ 1.82; what would be the price of a hogshead? Ans. \$ 70.56.

17. Bought $125\frac{1}{2}$yd. of lace for \$ 15.06; what was the price of 1 yard? Ans. \$ 0.12.

18. What cost 17cwt. 3qr. of wool, at \$ 35.75 per cwt.? Ans. \$ $634.56,2\frac{1}{2}$.

19. What cost 7hhd. 47gal. of wine, at \$ 87.25 per hogshead? Ans. \$ $675.84\frac{1}{63}$.

20. How many solid feet in a stick of timber 34ft. 9in. long, 1ft. 3in. wide, and 1ft. 6in. deep? Ans. 65.15625ft.

21. How many cwt. of coffee in $17\frac{3}{4}$ bags, each bag containing 2cwt. 1qr. 7lb.? Ans. 41cwt. 0qr. $5\frac{1}{4}$lb.

22. If 18yd. 1qr. of cloth cost \$ 36.50, what is the price of 1 yard? Ans. \$ 2.00.

23. If \$ 477.72 be equally divided among 9 men, what will be each man's share? Ans. \$ 53.08.

24. A man bought a barrel of flour for \$ 5.375, 7gal. of molasses for \$ 1.78, 9gal. of vinegar for \$ 1.1875, 1gal. of wine for \$ 1.125, 14lb. of sugar for \$ 1.275, and 5lb. of tea, for \$ 2.625; what did the whole amount to? Ans. \$ $13.36,7\frac{1}{2}$.

25. A man purchased 3 loads of hay; the first contained $2\frac{3}{8}$ tons, the second $3\frac{1}{8}$ tons, and the third $1\frac{1}{16}$ tons; what was the value of the whole, at \$ 17.625 a ton? Ans. \$ $128.88,2\frac{13}{16}$.

26. At \$ 13.625 per cwt., what cost 3cwt. 2qr. 7lb. of sugar? Ans. \$ 48.53,9$\frac{1}{16}$.

27. At \$125.75 per acre, what cost 37A. 3R. 35rd.? Ans. \$ 4774.57,0$\frac{5}{16}$.

28. At \$ 11.25 per cwt., what cost 17cwt. 2qr. 21lb. of rice? Ans. \$198.98,4$\frac{3}{8}$.

29. What cost 7$\frac{3}{4}$ bales of cotton, each weighing 3.37cwt., at \$ 9.37$\frac{1}{2}$ per cwt.? Ans. \$ 244.85,1$\frac{9}{16}$.

30. What cost 7hhd. 49gal. of wine, at \$ 97.625 per hogshead? Ans. \$ 759.30,5$\frac{35}{63}$.

31. What cost 7yd. 3qr. 3na. of cloth, at \$ 4.75 per yard? Ans. \$ 37.70,3$\frac{1}{8}$.

32. What cost 27T. 15cwt. 1qr. 3$\frac{1}{2}$lb. of hemp, at \$183.62 per ton? Ans. \$ 5098.03,7$\frac{5}{32}$.

33. What is the cost of constructing a railroad 17m. 3fur. 15rd., at \$1725.87,5 per mile? Ans. \$ 30067.97,8$\frac{33}{64}$.

34. When \$ 624.53125 are paid for 17A. 3R. 15p. of land, what is the cost of one acre? Ans. \$ 35.

35. Paid \$ 494.53125 for 19T. 15cwt. 2qr. 14lb. of hay; what was the cost per ton? Ans. \$ 25.

36. How much land, at \$ 40 per acre, can be obtained for \$1004.75? Ans. 25A. 0R. 19p.

37. Bought of Queen Victoria 9 acres of land, for which I paid 157.753125£. Required the price per acre. Ans. 17£. 10s. 6$\frac{3}{4}$d.

38. If \$198.984375 are paid for 17cwt. 2qr. 21lb. of rice, what is the value of 1cwt.? Ans. \$11.25.

Section XXIX.

EXCHANGE OF CURRENCIES.

It is well known, that, in different States of the Union, the American dollar has a different value as expressed in shillings and pence. The origin of this difference is thus explained: Previous to the formation of the Constitution, all accounts in this country were kept in the currency of Great Britain, and the dollar was reckoned at 4s. 6d. sterling. Owing, however, to the want of money, several States under the colonial government issued Bills of Credit, which were not received by the

British merchants in payment for goods at their par value. Holders of those bills were therefore obliged to pay a larger nominal amount than though they had paid in sterling. Thus eight shillings in the bills of New York passed for one dollar, or 4s. 6d. sterling. In the bills of the New England Colonies, where the depreciation was less, six shillings made a dollar, and in South Carolina and Georgia, four shillings and eight pence. In the ordinary reckonings of the people, shillings and pence are still considerably used, and their estimated value in different States is as follows.

In New England, Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, Mississippi, Texas, Alabama, and Florida, the dollar is valued at 6 shillings, $\$ 1 = \frac{6}{20}£. = \frac{3}{10}£.$

In New York, Ohio, and Michigan, the dollar is valued at 8 shillings, $\$ 1 = \frac{8}{20}£. = \frac{2}{5}£.$

In New Jersey, Pennsylvania, Delaware, and Maryland, the dollar is considered 7 shillings and 6 pence, $\$ 1 = \frac{90}{240}£. = \frac{3}{8}£.$

In North Carolina the dollar is reckoned at 10 shillings, $\$ 1 = \frac{10}{20}£. = \frac{1}{2}£.$

In South Carolina and Georgia 4 shillings 8 pence is the value of a dollar, $\$ 1 = \frac{56}{240}£. = \frac{7}{30}£.$

In Canada and Nova Scotia the dollar is valued at 5 shillings, $\$ 1 = \frac{5}{20}£. = \frac{1}{4}£.$

The following table exhibits the legal rates of interest in the United States, and the penalty of usury.

STATES.	RATE OF INTEREST.	PENALTY OF USURY.
Maine,	6 pr. ct.	Forfeit of the debt or claim.
N. Hampshire,	6 "	Forfeit of threefold the usury.
Vermont,	6 "	Recovery in an action, with costs.
Massachusetts,	6 "	Forfeit of threefold the usury.
Rhode Island,	6 "	Forfeit of the usury and interest on the debt.
Connecticut,	6 "	Forfeit of the whole debt.
New York,	7 "	Usurious contracts void.
New Jersey,	6 "	Forfeit of the whole debt.
Pennsylvania,	6 "	Forfeit of the whole debt.
Delaware,	6 "	Forfeit of the whole debt.
Maryland,	6 "	On tobacco contracts, 8 per ct. Usurious contracts void.
Virginia,	6 "	Forfeit double the usury.
North Carolina,	6 "	Contracts for usury void. Forfeit double the usury.
South Carolina,	7 "	Forfeit of interest and usury, with costs.

STATES.	RATE OF INTEREST.	PENALTY OF USURY.
Georgia,	8 per ct.	Forfeit of three times the usury, and contracts void.
Alabama,	8 "	Forfeit of interest and usury.
Mississippi,	8 "	By contract as high as 10. Recovery in action of debt.
Louisiana,	5 "	Bank 6; by agreement as high as 10; contracts void.
Tennessee,	6 "	Contracts void.
Kentucky,	6 "	Recovery with costs.
Ohio,	6 "	Contracts void.
Indiana,	6 "	A fine of double the usury.
Illinois,	6 "	By agreement as high as 12. Forfeit of threefold whole amount of interest.
Missouri,	6 "	By agreement as high as 10. Forfeit of the interest and usury.
Michigan,	7 "	Forfeit of the usury and one fourth the debt.
Arkansas,	6 "	By agreement as high as 10. Usury recoverable; contracts void.
Florida,	8 "	Forfeit of interest and usury.
Wisconsin,	7 "	By agreement as high as 12. Forfeit treble the excess.
Iowa,	7 "	By agreement as high as 12. Forfeit treble the excess.
Texas,	8 "	
Dist. Columbia,	6 "	Contracts void.

NOTE.—On debts or judgments in favor of the *United States*, interest is computed at the rate of 6 per cent. per annum.

In order, therefore, to change the preceding currencies to United States money, the shillings, pence, and farthings, if there be any, must first be reduced to decimals of a pound, and annexed to the pounds.

RULE.—*Divide the pounds by the value of a dollar in the given currency,* EXPRESSED BY A FRACTION OF A POUND; *that is, to change the old New England currency to United States money, divide by $\frac{3}{10}$; because 6 shillings is $\frac{3}{10}$ of a pound.*

To change the old currency of New York, &c., to United States money, divide by $\frac{4}{10}$; because 8 shillings is $\frac{4}{10}$ of a pound.

To change the old currency of Pennsylvania, &c., to United States money, divide by $\frac{3}{8}$; because 7 shillings and 6 pence is $\frac{3}{8}$ of a pound.

To change the old currency of South Carolina and Georgia to United States money, divide by $\frac{7}{30}$: because 4 shillings and 8 pence is $\frac{7}{30}$ of a pound.

To change Canada and Nova Scotia currency to United States money, divide by $\frac{1}{4}$; because 5 shillings is $\frac{1}{4}$ of a pound.

The old method of changing English sterling money to United States money was, to divide the pounds by $\frac{2}{10}$, and the quotient was dollars; and, to change dollars into English sterling money, to multiply the dollars by $\frac{2}{10}$, and the product was pounds sterling. But, as will be seen by a note on page 151, this process does not give the present value of a pound sterling.

EXAMPLES.

1. Change 18£. 4s. 6d. of the old New England currency to United States money.

$$18.225£. \div \tfrac{3}{10} = \$\,60.75 \text{ Ans.}$$

In this example we reduce the 4 shillings and 6 pence to a decimal of a pound, which we find to be .225. This decimal we annex to the pounds, and multiply the 18.225£. by 10, and divide by 3, and it produces the answer, $ 60.75. The reason for this process has already been shown.

2. Change $ 60.75 to the old currency of New England.

$$\$\,60.75 \times \tfrac{3}{10} = 18.225 = 18£.\ 4s.\ 6d. \text{ Ans.}$$

The decimal .225 is reduced to shillings and pence by Case IV. of Decimal Fractions.

3. Change 78£. 7s. 6d. of the old currency of New England to United States money. Ans. $ 261.25.

4. Change $ 261.25 to the old currency of New England. Ans. 78£. 7s. 6d.

5. Change 46£. 16s. 6d. of the old currency of New York to United States money. Ans. $117.06¼.

6. Change $117.06¼ to the old currency of New York. Ans. 46£. 16s. 6d.

7. Change 387£. of the old currency of Pennsylvania to United States money. Ans. $1032.

8. Change $1032 to the old currency of Pennsylvania, Delaware, and Maryland. Ans. 387£.

9. Change 12£. 12s. of the old currency of South Carolina and Georgia to United States money. Ans. $ 54.

10. Change $ 54 to the old currency of South Carolina and Georgia. Ans. 12£. 12s.

11. Change 128£. 18s. 6d. of Canada and Nova Scotia to United States money. Ans. $ 515.70.

12. Change $ 515.70 to Canada and Nova Scotia currency. Ans. 128£. 18s. 6d.

NOTE. — From time immemorial $ $\frac{40}{9}$ has been given in all our arithmetics as the value of the pound sterling in United States Money. It is time the *error* was corrected.

The *nominal par* of exchange with London, as expressed in reports of exchange, is 109.496+, or very nearly 109½, being 9½ above the computed par of $ 4.444$\frac{4}{9}$, represented by 100.

The Bank of England was established in 1694, by a company who advanced a loan of £1,200,000 sterling to government. Specie payment was suspended in 1797, and resumed, by act of Parliament, May 1, 1823.

The term *Sterling* is derived from the *Easterlings*, who were expert refiners from the eastern part of Germany, who came into England and first established the standard proportion of silver, — 11oz. 2dwt. fine silver, and 18dwt. alloy. The first sterling was coined in 1216. In the reign of Charles the Second (1666) a new gold coinage was minted, called *Guineas*, from the country from which the gold was originally brought. In 1816 (150 years after) the guinea was superseded by a new coin, called the *sovereign*, which represents the pound sterling. The guinea (old coinage) weighs 129$\frac{39}{89}$gr., standard. The sovereign (new coinage) weighs 123$\frac{171}{623}$gr., standard. The standard legal fineness of gold in England is 22 carats, or $\frac{916\frac{2}{3}}{1000}$. The standard of silver is 11oz. 2dwt. = $\frac{37}{40}$ = $\frac{925}{1000}$. The sovereign contains precisely 113$\frac{1}{623}$gr. of pure gold.

The coinage of the United States is regulated by Congress. By the last act of Congress, January 18, 1837, the standard for both gold and silver was fixed at $\frac{900}{1000}$, — that is, suppose any of our gold or silver coin to be divided into 1000 equal parts, 900 of those parts are pure gold or silver, and 100 parts are alloy.

By this act the eagle weighs 258gr. Troy, *standard*.
Containing, . . . 232.2gr. pure gold,
And . . . . 25.8gr. alloy.

Our gold coinage, then, is 21$\frac{3}{5}$ carats fine; our silver coinage is 10oz. 16dwt. fine, — 6dwt. short of sterling fineness, which is 11oz. 2dwt. The American dollar weighs 412½gr., standard, containing 371¼gr. pure silver, and 41¼gr. alloy. The alloy in our gold coins is mostly silver, and in our silver coin it is copper.

The ratio of gold to silver in our coinage is 15$\frac{2295}{2322}$ to 1, — that is, whatever an ounce of silver may be worth, an ounce of gold is worth 15$\frac{2295}{2322}$ times as much.

The pound sterling under the above act, as represented by the *sovereign* of *legal* weight and fineness, is $ $\frac{3520000}{723303}$ exactly, = $4.866+, which is the real gold par with London.

TO REDUCE STERLING MONEY TO UNITED STATES MONEY.

RULE. — *Express the shillings, pence, and farthings decimally; then multiply sterling by* $\frac{3520000}{723303}$, *and the product will be dollars, &c.*

NOTE. — This rule supposes the sovereign, which represents the pound sterling, to be $\frac{916\frac{2}{3}}{1000}$ fine, and to contain 113$\frac{1}{623}$gr. of pure gold. But the sovereign falls a little short of its legal weight and fineness. So that its

real value in our currency does not vary essentially from $4.84. This is the value assigned to it by act of Congress, in calculating *ad valorem* duties in our custom-houses on goods imported from England, which are invoiced in sterling money. Therefore, multiply sterling by $4.84 and we shall have the custom-house and market *par* value of sovereigns or pounds sterling.

N. B. — $$4.444\frac{4}{9}$ never represented the true value of the pound sterling in the United States currency.

Under the act of Congress of the 2d of April, 1792, establishing the mint and regulating the coins of the United States, the value of the pound sterling was $4.56+.

By the act of Congress of the 28th of June, 1834, called the *Gold Bill*, the value of the pound or sovereign was $$4.87\frac{1371}{1503}$. By the act of Congress of the 18th of January, 1837, supplementary to the act of 1834, the value of the pound sterling becomes $$\frac{3520000}{723303}+ = \$4.86\frac{474743}{723303}-$. Sovereigns are usually valued at $4.85 at the banks.

SECTION XXX.

INFINITE OR CIRCULATING DECIMALS.*

DEFINITIONS.

1. DECIMALS produced from Vulgar Fractions, whose denominators do not measure their numerators, and distinguished by the continual repetition of the same figure or figures, are called *infinite or circulating decimals.*

2. The circulating figures, that is, those that are continually repeated, are called *repetends.* If only the same figure is repeated, it is called a *single repetend*, as .11111 or .5555, and is expressed by writing the figure repeated with a point over it. Thus .11111 is denoted by $.\dot{1}$, and .5555 by $.\dot{5}$.

3. If the *same* figures circulate alternately, it is called a *compound repetend*, as .475475475, and is distinguished by putting a point over the first and last repeating figures; thus, .475475475 is written $.\dot{4}7\dot{5}$.

4. When other figures arise before those which circulate, it is called a *mixed repetend*; as $.12\dot{4}\dot{6}$, or $.17\dot{8}3\dot{5}$.

5. *Similar repetends* begin at the same place; as $.\dot{3}$ and $.\dot{6}$; or $5.1\dot{2}\dot{3}$ and $3.4\dot{7}\dot{8}$.

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* Infinite or circulating decimals being less important for use than many other rules, and somewhat difficult in their operation, the student can omit them until he reviews the Arithmetic.

6. *Dissimilar repetends* begin at different places; as $.\dot{9}8\dot{6}$ and $.46\dot{2}\dot{5}$.

7. *Conterminous repetends* end at the same place; as $.6\dot{3}\dot{1}$ and $.\dot{4}6\dot{5}$.

8. *Similar and conterminous repetends* begin and end at the same place; as $.1\dot{7}2\dot{8}$ and $.4\dot{9}8\dot{7}$.

REDUCTION OF CIRCULATING DECIMALS.

CASE I.

To reduce a simple repetend to its equivalent vulgar fraction.

If a unit with ciphers annexed to it be divided by 9 *ad infinitum*, the quotient will be one continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $.\dot{1}$; and since $.\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $.\dot{2}$ will be equivalent to $\frac{2}{9}$, $.\dot{3}$ to $\frac{3}{9}$, and so on, till $.\dot{9}$ is equal to $\frac{9}{9}$ or 1. Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure, and denominator 9. Again, $\frac{1}{99}$, or $\frac{1}{999}$, being reduced to decimals, makes .01010101, and .001001001 *ad infinitum*, $= .\dot{0}\dot{1}$ and $.\dot{0}0\dot{1}$; that is, $\frac{1}{99} = .\dot{0}\dot{1}$, and $\frac{1}{999} = .\dot{0}0\dot{1}$; consequently $\frac{2}{99} = .\dot{0}\dot{2}$, and $\frac{2}{999} = .\dot{0}0\dot{2}$; and, as the same will hold universally, we deduce the following

RULE. — *Make the given decimal the numerator, and let the denominator be a number consisting of as many nines as there are recurring places in the repetend.*

If there be integral figures in the circulate, as many ciphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

EXAMPLES.

1. Required the least vulgar fraction equal to $.\dot{6}$ and $.\dot{1}2\dot{3}$.

$.\dot{6} = \frac{6}{9} = \frac{2}{3}$ Ans. $\qquad .\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}$ Ans.

2. Reduce $.\dot{3}$ to its equivalent vulgar fraction. Ans. $\frac{1}{3}$.

3. Reduce $\dot{1}.6\dot{2}$ to its equivalent vulgar fraction. Ans. $1\frac{23}{37}$.

4. Reduce $.\dot{7}6923\dot{0}$ to its equivalent vulgar fraction. Ans. $\frac{10}{13}$.

CASE II.

To reduce a mixed repetend to its equivalent vulgar fraction.

1. What vulgar fraction is equivalent to $.13\dot{8}$?

OPERATION.

$$.13\dot{8} = \tfrac{13}{100} + \tfrac{8}{900} = \tfrac{117}{900} + \tfrac{8}{900} = \tfrac{125}{900} = \tfrac{5}{36} \text{ Ans.}$$

As this is a mixed circulate, we divide it into its finite and circulating parts; thus $.13\dot{8} = .13$, the finite part, and $.00\dot{8}$ the repetend or circulating part; but $.13 = \frac{13}{100}$; and $.00\dot{8}$ would be equal to $\frac{8}{9}$, if the circulate began immediately after the place of units; but, as it begins after the place of hundreds, it is $\frac{8}{9}$ of $\frac{1}{100} = \frac{8}{900}$. Therefore $.13\dot{8} = \frac{13}{100} + \frac{8}{900} = \frac{117}{900} + \frac{8}{900} = \frac{125}{900} = \frac{5}{36}$ Ans. Q. E. D.

RULE. — *To as many nines as there are figures in the repetend, annex as many ciphers as there are finite places for a denominator; multiply the nines in the denominator by the finite part, and add the repeating decimal to the product for the numerator. If the repetend begins in some integral place, the finite value of the circulating must be added to the finite part.*

2. What is the least vulgar fraction equivalent to $.5\dot{3}$?
Ans. $\frac{8}{15}$.

3. What is the least vulgar fraction equivalent to $.\dot{5}92\dot{5}$?
Ans. $\frac{16}{27}$.

4. What is the least vulgar fraction equivalent to $.008\dot{4}9713\dot{3}$?
Ans. $\frac{83}{9768}$.

5. What is the finite number equivalent to $3\dot{1}.6\dot{2}$?
Ans. $31\frac{23}{37}$.

CASE III.

To make any number of dissimilar repetends similar and conterminous.

1. Dissimilar made similar and conterminous.

OPERATION.

$$\begin{aligned}
9.1\dot{6}\dot{7} &= 9.61\dot{7}6767\dot{6} \\
14.6 &= 14.6\dot{0}00000\dot{0} \\
3.16\dot{5} &= 3.16\dot{5}5555\dot{5} \\
12.\dot{4}3\dot{2} &= 12.43\dot{2}4324\dot{3} \\
8.\dot{1}8\dot{1} &= 8.18\dot{1}8181\dot{8} \\
1.\dot{3}0\dot{7} &= 1.30\dot{7}3073\dot{0}
\end{aligned}$$

Any given repetend whatever, whether single, compound, pure, or mixed, may be transformed into another repetend, that shall consist of an equal or greater number of figures at pleasure; thus $.\dot{4}$ may be changed into $.\dot{4}\dot{4}$ or $.\dot{4}4\dot{4}$; and $.\dot{2}\dot{9}$ into $.\dot{2}92\dot{9}$ or $2\dot{9}2\dot{9}$. And as some of the circulates in this question consist of one, some of two, and others of three places; and as the least common multiple of 1, 2, and 3

is 6, we know that the *new* repetend will consist of 6 places, and will begin just so far from unity as is the farthest among the dissimilar repetends, which, in the present example, is the third place.

RULE. — *Change the given repetends into other repetends, which shall consist of as many figures as the least common multiple of the several number of places found in all the repetends contains units.*

2. Make $3.\dot{6}7\dot{1}$, $1.\dot{0}07\dot{1}$, $8.\dot{5}\dot{2}$, and $7.\dot{6}1632\dot{5}$ similar and con terminous.

3. Make $1.\dot{5}\dot{2}$, $8.\dot{7}15\dot{6}$, $3.56\dot{7}$, and $1.3\dot{7}\dot{8}$ similar and conterminous.

4. Make $.00\dot{0}\dot{7}$, $.141\dot{4}1\dot{4}$, and $887.\dot{1}$ similar and conterminous.

CASE IV.

To find whether the decimal fraction equal to a given vulgar fraction be finite or infinite, and of how many places the repetend will consist.

RULE. — *Reduce the given fraction to its least terms, and divide the denominator by* 2, 5, *or* 10, *as often as possible. If the whole denominator vanish in dividing by* 2, 5, *or* 10, *the decimal will be finite, and will consist of so many places as you perform divisions. If it do not vanish, divide* 9999, *&c., by the result till nothing remain, and the number of* 9*'s used will show the number of places in the repetend; which will begin after so many places of figures as there are* 10*'s,* 2*'s, or* 5*'s used in dividing.*

NOTE. — In dividing 1.0000, &c., by any prime number whatever, except 2 or 5, the quotient will begin to repeat as soon as the remainder is 1. And since 9999, &c., is less than 10000, &c., by 1, therefore 9999, &c., divided by any number whatever, will leave a 0 for a remainder, when the repeating figures are at their period. Now whatever number of repeating figures we have when the dividend is 1, there will be exactly the same number when the dividend is any other number whatever. For the product of any circulating number by any other given number will consist of the same number of repeating figures as before. Thus, let .378137813781, &c., be a circulate, whose repeating part is 3781. Now every repetend (3781), being equally multiplied, must produce the same product. For these products will consist of more places, yet the overplus in each, being alike, will be carried to the next, by which means each product will be equally increased, and consequently every four places will continue alike. And the same will hold for any other number whatever. Hence it appears, that the dividend may be altered at pleasure, and the number of places in the repetend will be still the same; thus, $\frac{1}{11} = .\dot{0}\dot{9}$, and $\frac{3}{11} = .\dot{2}\dot{7}$, where the number of places in each are alike; and the same will be true in all cases.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{210}{1120}$ be finite or infinite; and if infinite, of how many places the repetend will consist.

$\frac{210}{1120} = {}_{2)}\frac{3}{16} = \overset{(2)}{8} = \overset{(2)}{4} = \overset{(2)}{2} = 1$; therefore, because the denominator vanishes in dividing, the decimal is finite, and consists of four places; thus, $16)\frac{3.0000}{.1875}$.

2. Required to find whether the decimal equal to $\frac{475}{2800}$ be finite or infinite; and, if infinite, of how many places that repetend will consist.

$\frac{475}{2800} = \frac{19}{112}$ $2)112 = \overset{(2)}{56} = \overset{(2)}{28} = \overset{(2)}{14} = 7$. Thus, $7)\frac{999999}{142857}$; therefore, because the denominator, 112, did not vanish in dividing by 2, the decimal is infinite; and as six 9's were used, the circulate consists of six places, beginning at the fifth place, because four 2's were used in dividing.

3. Let $\frac{1}{11}$ be the fraction proposed.

4. Let $\frac{107}{253}$ be the fraction proposed.

SECTION XXXI.

ADDITION OF CIRCULATING DECIMALS.

EXAMPLE.

1. Let $3.\dot{5} + 7.\dot{6}5\dot{1} + 1.\dot{7}6\dot{5} + 6.1\dot{7}\dot{3} + 51.\dot{7} + 3.7 + 27.\dot{6}3\dot{1}$ and $1.\dot{0}0\dot{3}$ be added together.

OPERATION.

Dissimilar.	Similar and Conterminous.
$3.\dot{5}$	$= 3.5\dot{5}55555\dot{5}$
$7.\dot{6}5\dot{1}$	$= 7.6\dot{5}1651\dot{6}$
$1.\dot{7}6\dot{5}$	$= 1.7\dot{6}5765\dot{7}$
$6.1\dot{7}\dot{3}$	$= 6.1\dot{7}3737\dot{3}$
$51.\dot{7}$	$= 51.7\dot{7}7777\dot{7}$
3.7	$= 3.7\dot{0}0000\dot{0}$
$27.\dot{6}3\dot{1}$	$= 27.6\dot{3}1631\dot{6}$
$1.\dot{0}0\dot{3}$	$= 1.0\dot{0}3003\dot{0}$
	$103.2\dot{5}9122\dot{7}$

Having made all the numbers similar and conterminous by Sect. XXX., Case III., we add the first six columns, as in Simple Addition, and find the sum to be $3591224 = \frac{3591224}{999999} = 3.\dot{5}9122\dot{7}$. The repeating decimals $.\dot{5}9122\dot{7}$ we write in their proper place, and carry 3 to the next column, and then proceed as in whole numbers.

RULE. — *Make the repetends similar and conterminous, and find their sum, as in common Addition. Divide this sum by as many 9's as there are places in the repetend, and the remainder is the repetend of the sum, which must be set under the figures added, with ciphers on the left when it has not so many places as the repetends. Carry the quotient of this division to the next column, and proceed with the rest as with finite decimals.*

2. Add $2\dot{7}.5\dot{6} + 5.\dot{6}3\dot{2} + 6.\dot{7} + 16.3\dot{5}\dot{6} + .7\dot{1}$ and $6.\dot{1}23\dot{4}$ together. Ans. $63.\dot{1}6906708688\dot{8}$.

3. Add $2.7\dot{6}\dot{5} + 7.1\dot{6}67\dot{4} + \dot{3}.67\dot{1} + .\dot{7}$ and $.\dot{1}72\dot{8}$ together. Ans. $14.5\dot{5}43\dot{6}$.

4. Add $5.\dot{1}634\dot{5} + 8.\dot{6}98\dot{1} + 3.\dot{7}\dot{5}$ together. Ans. $17.\dot{5}59191208473740903\dot{0}\dot{2}$.

5. Reduce the following numbers to decimals, and find their sum : $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{9}$. Ans. $.\dot{5}8730\dot{1}$.

SECTION XXXII.

SUBTRACTION OF CIRCULATING DECIMALS.

EXAMPLE.

1. From $87.16\dot{4}\dot{5}$ take $19.47\dot{9}16\dot{7}$.

OPERATION.

$$\begin{array}{rcr} 87.16\dot{4}\dot{5} & = & 87.16\dot{4}54\dot{5} \\ 19.47\dot{9}16\dot{7} & = & \underline{19.47\dot{9}16\dot{7}} \\ & & 67.68\dot{5}37\dot{7} \end{array}$$

Having made the numbers similar and conterminous, we subtract as in whole numbers, and find the remainder of the circulate to be 5378, from which we subtract 1, and write the remainder in its place, and proceed with the other part of the question as in whole numbers. The reason why 1 should be added to the repetend may be shown as follows. The minuend may be considered $16\frac{4545}{9999}$, and the subtrahend $7\frac{9167}{9999}$; we then proceed with these numbers as in Case II. of Subtraction of Vulgar Fractions; and the numerator 5377 will be the repeating decimal. Q. E. D.

OPERATION.

$$\begin{array}{r} 16\frac{4545}{9999} \\ \underline{7\frac{9167}{9999}} \\ 8\frac{5377}{9999} \end{array}$$

RULE. — *Make the repetends similar and conterminous, and subtract as usual; observing, that if the repetend of the subtrahend be greater than the repetend of the minuend, then the remainder on the right must be less by unity than it would be if the expressions were finite.*

2. From $7.\dot{1}$ take $5.0\dot{2}$. Ans. $2.0\dot{8}$.
3. From $31\dot{5}.8\dot{7}$ take $78.\dot{0}37\dot{8}$. Ans. $237.\dot{8}3807209549\dot{7}$.
4. Subtract $\frac{1}{7}$ from $\frac{2}{9}$. Ans. $.\dot{0}7936\dot{5}$.
5. From $16.13\dot{4}\dot{7}$ take $11.08\dot{8}\dot{4}$. Ans. $5.04\dot{6}\dot{2}$.
6. From $18.16\dot{7}\dot{8}$ take $3.\dot{2}\dot{7}$. Ans. $14.89\dot{5}\dot{1}$.
7. From $3.\dot{1}2\dot{3}$ take $0.\dot{7}\dot{1}$. Ans. $2.\dot{4}0595\dot{1}$.
8. From $\frac{3}{7}$ take $\frac{2}{11}$. Ans. $.\dot{2}4675\dot{3}$.
9. From $\frac{4}{9}$ take $\frac{2}{7}$. Ans. $.\dot{1}5873\dot{0}$.
10. From $\frac{9}{17}$ take $\frac{6}{17}$. Ans. $.\dot{1}76470588235294\dot{1}$.
11. From $5.\dot{1}234\dot{5}$ take $2.3\dot{5}2345\dot{6}$.
Ans. $2.7\dot{7}110558216669277777\dot{9}888885999999\dot{4}$.

SECTION XXXIII.

MULTIPLICATION OF CIRCULATING DECIMALS.

1. Multiply $.\dot{3}\dot{6}$ by $.2\dot{5}$.

First Method.

OPERATION.

$.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$; $.2\dot{5} = \frac{2}{10} + \frac{5}{90} = \frac{23}{90}$.

$\frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = .0\dot{9}2\dot{9}$ Answer.

In the first method, we reduce the numbers to vulgar fractions, and then multiply and reduce them.

2. Multiply $582.\dot{3}4\dot{7}$ by $.08$.

Second Method.

OPERATION.

$582.\dot{3}4\dot{7} \times .08 = 46.58\dot{7}7\dot{8}$ Answer.

In the second method, we multiply as in whole numbers, but we add two units to the product; for $8 \times 347 = 2776 = \frac{2776}{999} = 2\frac{778}{999}$. Thus we see the repeating number is $\dot{7}7\dot{8}$.

RULE. — *Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual. Then change the vulgar fraction expressing the product into an equivalent decimal, and it will be the product required. But, if the multiplicand* ONLY *has a repetend, multiply as in whole numbers, and add to the right-hand place of the product as many units as there are tens in the product of the left-hand place of the repetend. The product will then contain a repetend whose places are equal to those in the multiplicand.*

3. Multiply $87.32\dot{5}8\dot{6}$ by 4.37. Ans. $381.6140\dot{3}3\dot{8}$.
4. Multiply $3.1\dot{4}\dot{5}$ by $4.\dot{2}9\dot{7}$. Ans. $13.5\dot{1}6953\dot{3}$.
5. What is the value of $.\dot{2}8571\dot{4}$ of a guinea? Ans. 8s.
6. What is the value of $.461607\dot{1}4285\dot{7}$ of a ton?
Ans. 9cwt. 0qr. 26lb.
7. What is the value of $.2\dot{8}493150\dot{6}$ of a year?
Ans. 104da.

SECTION XXXIV.

DIVISION OF CIRCULATING DECIMALS.

1. Divide $.\dot{5}\dot{4}$ by $.1\dot{5}$.

OPERATION.

$.\dot{5}\dot{4} = \frac{54}{99} = \frac{6}{11}$.
$.1\dot{5} = \frac{1}{10} + \frac{5}{90} = \frac{14}{90} = \frac{7}{45}$.
$\frac{6}{11} \div \frac{7}{45} = \frac{6}{11} \times \frac{45}{7} = \frac{270}{77}$.
$\frac{270}{77} = 3\frac{39}{77} = 3.\dot{5}0649\dot{3}$ Ans.

Having reduced the numbers to vulgar fractions, we divide one by the other, and change the quotient to a decimal.

RULE. — *Change both the divisor and the dividend into their equivalent vulgar fractions, and find their quotient as usual. Change the vulgar fraction expressing the quotient into its equivalent decimal, and it will be the quotient required.*

2. Divide $345.\dot{8}$ by $.\dot{6}$. Ans. $518.8\dot{3}$.
3. Divide $234.\dot{6}$ by $.\dot{7}$. Ans. $301.\dot{7}1428\dot{5}$.
4. Divide $.\dot{3}\dot{6}$ by $.2\dot{5}$. Ans. $1.\dot{4}22924901185770750988\dot{1}$.

SECTION XXXV.

MENTAL OPERATIONS IN FRACTIONS, &c.

IF any number be divided into two equal parts, and into two unequal parts, the product of the two unequal parts together with the square of half the difference of the two unequal parts is equal to the square of one of the equal parts. Also,

The product of any two numbers is equal to the square of

half their sum, less the square of half their difference. See Euclid's Elements, Book Second, Proposition Fifth.

NOTE. — A number is said to be *squared* when it is multiplied by itself; thus, the square of 5 is $5 \times 5 = 25$.

From the above proposition we deduce the following rules.

To multiply any number containing a half by itself.

RULE 1. — *Multiply the whole number given in the question by the next larger whole number, and to the product add the square of the half* $= \frac{1}{4}$.

1. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$.

OPERATION.

$$5 \times 6 = 30;\ \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4};\ 30 + \tfrac{1}{4} = 30\tfrac{1}{4} \text{ Ans.}$$

NOTE. — The whole number given is 5, and the next larger whole number is 6.

2. Multiply $7\frac{1}{2}$ by $7\frac{1}{2}$.	Ans. $56\frac{1}{4}$.
3. Multiply $3\frac{1}{2}$ by $3\frac{1}{2}$.	Ans. $12\frac{1}{4}$.
4. Multiply $9\frac{1}{2}$ by $9\frac{1}{2}$.	Ans. $90\frac{1}{4}$.
5. Multiply $11\frac{1}{2}$ by $11\frac{1}{2}$.	Ans. $132\frac{1}{4}$.
6. Multiply $20\frac{1}{2}$ by $20\frac{1}{2}$.	Ans. $420\frac{1}{4}$.
7. Multiply $30\frac{1}{2}$ by $30\frac{1}{2}$.	Ans. $930\frac{1}{4}$.

NOTE. — The same principle will hold good if we multiply any number by itself whose unit is a 5.

RULE 2. — *Take the next least number that ends in a cipher, and multiply it by the next larger number ending in a cipher, and add to the product the square of* $5 = 25$, *and the result will be the product.*

8. Multiply 25 by 25. Ans. 625.

The next less number ending in a cipher is 20, and the next larger is 30; $30 \times 20 = 600$; $5 \times 5 = 25$; $600 + 25 = 625$ Ans.

9. Multiply 35 by 35.	Ans. 1225.
10. Multiply 85 by 85.	Ans. 7225.
11. Multiply 95 by 95.	Ans. 9025.

To find the product of two mixed numbers, whose fractional part is a half, and whose difference is a unit.

RULE 3. — *Multiply the larger number without the fraction by itself, and from the product subtract the fractional part multiplied by itself, and the result will be the product.*

12. Multiply $6\frac{1}{2}$ by $7\frac{1}{2}$. Ans. $48\frac{3}{4}$.

OPERATION.

$7 \times 7 = 49$; $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; $49 - \frac{1}{4} = 48\frac{3}{4}$ Ans.

13. Multiply $8\frac{1}{2}$ by $9\frac{1}{2}$. Ans. $80\frac{3}{4}$.
14. Multiply $11\frac{1}{2}$ by $12\frac{1}{2}$. Ans. $143\frac{3}{4}$.
15. Multiply $19\frac{1}{2}$ by $20\frac{1}{2}$. Ans. $399\frac{3}{4}$.
16. Multiply $89\frac{1}{2}$ by $90\frac{1}{2}$. Ans. $8099\frac{3}{4}$.

NOTE. — If the fractional parts of the numbers approach within a $\frac{1}{3}$, $\frac{1}{4}$, or $\frac{1}{5}$, &c., of the larger number, the principle is the same.

17. What is the product of $4\frac{2}{3}$ multiplied by $5\frac{1}{3}$. Ans. $24\frac{8}{9}$.

OPERATION.

$5 \times 5 = 25$; $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$; $25 - \frac{1}{9} = 24\frac{8}{9}$ Ans.

18. Multiply $7\frac{3}{4}$ by $8\frac{1}{4}$. Ans. $63\frac{15}{16}$.
19. Multiply $9\frac{4}{5}$ by $10\frac{1}{5}$. Ans. $99\frac{24}{25}$.
20. Multiply $8\frac{1}{4}$ by $9\frac{3}{4}$. Ans. $80\frac{7}{16}$.
21. Multiply $19\frac{9}{10}$ by $20\frac{1}{10}$. Ans. $399\frac{99}{100}$.

To find the product of two numbers, one of which is as much *less* than either 20, 30, 40, &c., as the other is *more* than either of these numbers.

RULE 4. — *Multiply the* 20, 30, *or* 40, *as the case may be, by itself, and subtract from the product the square of half the difference of the two numbers to be multiplied, and the result will be the product.*

22. Multiply 28 by 32. Ans. 896.

We find that 28 is as much less than 30 as 32 is more than 30; we therefore multiply 30 by $30 = 900$, and from this product we subtract the square of $2 = 4$; $900 - 4 = 896$ Ans.

23. What is the product of 75 by 85? Ans. 6375.
24. What is the product of 83 by 77? Ans. 6391.
25. What is the product of 97 by 103? Ans. 9991.
26. What is the product of 17 by 23? Ans. 391.
27. What cost 18cwt. of steel, at \$22 per cwt.? Ans. \$396.
28. What cost 27 tons of hay, at \$33 per ton? Ans. \$891.
29. What cost 64 gallons of oil, at \$0.56 per gallon? Ans. \$35.84.
30. What cost 28 tons of hay, at \$32 per ton? Ans. \$896.
31. What cost 49 tons of iron, at \$51 per ton? Ans. \$2499.

SECTION XXXVI.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

1. If a man travel 48 miles in 12 hours, how far will he travel in 17 hours? Ans. 68 miles.

[The following is the most obvious solution of this question. If he travel 48 miles in 12 hours, in 1 hour he will travel $\frac{1}{12}$ of 48 miles, which is 4 miles. Then if he travel 4 miles in 1 hour, he will in 17 hours travel 17 times as far, which is 68 miles, the answer.]

2. If 72 pounds of beef cost \$6.48, what will 1 pound cost? what will 675 pounds cost? Ans. \$60.75.

3. If $\frac{1}{8}$ of a dollar buy 1 pound of sugar, how much may be bought for \$1? how much for \$$29\frac{7}{8}$? Ans. 239lb.

4. If a hogshead of wine cost \$73.50, what is the value of 1 gallon? what cost 17hhd. 45 gallons? Ans. \$1302.00.

5. Bought 11 bushels of rye for \$9.00; what cost 25 bushels? Ans. \$$20.45\frac{5}{11}$.

6. If a crew of 15 hands consume in 3 months 1620 pounds of beef, how much would be sufficient for 27 hands for the same time? Ans. 2916lb.

7. Bought 9 yards of flannel for \$7.00; what would be the value of $37\frac{11}{16}$ yards? Ans. \$$29.31\frac{1}{4}$.

8. If a certain field will pasture 8 horses nine weeks, how long will it pasture 23 horses? Ans. $3\frac{3}{23}$ weeks.

9. If $7\frac{3}{12}$ dozen of hats cost \$318.50, what will $19\frac{11}{12}$ dozen cost? Ans. \$$874.95\frac{85}{87}$.

10. If 1 ton of hay cost \$25.00, what will 17 tons 13cwt. 19 pounds cost? Ans. \$441.46+.

11. What part of $9\frac{3}{4}$ is 25? Ans. $2\frac{22}{39}$.

12. What part of 25 is $9\frac{3}{4}$? Ans. $\frac{39}{100}$.

13. If \$25 will pay for $7\frac{3}{8}$ yards of broadcloth, what would be the price of 97 yards? Ans. \$$328.81\frac{21}{59}$.

14. If \$47.25 will pay for the keeping of 7 horses 2 months, what would it cost to keep 43 horses for the same time? Ans. \$290.25.

15. If $7\frac{1}{2}$ yards of cloth a yard wide is sufficient to make a cloak, how many yards would it take if the cloth was $1\frac{3}{4}$ yards wide? Ans. $4\frac{2}{7}$ yards.

16. If a barrel of beer will last 10 men a week, how long would it last 1 man? how long 37 men? Ans. $1\frac{33}{37}$ days.

17. If 5 calves are worth 9 sheep, how many calves will purchase 108 sheep? Ans. 60 calves.

18. If 11 yards of cotton 3 quarters wide are sufficient to line a garment, how many yards would it require that were 5 quarters wide? Ans. $6\frac{3}{5}$ yards.

19. If 7 pairs of shoes will purchase 2 pairs of boots, how many pairs of shoes would it take to buy 18 pairs of boots? Ans. 63 pairs.

20. If four gallons of vinegar be worth 7 gallons of cider, what quantity of vinegar would it take to buy 47 gallons of cider? Ans. $26\frac{6}{7}$ gallons.

21. If a man travel 377 miles in 15 days, how far would he travel in 1 day? how far in 100 days? Ans. $2513\frac{1}{3}$ miles.

22. If 12 men can dig a ditch 50 feet long, 4 feet wide, and 3 feet deep, in 30 days, how long would it take 1 man? how long 47 men? Ans. $7\frac{31}{47}$ days.

23. If 5 barrels of flour cost $25.75, what will 39 barrels cost? Ans. $200.85.

24. Bought 17 acres of land for $791.01; what would 98 acres 3 roods and 14 perches cost? Ans. $4598.90,$8\frac{7}{8}$.

25. Bought $97\frac{3}{8}$ yards of cloth for $275.20; what is the price of 7 yards? Ans. $19.78+.

26. If a pole 7 feet long cast a shadow of 5 feet, how high is that steeple whose shadow is 97 feet? Ans. $135\frac{4}{5}$ feet.

27. Gave $3\frac{3}{7}$cwt. of sugar, at $9, for $\frac{11}{17}$ of an acre of land; how much sugar would it have required to purchase an acre? Ans. $5\frac{23}{77}$cwt.

28. If a vessel sail $47\frac{3}{7}$ miles in $3\frac{2}{7}$ hours, how far would it sail in 1 week? Ans. $2425\frac{1}{23}$m.

29. James can mow a field in 7 days, by laboring 10 hours a day; how many days would it take him to perform the work by laboring 12 hours a day? Ans. $5\frac{5}{6}$ days.

30. If $\frac{7}{11}$ of a lot of land is worth $42.12, what is the value of $\frac{4}{9}$ of it? Ans. 29.41\frac{5}{7}$.

31. If a man can earn $10.27 in $\frac{4}{7}$ of a week, how much would he earn in a month? Ans. $71.89.

32. If 18 men can reap 72 acres in 5 days, how long would it take 6 men to perform the labor? Ans. 15 days.

33. If 19 gallons of wine can be bought for $25, how many gallons will $71.25 buy? Ans. $54\frac{3}{20}$ gallons.

34. If a penny loaf weighs 8 ounces when wheat is $1 a bushel, what should it weigh when wheat is sold for $1.25 a bushel? Ans. $6\frac{2}{5}$ ounces.

35. If a basket which contains $1\frac{1}{8}$ bushels must be filled with apples 7 times to make one barrel of cider, how many barrels may be made by its being filled 126 times? Ans. 18bbl.

36. If a cloak can be made of $4\frac{1}{8}$ yards of cloth that is $1\frac{3}{8}$ yards wide, how many yards would it take of cloth that is $\frac{7}{8}$ of a yard wide? Ans. 6yd. 1qr. $3\frac{5}{7}$na.

37. If a box 4 feet long, 2 feet wide, $1\frac{1}{2}$ feet high, contains 300 pounds of sugar, how much will a box that is 8 feet long, 4 feet wide, and 3 feet high, contain? Ans. 2400lb.

38. How much in length, that is $12\frac{4}{7}$ rods in breadth, will make an acre? Ans. $12\frac{8}{11}$rd.

39. Sound, uninterrupted, moves 1142 feet in a second; how long, after a cannon's being discharged at Boston, is the time before it is heard at Bradford, the nearest distance being $25\frac{21}{22}$ miles? Ans. 2 minutes.

40. Bought $\frac{7}{11}$ of a ton of potash, and sold $\frac{11}{14}$ of it for \$46.70; what was the value of a ton? Ans. \$93.40.

41. If $\frac{4}{11}$ of a lot of land be worth \$97, what would the whole lot be worth? Ans. \$266.75.

42. If 19 pounds of salmon be worth 50 pounds of beef, how much salmon would buy 77 pounds of beef? Ans. $29\frac{13}{50}$lb.

43. What part of $17\frac{3}{11}$ is $4\frac{1}{4}$? Ans. $\frac{187}{760}$.

44. What part of 11s. 3d. is 15s.? Ans. $\frac{4}{3}$.

45. What part of $7\frac{1}{4}$ yards is $3\frac{3}{5}$ ells English? Ans. $\frac{18}{29}$.

Section XXXVII.

SIMPLE INTEREST.

Interest is the compensation which the *borrower* of money makes to the *lender*.

Principal is the sum lent.

Amount is the interest added to the principal.

Per cent., a contraction of *per centum*, is the rate established by law, or that which is agreed on by the parties, and is so much for a hundred dollars for one year.

CASE I.

General Rule. — *Let the per cent. be considered as a decimal of the place of hundreds, and multiply the principal by it, and the product is the interest for one year. But, if it be required to find the interest for more than one year, multiply the product by the number of years.*

Note. — The decimal for 6 per cent. is .06; for 7 per cent., .07; for 8 per cent., .08; for $9\frac{1}{4}$ per cent., .0925; for $2\frac{1}{2}$ per cent., .025, &c. The decimal must be pointed off as in Multiplication of Decimal Fractions.

This rule is obvious from the fact that the rate per cent. is such a part of every hundred dollars. Thus 6 per cent, is $\frac{6}{100}$ of the principal.

NOTE. — When no particular per cent. is named, 6 per cent. is to be understood, as it is the legal interest in the New England States. See page 148.

1. What is the interest of \$ 144 for 1 year? Ans. \$ 8.64.

OPERATION.

\$ 144
.06
\$ 8.64 Ans.

There being two places of decimals in the multiplier, we point off two in the product.

2. What is the interest of \$ 78.78 for 3 years?

OPERATION.

\$ 78.78
.06
4.72,68
3
\$ 14.18,04 Ans.

There being two places of decimals in the multiplicand, and two in the multiplier, we point off four places in the product.

NOTE. — It is a custom with merchants to reject mills in their computations, but when the decimal of a cent exceeds 5 they add 1 to the number of cents. Thus, they would reckon \$ 81.93,8 to be in value \$ 81.94.

3. What is the interest of \$ 675 for 1 year? Ans. \$ 40.50.
4. What is the interest of \$ 1728 for 1 year? Ans. \$ 103.68.
5. What is the interest of \$ 19.64 for 2 years? Ans. \$ 2.35,6.
6. What is the interest of \$ 896.28 for 3 years? Ans. \$ 161.33.
7. What is the interest of \$ 349.25 for 10 years? Ans. \$ 209.55.
8. What is the interest of \$ 3967.87 for 2 years? Ans. \$ 476.14,4.
9. What is the interest of \$ 123.45 for 6 years? Ans. \$ 44.44,2.
10. What is the interest of \$ 89.25 for 50 years? Ans. \$ 267.75.
11. What is the interest of \$ 17.25 for 7 years? Ans. \$ 7.24,5.
12. What is the interest of \$ 29.19 for 9 years? Ans. \$ 15.76,2.
13. What is the interest of \$ 617.56 for 25 years? Ans. \$ 926.34.

14. What is the amount of $ 31.75 for 100 years?
Ans. $ 222.25.

15. What is the amount of $ 76.47 for 7 years?
Ans. $ 108.58,7

16. What is the amount of $ 716.57 for 4 years?
Ans. $ 888.54,6.

17. What is the amount of $ 178.56,5 for 30 years?
Ans. $ 499.98,2.

18. What is the interest of $ 97.06 for 9 years?
Ans. $ 52.41,2.

19. What is the interest of $ 0.75 for 75 years?
Ans. $ 3.37,5.

20. What is the interest of $ 750 for 12 years?
Ans. $ 540.

CASE II.

To find the interest for months at 6 per cent.

RULE. — *Multiply the principal by half the number of months, expressed decimally as a per cent.; that is, for* 12 *months multiply by* .06; *for* 8 *months multiply by* .04; *for* 7 *months*, .035; *for* 1 *month*, .005; *and point for decimals as in the last rule.*

NOTE 1. — It is obvious, that, if the rate per cent. were 12, it would be 1 per cent. a month. If, therefore, it be 6 per cent. it will be a half per cent. a month, that is, half the months will be the per cent.

NOTE 2. — If any other per cent. is wanted, proceed as above, and then multiply by the given rate per cent. and divide by 6, and the quotient is the interest.

1. What is the interest of $ 368 for 8 months?

$ 368
.04 = half the months.
$ 14.72 = Answer.

2. What is the interest of $ 637 for 10 months?
Ans. $ 31.85.

3. What is the interest of $1671.32 for 14 months?
Ans. $ 116.99.

4. What is the interest of $ 891.24 for 9 months?
Ans. $ 40.10.

5. What is the interest of $ 819.75 for 11 months?
Ans. $ 45.08,6.

6. What is the interest of $ 3671.25 for 18 months?
Ans. $ 238.63.

7. What is the interest of $ 61.18 for 15 months?
Ans. $ 4.58.

8. What is the interest of $ 3181.29 for 18 months?
Ans. $ 286.31.

9. What is the interest of $ 11.39 for 19 months? Ans. $ 1.08.

10. What is the interest of $ 9.98 for 23 months? Ans. $ 1.14.

11. What is the interest of $ 87.19 for 27 months? Ans. $ 11.77.

12. What is the interest of $ 32.18 for 36 months? Ans. $ 5.79.

13. What is the interest of $ 167.18 for 50 months? Ans. $ 41.79.

14. What is the interest of $ 386.19 for 100 months? Ans. $ 193.09.

CASE III.

To find the interest of any sum for months and days, at 6 per cent.

RULE.* — *Find the interest for the number of months, as under Case II. Then to find it for the number of days, multiply the principal by one sixth the number of days, and, if the principal be dollars, cut off three figures from the right hand, and those at the left will be the interest in dollars, and those at the right will be cents and mills. But if the principal be dollars and cents, five figures must be cut off from the right hand, and those at the left will be the interest, &c., as before.*

---

* The reason for this rule, so far as it relates to the interest for any number of months, was explained above. But the reason for the operation in the case of days is not so obvious. It will be seen, however, that it is nothing but an abridgment of the general rule for calculating interest, as given on page 164, which will appear from what follows.

To find the interest for any number of *years*, we multiply the principal by the *annual* per cent., and the product thus obtained by the number of years, and cut off two figures, &c. In like manner, it is evident that to find the interest for any number of *days*, we have only to multiply by the *daily* per cent., and the product thus obtained by the number of days, and cut off as in the case of years. But 6 per cent. *per annum* is $\frac{1}{60}$ per cent *per diem*, allowing 360 days to a year. Now, to multiply the principal by $\frac{1}{6}$ of the days, and cut off *one* figure from the product at the right, is the same as to multiply by $\frac{1}{60}$ (the *daily* per cent.), and the product thus obtained by the whole number of days. Where the principal is dollars only, the rule directs to cut off *three* figures. Two of them are cut off according to the general rule, on page 164, and the other, because in the operation we have multiplied by $\frac{1}{6}$ the number of days, instead of $\frac{1}{60}$ of them, which would have been the proper multiplier.

The above may be illustrated concisely by the following operation. Let it be required to find the interest of 75 dollars for 180 days. By the general rule we have $75 \times \frac{1}{60} \div 100 \times 180 = \$2.25$. By the rule under Case III. we have $75 \times \frac{1}{6}$ of $180 = \$2.25$.

NOTE 1. — If one half the number of months be expressed by a single figure, we have only to *annex* to this figure $\frac{1}{6}$ the number of days, and multiply the principal by the number thus found, cutting off as above, and we obtain, by a single operation, the interest for the months and days.

NOTE 2. — If any other per cent. than 6 is given, we may proceed as above, and then multiply by the given rate and divide by 6, and the result will be the interest sought.

1. What is the interest of $ 68.25 for 8 months and 24 days?
Ans. $ 3.00,3.

OPERATION.

$$\begin{array}{r} \$68.25 \\ .044 \\ \hline 27300 \\ 27300 \\ \hline \$3.00,300 \end{array}$$

The first 4 in the multiplier is half of the 8 months; the second 4 is one sixth of the 24 days.

2. What is the interest of $ 637.28 for 17 months and 19 days, at 8 per cent.? Ans. $ 74.91,5.

3. What is the interest of $ 396.15 for 13 months and 9 days? Ans. $ 26.34,3.

4. What is the interest of $ 16.75 for 7 months and 17 days, at 7 per cent.? Ans. $ 0.73,9.

5. What is the interest of $ 976.18 for 29 months and 23 days, at 9 per cent.? Ans. $ 217.93,2.

6. What is the interest of $ 36.18 for 3 months and 7 days?
Ans. $ 0.58,4.

7. What is the interest of $ 51.17 for 9 months and 29 days, at 4 per cent.? Ans. $ 1.69,9.

8. What is the interest of $ 365.19 for 33 months and 4 days, at 2 per cent.? Ans. $ 20.16,6.

9. What is the interest of $ 125.75 for 5 months and 4 days? Ans. $ 3.22,7.

10. What is the interest of $ 35.49 for 1 month and 2 days, at $7\frac{1}{2}$ per cent.? Ans. $ 0.23,6.

11. What is the interest of $ 112.50 for 3 months and 1 day, at $9\frac{1}{2}$ per cent.? Ans. $ 2.70,1.

12. What is the interest of $ 97.15 for 35 months and 27 days? Ans. $ 17.43,8.

13. What is the interest of $ 47.15 for 1 month and 19 days, at $13\frac{1}{2}$ per cent.? Ans. $ 0.86,6.

14. What is the interest of $ 678.75 for 87 months and 20 days? Ans. $ 297.51,8.

15. What is the interest of $ 86 for 99 months and 29 days, at 25 per cent.? Ans. $ 179.10,6.

16. What is the interest of $33.35,8 for 15 months and 17 days? Ans. $2.59,6.

17. What is the interest of $144 for 5 days?
Ans. $0.12,0.

CASE IV.

When the interest is required on any sum, from a certain day of the month in a year, to a particular day of a month in the same, or in another year.

RULE. — *Find the time, by placing the latest date in an upper line, and the earliest date under it. Let the year be placed first; the number of months that have elapsed since the year commenced at its right hand, and the day of the month next; then subtract the earlier from the latest date, and the remainder is the time for which the interest is required. Then proceed as in the last rule. Or, the months may be reckoned by their ordinal number, as in Operation Second.*

NOTE. — Many *practical* men prefer reckoning interest by the second method.

EXAMPLES.

1. What is the interest of $84.97, from Sept. 25, 1833, to March 8, 1835? Ans. $7.40,6.

Operation First.

Yrs.	m.	d.
1835	2	8
1833	8	25
1	5	13

Operation Second.

Yrs.	m.	d.
1835	3	8
1833	9	25
1	5	13

First Method.

$84.97
.087$\frac{1}{6}$
―――
59479
67976
1416
―――
$7.40,655

Second Method.

		$84.97
		.06
1 year,	=	5.0982
4 months,	$\frac{1}{3}$ =	1.6994
1 month,	$\frac{1}{4}$ =	.4248
10 days,	$\frac{1}{3}$ =	.1416
2 days,	$\frac{1}{5}$ =	283
1 day,	$\frac{1}{2}$ =	141

$7.40,64, interest as before.

It is evident that 4 months' interest is $\frac{1}{3}$ of a year's interest; and for the same reason, 1 month's interest is $\frac{1}{4}$ of 4 months' interest; and as 10 days is $\frac{1}{3}$ of a month, the interest for that time is $\frac{1}{3}$ of a month's interest; and if the interest of 10 days be divided by 5, the quotient will be 2 days' interest, and the half of this will be 1 day's interest.

2. What is the interest of $786.75, from Dec. 9, 1831, to May 11, 1833? Ans. $67.13,6.

3. What is the interest of $98.25, from July 4, 1826, to Oct. 19, 1829? Ans. $19.40,4.

4. What is the interest of $76.89,5, from Jan. 11, 1822, to July 27, 1833? Ans. $53.26,2.

5. What is the interest of $22.76,3, from Feb. 19, 1806, to July 18, 1830? Ans. $33.34,4.

6. What is the interest of $76.35, from August 17, 1830, to May 5, 1832? Ans. $7.86,4.

7. What is the interest of $97.86, from May 17, 1821, to Dec. 19, 1828? Ans. $44.55,8.

8. What is the interest of $1728.75, from Nov. 19, 1823, to June 18, 1826? Ans. $267.66,8.

9. What is the interest of $99.99,9, from Jan. 1, 1800, to Feb. 29, 1832? Ans. $192.96,4.

10. What is the interest of $16.76, from Dec. 17, 1811, to June 17, 1822? Ans. $10.55,8.

11. What is the interest of $35.61, from Nov. 11, 1831, to Dec. 15, 1833? Ans. $4.47,4.

12. What is the interest of $786.97, from Oct. 19, 1827, to August 17, 1831, at 7½ per cent.? Ans. $225.92,5.

13. What is the interest of $96.84, from Nov. 27, 1829, to July 3, 1832, at 7½ per cent.? Ans. $18.88,3.

14. What is the interest of $11.10,5, from April 17, 1832, to Dec. 7, 1832, at 7 per cent.? Ans. $0.49,6.

15. What is the interest of $117.21, from June 19, 1806, to June 17, 1819, at 8½ per cent.? Ans. $129.46,1.

16. What is the interest of $17869.75, from Feb. 7, 1830, to Jan. 11, 1832, at 5 per cent.? Ans. $1722.44,5.

17. What is the interest of $71.09,1, from July 29, 1823, to June 19, 1827, at 12 per cent.? Ans. $33.17,5.

18. What is the interest of $83.47, from Nov. 8, 1830, to July 11, 1833, at 8¾ per cent.? Ans. $19.53,7.

19. What is the interest of $79.25, from Dec. 8, 1831, to July 17, 1833? Ans. $7.64,7.

20. What is the interest of $175.07, from Jan. 7, 1825, to Oct. 12, 1829? Ans. $50.04.

21. What is the interest of $12.75, from June 16, 1831, to August 20, 1833? Ans. $1.66,6.

22. What is the interest of $197.28,5, from Dec. 6, 1832, to Jan. 11, 1834? Ans. $12.98,7.

23. What is the interest of $12.69, from Jan. 2, 1833, to August 30, 1834, at 7 per cent.? Ans. $1.47,5.

24. What is the interest of $79.15, from Feb. 11, 1831, to June 10, 1833, at 7¼ per cent.? Ans. $13.37,3.

25. What is the amount of $ 83.33, from March 11, 1831, to Jan. 1, 1833, at 7½ per cent.? Ans. $ 94.61,4.

26. What is the amount of $100.25, from March 2, 1831, to June 1, 1831, at 4 per cent.? Ans. $101.24,1.

27. What is the amount of $ 369.29, from April 30, 1830, to July 31, 1832, at 9 per cent.? Ans. $ 444.16,3.

28. What is the interest of $ 769.87, from Jan. 1, 1830, to June 17, 1835, at 9½ per cent.? Ans. $ 399.41,2.

29. What is the interest of $ 69.75, from Jan. 11, 1833, to June 29, 1833, at 17½ per cent.? Ans. $ 5.69,6.

30. What is the interest of $ 368.18, from April 2, 1816, to June 19, 1835, at 2 per cent.? Ans. $141.48,3.

31. What is the interest of $16.16, from March 3, 1831, to Dec. 6, 1833, at 1 per cent.? Ans. $ 0.44,5.

32. What is the interest of $1728.19, from May 7, 1824, to July 17, 1830, at ½ per cent.? Ans. $ 26.76,2.

33. What is the interest of $ 397.16, from Dec. 29, 1831, to June 30, 1833, at 5½ per cent.? Ans. $ 32.82,6.

34. What is the amount of $1760.07, from Feb. 17, 1831, to Dec. 19, 1832, at ½ per cent.? Ans. $1776.25,2.

35. G. K. M., of Bradford, has sent shoes at several times to Spofford & Tileston, New York, as follows: —

January 16, 1834,	were sent shoes	to the value	of	$ 865.00
Feb. 17, 1834,	"	"	"	386.27
March 29, 1834,	"	"	"	769.25
May 25, 1834,	"	"	"	183.75
June 19, 1834,	"	"	"	396.81

The above were sold on six months' credit. G. K. M. has received of Spofford & Tileston as follows: — Sept. 1, 1834, $1000; Oct. 19, 1834, $ 375.25; Nov. 15, 1834, $ 681.29; Dec. 8, 1834, $100; March 12, 1835, $ 275.28. Required the balance at the time of settlement, Sept. 9, 1835.

Ans. Spofford & Tileston owe to G. K. M. $195.51+.

CASE V.

To find the interest for any sum for days.

RULE.* — *If the rate per cent. be* .06, *multiply the principal by the number of days, and divide by* 6083⅓, *and the quotient is the interest; but if the rate per cent. be* .05, *divide by* 7300.

* This rule is but an abridgment of the obvious method for obtaining the interest for any number of days, which is, first to find it for one year, then divide by 365, which gives it for one day, and then multiply the

EXAMPLES.

1. What is the interest of $1835 for 35 days? Ans. $10.55,7.

2. What is the interest of $165.37 for 165 days? Ans. $4.48,5.

3. What is the interest of $16.87 for 79 days, at 5 per cent.? Ans. $0.18,2.

4. What is the interest of $167 for 87 days, at 5 per cent.? Ans. $1.99.

5. What is the interest of $761.81 for 165 days? Ans. $20.66,2.

6. What is the interest of $76.18,5 for 315 days, at 5 per cent.? Ans. $3.28,7.

7. What is the interest of $178.69,7 for 271 days? Ans. $7.96.

8. What is the interest of $1728.79 for 318 days, at 5 per cent.? Ans. $75.30,8.

9. What is the interest of $73 for 73 days, at 5 per cent.? Ans. $0.73.

10. What is the interest of $96.10 for 54 days? Ans. $0.85,3.

11. What is the interest of $144.50 for 144 days, at 5 per cent.? Ans. $2.85.

12. What is the amount of $1728 for 200 days? Ans. $1784.81.

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result by the number of days. The full operation of finding the interest of any sum, say $1835 for 35 days, will be as follows:

$$\frac{1835 \times .06}{365} \times 35 = 10.55,7+.$$

Now it is evident, that, instead of multiplying 1835 (the numerator) by .06, we may divide 365 (the denominator) by it, without affecting the result. But $365 \div .06 = 6083+$, so that we have only to multiply the principal by the number of days, and divide the product by $6083\frac{1}{3}$, when the rate is 6 per cent., and we have the interest required. For the same reason, we divide by 7300 when the rate is 5 per cent. In the same way a divisor may be found, answering to any given rate.

All the foregoing rules are on the principle, that there are only 360 days in a year; yet they are adopted by all mercantile men, and also by banks. But if a note be written for 144 days, the holder of it can obtain interest for only $\frac{144}{365}$ of a year by the law of Massachusetts.

If a note be written for months, calendar months are understood, whether the months have 28 or 31 days.

If a note be dated April 21st, for one month, it will be due May 21st but if it be dated Jan. 28th, 29th, 30th, or 31st, it being for one month, it will be due Feb. 28th, it being the last day of the month, unless it be leap year.

SECTION XXXVIII.

PARTIAL PAYMENTS.

WHEN notes are paid within one year from the time they become due, it has been the usual custom to find the amount of the principal from the time it became due until the time of payment; and then to find the amount of each indorsement from the time it was paid until settlement, and to subtract their sum from the amount of the principal.

EXAMPLES.

(1.) $1728.00. Baltimore, January 1, 1833.

For value received, I promise Riggs, Peabody, & Co. to pay them, or order, on demand, one thousand seven hundred and twenty-eight dollars, with interest. John Paywell, Jr.

On this note are the following indorsements. March 1, 1833, received three hundred dollars. May 16, 1833, received one hundred and fifty dollars. Sept. 1, 1833, received two hundred and seventy dollars. December 11, 1833, received one hundred and thirty-five dollars.

What was due at the time of payment, which was December 16, 1833? Ans. $948.03.

OPERATION.

Principal,		$1728.00
Interest for 11 months and 15 days,		99.36
		$1827.36
First payment,	$300.00	
Interest for 9 months and 15 days,	14.25	
Second payment,	150.00	
Interest for 7 months,	5.25	
Third payment,	270.00	
Interest for 3 months and 15 days,	4.72	
Fourth payment,	135.00	
Interest for 5 days,	.11	
		$879.33
Balance remaining due, Dec. 16, 1833,		$948.03

(2.) $700.00. Concord, Feb. 4, 1834.

For value received, we jointly and severally promise James Thomas to pay him, or order, on demand, seven hundred dollars, with interest. Sampson Phillips.
Attest, Henry Dix. Richard Fletcher.

On this note are the following payments. March 18, 1834, received one hundred and sixty dollars. June 24, 1834, received two hundred dollars. September 11, 1834, received one hundred and twenty dollars. October 5, 1834, received sixty dollars.

What was due on this note Nov. 28, 1834? Ans. $180.43.

(3.) $600.00. Portland, May 16, 1834.

For value received, we, Luke A. Homer, as principal, and Daniel D. Snow and Ichabod Frost, as sureties, promise John Webster to pay him, or order, in one year, six hundred dollars, with interest after three months. Luke A. Homer.
Daniel D. Snow.
Attest, M. Peters. Ichabod Frost.

On this note are the following indorsements. Sept. 18, 1834, received one hundred and thirty-six dollars. December 5, 1834, received one hundred and ninety-seven dollars. February 11, 1835, received two hundred dollars. April 19, 1835, received forty dollars.

What was due August 1, 1835? Ans. $40.31,2.

In the United States courts, and in most of the courts of the several States, the following rule is adopted for estimating interest on notes and bonds, when partial payments have been made.

RULE. — *Compute the interest on the principal sum, from the time when the interest commenced to the first time when a payment was made, which exceeds, either alone or in conjunction with the preceding payments, if any, the interest at that time due; add that interest to the principal, and from the sum subtract the payment made at that time, together with the preceding payments, if any, and the remainder forms a new principal; on which compute and subtract the interest, as upon the first principal, and proceed in the same manner to the time of judgment.*

The following example will illustrate the above rule.

(4.) $165.18. Boston, June 17, 1827.

For value received, I promise James E. Snow to pay him, or order, on demand, one hundred and sixty-five dollars and eighteen cents, with interest. James Y. Frye.
Attest, John True.

On this note are the following indorsements. December 7, 1827, received eighteen dollars and thirteen cents of the within note. October 19, 1828, received twenty-eight dollars and sixteen cents. September 25, 1829, received thirty-six dollars and twelve cents. July 10, 1830, received three dollars and eighteen cents. June 6, 1831, received thirty-

six dollars and twenty-eight cents. December 28, 1832, received thirty one dollars and seventeen cents. May 5, 1833, received three dollars and eighteen cents. September 1, 1833, received twenty-five dollars and eighteen cents. October 18, 1834, received ten dollars.

How much remains due September 27, 1835? Ans. $15.41,7.

METHOD OF OPERATION.

Principal, carrying interest from June 17, 1827,	$165.18,0
Interest from June 17, 1827, to Dec. 7, 1827, 5mo. 20 days,	4.68,0
Amount,	169.86,0
First payment, Dec. 7, 1827, - - - -	18.13,0
Balance for new principal, - - - -	151.73,0
Interest from Dec. 7, 1827, to Oct. 19, 1828, 10mo. 12da.,	7.88,9
Amount,	159.61,9
Second payment, Oct. 19, 1828, - - - -	28.16,0
Balance for new principal, - - - -	131.45,9
Interest from Oct. 19, 1828, to Sept. 25, 1829, 11mo. 6da.,	7.36,1
Amount,	138.82,0
Third payment, Sept. 25, 1829, - - - -	36.12,0
Balance for new principal, - - - -	102.70,0
Interest from Sept. 25, 1829, to June 6, 1831, 20mo. 11da.,	10.45,8
Amount,	113.15,8
Fourth pay't, July 10, 1830, a sum less than int'st, 3.18	
Fifth pay't, June 6, 1831, a sum greater than int'st, 36.28	
	39.46,0
Balance for new principal, - - - -	73.69,8
Interest from June 6, 1831, to Dec. 28, 1832, 18mo. 22da.,	6.90,3
Amount,	80.60,1
Sixth payment, Dec. 28, 1832, - - - -	31.17,0
Balance for new principal, - - - -	49.43,1
Interest from Dec. 28, 1832, to May 5, 1833, 4mo. 7da.,	1.04,6
Amount,	50.47,7
Seventh payment, May 5, 1833, - - - -	3.18,0
Balance for new principal, - - - -	47.29,7
Interest from May 5, 1833, to Sept. 1, 1833, 3mo. 26da.,	91,4
Amount,	48.21,1
Eighth payment, Sept. 1, 1833, - - - -	25.18,0
Balance for new principal, - - - -	23.03,1
Interest from Sept. 1, 1833, to Oct. 18, 1834, 13mo. 17da.,	1.56,2
Amount,	24.59,3

Amount brought forward,	$ 24.59,3
Ninth payment, - - - - - - - -	10.00,0
Balance for new principal, - - - - -	14.59,3
Interest from Oct. 18, 1834, to Sept. 27, 1835, 11mo. 9da.,	.82,4
Balance due at the time of payment, - - - -	$ 15.41,7

(5.) $ 769.87. Salem, June 17, 1829.

For value received, I promise L. Swan to pay him, or order, on demand, seven hundred and sixty-nine dollars and eighty-seven cents, with interest. Samuel Q. Peters.

Attest, Moses Haynes.

On this note are the following payments. March 1, 1830, received seventy-five dollars and fifty cents. June 11, 1831, received one hundred and sixty-five dollars. September 15, 1831, received one hundred and sixty-one dollars. Jan. 21, 1832, received forty-seven dollars and twenty-five cents. March 5, 1833, received twelve dollars and seventeen cents. December 6, 1833, received ninety-eight dollars. July 7, 1834, received one hundred and sixty-nine dollars.

What remains due Sept. 25, 1835? Ans. $ 226.29,7.

(6.) $ 300.00. Haverhill, April 30, 1831.

For value received, I promise Kimball & Hammond to pay them, or order, on demand, three hundred dollars, with interest. Simpson W. Leavet.

Attest, James Quintire.

The following partial payments were made on this note. June 27, 1832, received one hundred and fifty dollars. December 9, 1832, received one hundred and fifty dollars.

What was due, Oct. 9, 1833? Ans. $ 26.73,5.

(7.) $ 54.18. New York, Feb. 11, 1832.

For value received, I promise John Trow to pay him, or order, on demand, fifty-four dollars and eighteen cents, with interest. Luke M. Sampson.

On this note are the following payments. July 11, 1833, received twelve dollars and twenty-five cents. August 15, 1834, received two dollars and ten cents. July 9, 1835, received three dollars and twelve cents. August 21, 1835, received thirty-seven dollars and eighteen cents.

What was due Dec. 17, 1835. Ans. $10.22,2.

(8.) $1728.00. Boston, Jan. 7, 1831.

For value received, we jointly and severally promise Jones, Oliver, & Co. to pay them, or order, on demand, one thousand seven hundred and twenty-eight dollars, with interest.

John Bountiful.

Attest, Timothy True. James Trusty.

On this note are the following payments. February 9, 1832, received seven hundred and sixty dollars and twenty-eight cents and five mills. March 5, 1833, received sixty-eight dollars and fifty cents. December 28, 1833, received eight hundred and seventy-six dollars and twenty-eight cents. July 17, 1834, received sixty dollars.

What was due at the time of payment, which was Oct. 1, 1834? Ans. $209.22,9.

(9.) $500.00. Philadelphia, May 7, 1829.

For value received, I promise John Jordan to pay him, or order, on demand, five hundred dollars, with interest.

Thomas C. True.

The following partial payments were indorsed on this note. June 29, 1830, received one hundred dollars. December 5, 1831, received one hundred dollars. March 12, 1832, received five dollars. July 4, 1833, received ninety-five dollars. December 1, 1834, received two hundred dollars.

What was due Jan. 1, 1836? Ans. $141.50,4.

(10.) $89.75. Newburyport, March 19, 1831.

For value received, we jointly and severally promise John Frost to pay him, or order, on demand, eighty-nine dollars and seventy-five cents, with interest. Henry Augustus.

Attest, James Snow. Marcus T. Cicero.

On this note are the following indorsements. December 6, 1831, received twelve dollars and twelve cents. February 17, 1832, received twelve dollars and twelve cents. March 19, 1833, received three dollars and sixteen cents. December 28, 1834, received two dollars and eighteen cents. January 1, 1835, received twenty-five dollars and twenty-five cents. March 11, 1835, received thirty-one dollars and eighteen cents. July 17, 1835, received five dollars and eighteen cents. September 1, 1835, received six dollars and twenty-nine cents.

What was due Dec. 29, 1835? Ans. $10.57.

(11.) $1000.00. New York, January 1, 1840.

For value received, I promise to pay James Johnson, or order, on demand, one thousand dollars, with interest at seven per cent. Samuel T. Fortune.

Attest, Job Harris.

On this note are the following indorsements. Sept. 28, 1840, received one hundred and forty-four dollars. March 1, 1841, received twenty dollars. July 17, 1841, received three hundred and sixty dollars. Aug. 9, 1841, received one hundred and ninety dollars. Sept. 25, 1842, received one hundred and seventy dollars. Dec. 11, 1843, received two hundred dollars. July 4, 1845, received seventy-five dollars.

What was due June 1, 1847? Ans. $7.61.

The following is the rule established by the Supreme Court of the State of Connecticut.

RULE. — *Compute the interest to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; provided the time between one payment and another be one year or more. But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added together.**

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.

(12.) $900.00. New Haven, June 1, 1828.

For value received, I promise J. D. to pay him, or order, nine hundred dollars, on demand, with interest.

James L. Emerson.

On this note are the following indorsements. June 16, 1829, received two hundred dollars of the within note. Aug. 1, 1830, received one hundred and sixty dollars. Nov. 16, 1830, received seventy-five dollars. Feb. 1, 1832, received two hundred and twenty dollars.

What was due August 1, 1832? Ans. $417.82,2.

OPERATION.

Principal, - - - - - - -	$900.00
Interest from June 1, 1828, to June 16, 1829, 12½ months,	56.25
	956.25
First payment, - - - - - -	200.00
	756.25
Interest from June 16, 1829, to Aug. 1, 1830, 13½ months,	51.04,6
	807.29,6
Second payment, - - - - - -	160.00,0
	647.29,6
Interest for one year, - - - - - -	38.83,7
	686.13,3

* If the year extends beyond the time of payment, find the amount of the remaining principal to the time of payment; find also the amount of indorsement, or indorsements, and subtract their sum from the amount of the principal.

Amount brought forward,	$ 686.13,3
Am't of 3d pay't, from Nov. 16, 1830, to Aug. 1, 1831, 8½mo.,	78.18,7
	607.94,6
Interest from Aug. 1, 1831, to Aug. 1, 1832, 12 months,	36.47,6
	644.42,2
Am't of 4th pay't, from Feb. 1, 1832, to Aug. 1, 1832, 6mo.,	226.60,0
Balance due August 1, 1832, - - - -	$ 417.82,2

Method of computing annual interest in the State of Vermont.

When the contract is for the payment of interest *annually*, and no payments have been made, we adopt the following

RULE. — *Find the interest of the principal for each year, separately, up to the time of payment; we must then find the* simple *interest of these interests, severally, from the time they become due up to the time of payment, and the sum of all the interests added to the principal will be the amount.*

Should payments have been made, we find the amount of the principal and from this we subtract the amount of the indorsement or indorsements to the end of the first year. We proceed in the same manner to the time of payment.

But when the contract is for a sum payable at a specified *time, with* annual *interest, and payments are made before the debt becomes due, we find the interest of the principal up to the time of the first payment, and this interest we reserve; we then subtract the payment from the principal, and find the interest of the remainder up to the time of the next payment, which interest we add to the other interest, and so continue up to the time the debt becomes due, and the sum of the interests, added to the last principal, will be the amount due at that time. After the debt becomes due, the interest is to be extinguished annually, if the payments are sufficient for that purpose.*

13. J. Jones has J. Smith's note, dated January 1, 1840, for $ 500, with interest, to be paid annually, at 6 per cent. What was due January 1, 1844? Ans. $ 630.80.

OPERATION.

1st year,	$ 500 × .06 = $ 30.	$ 30 × .18 = $ 5.40
2d "	$ 500 × .06 = $ 30.	$ 30 × .12 = $ 3.60
3d "	$ 500 × .06 = $ 30.	$ 30 × .06 = $ 1.80
4th "	$ 500 × .06 = $ 30.	$ 10.80 interest of interest.
	$ 120 interest of principal.	

Principal,	$ 500.00.
Interest of principal,	120.00.
Interest of interest,	10.80.
Amount,	$ 630.80 Ans.

This is the method, when there are no indorsements.

(14.) $300.00. Thetford, Vt., January 1, 1843.

For value received, I promise to pay Hiram Orcutt, Esq. or order, on demand, three hundred dollars, with interest annually. M. T. Cicero.

Attest, P. M. Virgil.

On this note are the following indorsements. Sept. 1, 1843, received eighty dollars. July 1, 1844, received one hundred dollars. April 1, 1845, received fifty dollars.

What was due January 1, 1847?

OPERATION.

Principal,	$300.00
Interest for one year,	18.00
Amount,	$318.00
First payment,	80.00
Interest on first payment, 4 months,	1.60
Amount of payment,	81.60
New principal,	236.40
Interest on new principal, one year,	14.18
Amount,	250.58
Second payment,	100.00
Interest on second payment, 6 months,	3.00
Amount of payment,	103.00
New principal,	147.58
Interest on new principal, one year,	8.85
Amount,	156.43
Third payment,	50.00
Interest on third payment, 9 months,	2.25
Amount of payment,	52.25
New principal,	104.18
Interest on new principal, one year,	6.25
Remains due January 1, 1847,	$110.43

The above is the method of computing *annual* interest, when there are indorsements on the note, and it is not payable at a *specified* time.

(15.) $600.00. Montpelier, Vt., June 1, 1844.

For value received, I promise to pay John Smith, or order, six hundred dollars, in three years, with interest annually.

Attest, S. Morse. John Y. Jones.

On this note are the following indorsements. Sept. 1, 1845, received two hundred and fifty dollars. Nov. 1, 1846, received two hundred dollars.

What was due June 1, 1847? Ans. $242.75.

OPERATION.

$600 × .105 = 63.00, interest from June 1, 1844, to Sept. 1, [1845.
250
350 × .07 = 24.50, interest from Sept. 1, 1845, to Nov. 1, [1846.
200
150 × .035 = 5.25, interest from Nov. 1, 1846, to June 1, [1847.
92.75 $92.75, amount of interest.
$242.75 remains due June 1, 1847.

The above is the method of computing *annual* interest, when the contract is for a *specified* time.

NOTE. — The above methods of computing annual interest are not only practised in the courts of Vermont, but a similar method has been adopted in some of the courts of New Hampshire; but it is *not* the method in Massachusetts.

SECTION XXXIX.

MISCELLANEOUS PROBLEMS IN INTEREST.

PRINCIPAL, interest, and time given, to find the rate per cent.

1. At what rate per cent. must $500 be put on interest to gain $120 in 4 years?

OPERATION.

$500
.01
5.00
4
20.00)120.00(6 per cent. Ans.
120.00

BY ANALYSIS.

The interest of $1 for the given time at one per cent. is 4 cents. $500 will be 500 times as much, = $500 × .04 = $20.00. Then, if $20 give 1 per cent. $120 will give $\frac{120}{20}$, = 6 per cent.

RULE. — *Divide the given interest by the interest of the given sum at 1 per cent. for the given time, and the quotient will be the rate per cent. required.*

2. At what rate per cent. must $120 be on interest to amount to $133.20 in 16 months? Ans. 8¼ per cent.

3. At what rate per cent. must \$280 be on interest to amount to \$411.95 in $6\frac{1}{2}$ years? Ans. $7\frac{1}{4}$ per cent.

Principal, interest, and rate per cent. given, to find the time.

4. How long must \$500 be on interest at 6 per cent. to gain \$120?

OPERATION.

\$500
.06
30.00)120.00(4 years, Ans.
120.00

BY ANALYSIS.

We find the interest of \$1.00 at the given rate for one year is 6 cents. \$500 will, therefore, be 500 times as much, $=$ \$500 $\times$ .06 $=$ \$30.00. Now, if it take 1 year to gain \$30, it will require $\frac{120}{30}$ to gain \$120, $=$ 4 years, Ans.

RULE. — *Divide the given interest by the interest of the principal for 1 year, and the quotient is the time.*

5. How long must \$120 be on interest at $8\frac{1}{4}$ per cent. to amount to \$133.20? Ans. 16 months.

6. How long must \$280 be on interest at $7\frac{1}{4}$ per cent. to amount to \$411.95? Ans. $6\frac{1}{2}$ years.

Interest, time, and rate per cent. given, to find the principal.

7. What principal at 6 per cent. is sufficient in 4 years to gain \$120?

OPERATION.

\$1
.06
.06
4
.24)120.00(\$500, Ans.
120.00

BY ANALYSIS.

The interest of \$1 for the given rate and time is 24 cents. If 24 cents, then, give \$1 for principal, \$120.00 will give $\frac{120.00}{.24}$ times as much, $=$ \$500, Ans.

RULE. — *Divide the given interest or amount by the interest or amount of \$1 for the given rate and time, and the quotient is the principal.*

8. What principal at $8\frac{1}{4}$ per cent. is sufficient in 16 months to gain \$13.20? Ans. \$120.00.

9. What principal at $7\frac{1}{4}$ per cent. is sufficient in $6\frac{1}{2}$ years to amount to \$411.95? Ans. \$280.00.

SECTION XL.

COMPOUND INTEREST.

THE law specifies that the borrower of money shall pay a certain number of dollars, called *per cent.*, for the use of one hundred dollars for a year. Now, if this *borrower* does not pay to the *lender* this *per cent.* at the *end* of the year, it is no more than just that he should pay interest for the use of it, so long as he shall keep it in his possession; and this is called *Compound Interest.*

1. What is the compound interest of $300 for 3 years?
Ans. $57.30,4.

First Method.

$300, principal.
1.06
18.00, interest for 1 year.
300
318.00, amount for 1 year.
1.06
19.08, int. for second year.
318
337.08, amount for 2 years.
1.06
20.22,48, int. for third year.
337.08
357.30,48, amount for 3 years.
300
$57.30,48, compound int. for 3yrs.

Second Method.

5 = $\frac{1}{20}$) 300
1 = $\frac{1}{5}$) 15
3
5 = $\frac{1}{20}$) 318
1 = $\frac{1}{5}$) 15.90
3.18
5 = $\frac{1}{20}$) 337.08
1 = $\frac{1}{5}$) 16.85,4
3.37,0
357.30,4
300
$57.30,4

NOTE. — 5 per cent. is $\frac{1}{20}$ of the principal, and 1 per cent. is $\frac{1}{5}$ of 5 per cent.

RULE. — *Find the interest of the given sum for one year, and add it to the principal; then find the interest of this amount for the next year; and so continue until the time of settlement. Subtract the principal from the last amount, and the remainder is the compound interest*

2. What is the amount of $500 for 3 years?
Ans. $595.50,8.

3. What is the compound interest of $345 for 10 years?
Ans. $272.84,2.

4. What is the compound interest of $316 for 3 years 4 months and 18 days? Ans. $69.01,7.

By the aid of the following Table, calculations are more easily effected than by the preceding rule.

TABLE,

Showing the *amount* of 1 dollar, or 1 pound, for any number of years under 40, at 3, 4, 5, 6, and 7 per cent., compound interest.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	1.030000	1.040000	1.050000	1.060000	1.070000	1
2	1.060900	1.181600	1.102500	1.123600	1.144900	2
3	1.092727	1.124864	1.157625	1.191016	1.225043	3
4	1.125508	1.169858	1.215506	1.262476	1.310796	4
5	1.159274	1.216652	1.276281	1.338225	1.402552	5
6	1.194052	1.265319	1.340095	1.418519	1.500730	6
7	1.229873	1.315931	1.407100	1.503630	1.605781	7
8	1.266770	1.368569	1.477455	1.593848	1.718186	8
9	1.304773	1.423311	1.551328	1.689478	1.838459	9
10	1.343916	1.480284	1.628894	1.790847	1.967151	10
11	1.384233	1.539454	1.710339	1.898298	2.104852	11
12	1.425760	1.601032	1.795856	2.012196	2.252191	12
13	1.468533	1.665073	1.885649	2.132928	2.409845	13
14	1.512589	1.731676	1.979931	2.260903	2.578534	14
15	1.557967	1.800943	2.078928	2.396558	2.759032	15
16	1.604706	1.872981	2.182874	2.540351	2.952164	16
17	1.652847	1.947900	2.292018	2.692772	3.158815	17
18	1.702433	2.025816	2.406619	2.854339	3.379932	18
19	1.753506	2.106849	2.526950	3.025599	3.616528	19
20	1.806111	2.191123	2.653297	3.207135	3.869685	20
21	1.860294	2.278768	2.785962	3.399563	4.140563	21
22	1.916103	2.369918	2.925260	3.603537	4.430403	22
23	1.973586	2.464715	3.071523	3.819749	4.740530	23
24	2.032794	2.563304	3.225099	4.048934	5.072367	24
25	2.093777	2.665836	3.386354	4.291870	5.427434	25
26	2.156591	2.772469	3.555672	4.549382	5.807352	26
27	2.221289	2.883368	3.733456	4.822345	6.213868	27
28	2.287927	2.998703	3.920129	5.111686	6.648838	28
29	2.356565	3.118651	4.116135	5.418387	7.114257	29
30	2.427262	3.243397	4.321942	5.743491	7.612255	30
31	2.500080	3.373133	4.538039	6.088100	8.145112	31
32	2.575082	3.508058	4.764941	6.453386	8.715270	32
33	2.652335	3.648381	5.003188	6.840589	9.325339	33
34	2.731905	3.794316	5.253347	7.251025	9.978113	34
35	2.813862	3.946088	5.516015	7.686086	10.676581	35
36	2.898278	4.103932	5.791810	8.147252	11.423942	36
37	2.985226	4.268089	6.081406	8.636087	12.223618	37
38	3.074783	4.438813	6.385477	9.154252	13.079271	38
39	3.167026	4.616365	6.704751	9.703507	13.994820	39
40	3.262037	4.801020	7.039988	10.285717	14.974457	40

To perform questions by the Table, multiply the amount of one dollar

for the given rate and time found in the Table by the principal, and from the product subtract the principal, and the remainder is the compound interest.

NOTE.—If there be months and days, find the amount for them on the number taken from the Table, *before* it is multiplied by the principal.

EXAMPLES.

5. What is the compound interest of $ 360 for 5 years, 6 months, and 24 days? Ans. $138.14.

OPERATION.

```
   1.338225, amount of 1 dollar for 5 years.
      .034, ratio for 6 months and 24 days.
   --------
   5352900
  4014675
  ---------
 .045499650
1.338225
-----------
1.383724650, amount of 1 dollar for 5yrs. 6mo. 24 days.
        360
-----------
 83023479000
415117395
-----------
498.14,0874000, amount of principal for 5yrs. 6mo. 24 days.
360
------
$ 138.14, compound interest of the principal for do.
```

6. What is the interest of $ 890 for 30 years? Ans. $ 4221.70,6.

7. What is the amount of $ 480 for 40 years? Ans. $ 4937.14,4.

8. What is the interest of $ 300 for 10 years, 7 months, and 15 days? Ans. $ 257.40,1.

9. What is the amount of $ 586 for 12 years, 1 month, and 29 days, at 5 per cent.? Ans. $1060.99,5.

10. The probable number of blacks at this time (1835) in the United States is 2,500,000. Now, supposing their increase to be 25 per cent. for every 10 years, what will be their number in the year 1935? Ans. 23,283,057.

11. Supposing the annual increase of the people of color to be 2 per cent., how many must be sent out of the country each year, that their numbers might not increase? Ans. 50,000.

12. What is the amount of $ 900 for 7 years, at 5 per cent.? Ans. $1266.39.

13. What is the interest of $350 for 3 years, 3 months, and 24 days? Ans. $74.77,5+.

14. What is the interest of $970 for 2 years, 9 months, and 24 days? Ans. $173.29,5.

To find the amount of a note by compound interest, when there have been partial payments.

RULE. — *Find the amount of the principal, and from it subtract the amount of the indorsements.*

EXAMPLES.

15. A, by his note dated January 1, 1830, promises to pay B $500 on demand.

On this note are the following indorsements. July 16, 1830, received two hundred dollars. August 21, 1831, received two hundred dollars. December 1, 1832, received one hundred dollars.

What was the balance Sept. 1, 1834? Ans. $52.73.

(16.) $100.00. Boston, Sept. 25, 1833.

For value received, I promise Peter Absalom to pay him, or order, on demand, one hundred dollars, with interest after six months. J. P. Jay.

On this note are the following indorsements. June 11, 1834, received fifty dollars. Sept. 25, 1834, received fifty dollars.

What was due August 25, 1835? Ans. $2.24,7.

(17.) $1000.00. New York, January 1, 1840.

For value received, I promise to pay J. R. Montgomery, or order, on demand, one thousand dollars, with interest at seven per cent. John Q. Smith.

Attest, J. Page.

On this note are the following indorsements. June 10, 1840, received seventy dollars. Sept. 25, 1841, received eighty dollars. July 4, 1842, received one hundred dollars. Nov. 11, 1843, received thirty dollars. June 5, 1844, received fifty dollars.

What remains due April 1, 1845, at 7 per cent. compound interest? Ans. $1022.34.

(18.) $1700.00. Bradford, July 4, 1841

Lent A. Brown seventeen hundred dollars. Sept. 1, 1843, received one thousand dollars.

What is due July 4, 1847, at 5 per cent. compound interest. Ans. $1071.81,9.

SECTION XLI.

DISCOUNT.

THE object of Discount is to show us what allowance should be made, when any sum of money is paid before it becomes due.

The *present worth* of any sum is the principal that must be put at interest to amount to that sum in the given time. That is, $ 100 is the *present worth* of $ 106 due one year hence; because $ 100 at 6 per cent. will amount to $ 106, and $ 6 is the *discount.*

1. What is the present worth of $ 12.72, due one year hence?

First Method.

```
     $ 12.72
         100
106)1272.00($ 12 Ans.
    106
    ---
     212
     212
     ---
```

Second Method.

```
     $
1.06)12.72($ 12 Ans.
     10.6
     ----
      2.12
      2.12
      ----
```

As $ 100 will amount to $ 106 in one year at 6 per cent., it is evident, that, if $\frac{100}{106}$ of any sum be taken, it will be its present worth for one year, and that $\frac{6}{106}$ will be the discount. And, as $ 1 is the present worth of $ 1.06 due one year hence, it is evident that the present worth of $12.72 must be equal to the number of times $ 12.72 will contain $1.06.

RULE. — *Divide the given sum by the amount of $ 1 for the given rate and time, and the quotient will be the present worth. If the present worth be subtracted from the given sum, the remainder will be the discount.*

2. What is the present worth of $ 117.60, due one year hence, at 12 per cent.? Ans. $ 105.00.

3. What is the discount of $ 802.50, at 7 per cent., due one year hence? Ans. $ 52.50.

4. What is the present worth of $ 769.60, due 3 years and 5 months hence? Ans. $ 638.67,$2\frac{48}{241}$.

5. What is the present worth of $ 986.40, due 7 years 9 months and 20 days hence? Ans. $ 671.78,$2\frac{58}{881}$.

6. How much grain must be sent to the miller that a bushel of meal may be returned, the miller taking $\frac{1}{16}$ part for toll? Ans. $34\frac{2}{15}$ quarts.

7. What is the present worth of \$ 678.75, due 3 years 7 months hence, at $7\frac{1}{2}$ per cent.? Ans. \$ 534.97,$5\frac{75}{203}$.

8. I have given my note for \$ 1000, to be paid Dec. 18, 1835. What is the note worth June 7, 1834?

Ans. \$ 915.89$\frac{461}{6551}$.

9. James has a note against Samuel for \$ 715.50, dated August 17, 1834, which becomes due January 11, 1835. What ready money will pay the note Sept. 25, 1834?

Ans. \$ 703.07,8+.

10. A has B's note, which becomes due Nov. 25, 1835, for \$ 914.75. What is this note worth Jan. 1, 1835?

Ans. \$ 867.88,4+.

11. A merchant has given two notes; the first for \$ 79.87, to be paid Jan. 21, 1836; the second for \$ 87.75, to be paid Dec. 17, 1836. What ready money will discharge both notes Feb. 10, 1835? Ans. \$ 154.54,4+.

12. It being now Oct. 14, 1833, and A owing me \$ 1728, to be paid Dec. 17, 1837, what ought I now to receive as an equivalent? Ans. \$ 1381.84,$7\frac{658}{2501}$.

13. Bought cloth in Boston at \$ 5.00 per yard. What must be my "asking price," in order that I may *fall* on it 10 per cent. and still make 10 per cent. on my purchase?

Ans. \$ 6.11$\frac{1}{9}$.

14. James Ober owes Samuel Hall as follows: \$ 365.87, to be paid Dec. 19, 1835; \$ 161.15, to be paid July 16, 1836; \$ 112.50, to be paid June 23, 1834; \$ 96.81, to be paid April 19, 1838. What should he receive as an equivalent, Jan. 1, 1834? Ans. \$ 653.40+.

SECTION XLII.

PER CENTAGE.

THIS term is used to express so much by the hundred. It is derived from two Latin words, *per* and *centum*, and means *by the hundred*. It is not only applied to money, but to any commodity.

The process of operation is similar to Interest.

EXAMPLES.

1. Received a legacy of \$ 1728. I gave 10 per cent. of it to a benevolent society. How much had I remaining?

Ans. \$ 1555.20.

2. Bought a horse for $120, and sold him at 6 per cent. advance. What did I gain? Ans. $7.20.

3. Sent 1728 barrels of flour to Liverpool, but in a storm 25 per cent. of them were thrown overboard; how many remained? Ans. 1296 barrels.

4. A certain colonel, whose regiment consisted of 900 men, lost 8 per cent. of them in battle, and 50 per cent. of the remainder by sickness. How many had he remaining?
Ans. 414 men.

5. A merchant, having $1728 in the Union Bank, wishes to withdraw 15 per cent.; how much will remain?
Ans. $1468.80.

6. A gentleman, who had an estate of $25,000, gave in his will, to his wife, 40 per cent. of his property, and to his son Samuel, 30 per cent. of the remainder. The residue he divided equally between his daughters, Marcia, Isabella, and Clara, after having deducted $60 as a present to his clergyman. What did each receive?

Ans. Wife, $10,000; son, $4,500; daughters, $3,480 each.

7. What is 15 per cent. on 500 bushels? Ans. 75 bushels.
8. What is 20 per cent. on 75cwt.? Ans. 15cwt.
9. What is 30 per cent. on 150 tons? Ans. 45 tons.
10. What is 75 per cent. on $500? Ans. $375.
11. What is 95 per cent. on 700 chaldrons?
Ans. 665 chaldrons.
12. What is 2 per cent. on 40 miles? Ans. .8 miles.
13. What is 99 per cent. on $1000? Ans. $990.
14. What is $33\frac{1}{3}$ per cent on 144 barrels? Ans. 48bbl.
15. What is $66\frac{2}{3}$ per cent. on 90 hogsheads?
Ans. 60 hogsheads.
16. What is $\frac{1}{4}$ per cent. on $100? Ans. $0.25.
17. What is $\frac{7}{8}$ per cent. on 1728lb.? Ans. 15.12lb.

SECTION XLIII.

COMMISSION AND BROKERAGE.

COMMISSION AND BROKERAGE are compensations made to factors, brokers, and other agents, for their services, either for buying or selling goods.

NOTE. — A factor is an agent, employed by merchants *residing in other*

places to buy and sell, and to transact business on their account. A broker is employed by merchants to transact business.

Rule. — *The method of operation is the same as in Interest and Discount.*

EXAMPLES.

1. My agent in New Orleans has purchased cotton, on my account, to the amount of $18,768; what is his commission, at $1\frac{3}{4}$ per cent.? Ans. $328.44.

2. If a broker sells goods for me to the amount of $896, what is his commission, at 2 per cent.? Ans. $17.92.

3. My factor in London writes that he has purchased for me to the amount of 395£. 15s. 5d.; what is his commission, at $2\frac{1}{4}$ per cent.? Ans. 8£. 18s. $1\frac{65}{100}$d.

4. A factor receives $1976, which he is to lay out for goods; having deducted his commission of 4 per cent. how much will remain to be laid out? Ans. $1900.

Note. — As his commission is to be taken out of the sum *remitted* he will *not* receive 4 dollars on every 100, but 4 on every 104; that is, he will receive $\frac{4}{104}$.

5. What is a broker's commission for purchasing goods to the amount of $7658.75, at $1\frac{1}{2}$ per cent? Ans. $$114.88\frac{1}{8}$.

6. My factor at New Orleans advises me that he has purchased on my account 37 bales of cotton, at $107.75 per bale; what is his commission, at $\frac{3}{8}$ per cent.? Ans. $$14.95\frac{1}{32}$.

7. I have engaged a broker to purchase for me 12 shares in the Boston and Maine Railroad, at $112.25 per share; what is his commission, at $\frac{1}{4}$ per cent.? Ans. $$3.36\frac{3}{4}$.

8. My agent, S. Cloon, at Cincinnati, advises me that he has purchased on my account a cargo of pork, consisting of 700 barrels, at $12.25 per barrel; what is his commission, at $1\frac{3}{4}$ per cent.? Ans. $$150.06\frac{1}{4}$.

9. Sent to my agent, John Crowell, at Rochester, N. Y., $8960, to purchase a quantity of flour; his commissions are 2 per cent. on the purchase, which he is to deduct from the money sent him; what is his commission? Ans. $$175.68\frac{32}{51}$.

10. What is a broker's commission on the sale of 700 barrels of flour, at $5.75 per barrel, at $1\frac{3}{4}$ per cent.? Ans. $$70.43\frac{3}{4}$.

11. What is the commission on the sale of 173cwt. of sugar, at $8.95 per cwt., at $1\frac{7}{8}$ per cent.? Ans. $$29.03\frac{5}{32}$.

12. My agent in London has purchased goods for me to the value of 879£. 12s. 9d; what is his commission, at $3\frac{3}{8}$ per cent.? Ans. 29£. 13s. $9\frac{51}{800}$d.

13. Sent a cargo of flour to Liverpool, which my factor sold

for 987£. 18s. 6d. He invested this sum in broadcloths, at 1£. 3s. 8d. per yard. His commission for selling the flour is $2\frac{1}{4}$ per cent., and for purchasing the broadcloth $1\frac{7}{8}$ per cent., and he is to receive his commissions, for selling and buying, out of the proceeds of the flour. Required the number of yards of broadcloth that I should receive. Ans. $801\frac{3576}{57865}$yd.

14. I have remitted to my correspondent a certain sum of money, which he is to lay out for me in iron, and having reserved to himself $2\frac{1}{2}$ per cent. on the purchase, which amounted to $ 90, he buys for me the iron, at $ 95 per ton. Required the sum remitted, and the quantity of iron purchased.

Ans. { Sum remitted, $ 3690.
Iron purchased, 37T. 17cwt. 3qr. $16\frac{4}{19}$lb.

Section XLIV.

STOCKS.

Stocks is a general name used for funds established by government, or individuals, in their corporate capacity, the value of which is often variable.

When stocks will bring more in the market than their original cost, their value is said to be *above par*, and when, from any circumstance, their value is less than the original cost, they are said to be *below par*.

The method for computation is the same as in Interest.

EXAMPLES.

1. What is the value of $ 24360 of the National Bank stock, at 135 per cent.? $ 24360 × 1.35 = $ 32886 Ans.

2. Sold 15 shares, $ 100 each, of the Boston Bank, at 13 per cent. advance. To what did they amount? Ans. $1695.

3. What must I give for 12 shares in the Haverhill Bank, at 15 per cent. advance, shares being $100 each? Ans. $1380.

4. What is the purchase of 1058£. 12s. bank stock, at $115\frac{3}{4}$ per cent.? Ans. 1225£. 6s. $7\frac{2}{25}$d.

5. Sold 30 shares, $100 each, in the Boston and Providence Railroad, at $8\frac{3}{4}$ per cent. advance. To what did they amount? Ans. $ 3262.50.

6. What is the value of 10 shares in the Philadelphia and Trenton Railroad stock, at 85 per cent., original shares being $100? Ans. $ 850.

7. What must be given for 5 shares of the stock in the Ocean Insurance Company, at 7 per cent. advance, the original shares being $100? Ans. $535.

SECTION XLV.

INSURANCE AND POLICIES.

INSURANCE is a security, by paying a certain sum, to indemnify the secured against such losses as shall be specified in the policy.

POLICY is the name of the writ, or instrument, by which the contract or indemnity is effected between the parties.

NOTE. — If the average loss does not exceed 5 per cent. the *underwrit ers* are free, and the insured bears the loss himself.

RULE. — *The same as in Interest and Discount.*

EXAMPLES.

1. What would be the amount of the insurance on my house, valued at $5728, at 1¾ per cent.? Ans. $100.24.

2. What would be the premium for insuring my good ship Betsey, for a voyage from Boston to Liverpool, at 1¼ per cent., the vessel being valued at $17,289. Ans. $216.11¼.

3. My ship Massachusetts is valued at $50,765. If I insure $10,000 at the Columbian Office at 4¾ per cent., $12,000 at the Atlas Office at 3⅞, and the vessel is cast away on her voyage, what is the amount of my loss? Ans. $29,705.

4. A gentleman in Boston effected an insurance on his store and goods, valued at $47,600, for 5 years. For the first year he is to pay 4¼ per cent., for the second year 3⅞ per cent., for the third year 4⅔ per cent., for the fourth year 5 per cent., and for the fifth year 5⅓ per cent. What is the amount of his whole insurance? Ans. $11,007.50.

5. What is the premium on $1728 at 7¼ per cent? Ans. $125.28.

6. My ship, the Julia Ann, is valued at $35,000, and her cargo at $75,000. I procure an insurance on ⅗ the value of the ship, at 3¼ per cent., and on ⅔ of her cargo, at 2½ per cent. What is the amount of premium? Ans. $1932.50.

7. The good ship Marcia Demming is valued at $18,750; her cargo cost $37,960, and, being bound to Canton, I have got

$10,000 insured on the vessel, at $3\frac{3}{8}$ per cent., and $20,000 on her cargo, at $4\frac{3}{4}$ per cent. What would be my loss if the vessel should founder at sea? Ans. $27,997.50.

8. My library consists of 2691 volumes, valued at $3675. If I get this insured, at $4\frac{7}{8}$ per cent, what is my actual loss if it be destroyed? Ans. 179.15\frac{5}{8}$.

9. What is the premium on $896, at 12 per cent.? Ans. $107.52.

10. What is the premium on $850, at $18\frac{1}{2}$ per cent.? Ans. $157.25.

11. What is the premium for insuring $9870, at 7 per cent.? Ans. $690.90.

12. What sum will be secured for a policy of $1728, deducting 15 per cent.? Ans. $1468.80.

13. What sum must a policy be taken out for, to cover $2475, when the premium is 10 per cent.? Ans. $2750.

NOTE. — As 10 per cent. is already taken out, the sum covered must be $\frac{90}{100}$ of the policy.

14. A certain company own a cotton factory, valued at $26,250. For what sum must a policy be taken out to cover the whole property, at $12\frac{1}{2}$ per cent.? Ans. $30,000.

15. If a policy be taken out for $3600, at 40 per cent., what is the sum covered? Ans. $2160.

NOTE. — It is evident, that, if 40 per cent. be taken from any sum, 60 per cent. will be left.

16. If a policy be taken out for $600, at 10 per cent., what is the sum covered? Ans. $540.

17. A merchant adventured $1000 from Boston to New Orleans, at 3 per cent.; from thence to Chili, at 5 per cent.; from thence to Canton, at 6 per cent.; and from thence to Boston, at 7 per cent. For what sum must he take out a policy, to cover his adventure the voyage round? Ans. $1241.34,8+.

SECTION XLVI.

BANKING.

A BANK is a place of deposit for money, which is usually divided into shares, and owned by persons called *stockholders.*

Its concerns are managed by a board of directors. It issues notes or bills of its own, intended to be a circulating medium of exchange or currency, instead of gold and silver. These bills are obtained from the bank by loans. When money is hired from a bank, it is the usual custom to deduct the interest at the time of receiving it. A man, therefore, who hires money from a bank, gives his note for a sum as much larger than he receives, as is the interest of the note for the given time. If a man, therefore, gives his note for $100, payable in 63 days, he *receives* only $ 98.95.

A promissory note is said to be *discounted*, when it is received at a bank as a security for money taken from it; and the interest deducted is the discount. The interest on every note discounted at a bank is computed for 3 days more than the time specified in the note; that is, if the note is payable in 60 days, the interest is taken for 63 days; for the law allows 3 days to the debtor, after the time has expired for payment, which are called *days of grace*.

The rule for computing the discount is the same as in simple interest.

EXAMPLES.

1. What is the bank discount on $ 476, for 30 days and grace? Ans. $ 2.61,8.

2. What is the bank discount on $1000, for 60 days and grace? Ans. $10.50.

3. What is the bank discount on $ 7800, for 90 days and grace? Ans. $120.90.

4. What is the bank discount on $ 8000, for 60 days and grace? Ans. $ 84.00.

5. How much money should be received on a note for $ 760, payable in 5 months, discounted at a bank when the interest is 6 per cent.? Ans. $ 740.62.

6. What sum is paid at a bank for a note of $1728, payable in 3 months? Ans. $1701.21,6.

7. A merchant sold a cargo of hemp for $ 7860, for which he received a note payable in 6 months. How much money will he receive at a bank for this note? Ans. $ 7620.27.

8. A merchant bought 450 quintals of fish at $ 3.50 cash, and sold them immediately for $ 4.00 on 6 months' credit, for which he received a note. If he should get this note discounted at a bank, what will he gain on the fish? Ans. $170.10.

SECTION XLVII.

BARTER.

BARTER is the exchange of one kind of merchandise for another, without loss to either party.

Questions in this rule are solved by finding what quantity of goods, at a given price, of one kind, are equal in value to another kind of goods whose price is also given.

EXAMPLES.

1. How much sugar, at $12\frac{1}{2}$ cents per lb., must be given in barter for 760lb. of raisins, at 8 cents per lb.? Ans. $486\frac{2}{5}$lbs.

2. What quantity of coffee, at 17 cents per lb., must be given in barter for 760lb. of tea, at $62\frac{1}{2}$ cents per lb.? Ans. $2794\frac{2}{17}$lbs.

3. A merchant delivered 3 hogsheads of wine, at $1.10 per gal., for 126yd. of cloth; what was the cloth per yd.? Ans. $1.65.

4. A has 12cwt. of sugar, worth 8 cents per lb., for which B gave him $1\frac{3}{4}$cwt. of cinnamon; at what did B value his cinnamon? Ans. $ $0.54\frac{6}{7}$ per lb.

5. A had 41cwt. of hops, at $ 6.70 per cwt., for which B gave him 17cwt. 3qr. 4lb. of prunes, and $ 88; what were the prunes valued at per lb.? Ans. $ $0.09\frac{371}{996}$.

6. A has sugar which he barters with B, for 4 cents per lb. more than it cost him, against tea which cost B 40 cents per lb., but which he puts in barter at 50 cents. What did A's sugar cost him per lb.? Ans. $0.16.

7. How many staves, at $25 per thousand, must a merchant receive for 15 hogsheads of wine, at $1.25 per gallon? Ans. $47\frac{1}{4}$M. staves.

8. Q has 670 bushels of oats, which cost him 35 cents per bushel; these he barters with Z, at 50 cents per bushel, for flour that cost Z $ 5.00 per barrel. What is the bartering price of the flour, and how much will Q receive? Ans. $46\frac{9}{10}$ barrels of flour, at $ $7.14\frac{2}{7}$ per barrel.

9. S. Jenkins has $73\frac{92}{107}$ bushels of corn, which is worth 7s. per bushel; but in barter he is willing to put it at 6s. 8d., provided he can have wheat worth 7s. 6d. per bushel for 7s. 3d. Will he gain or lose, and how much per cent.? Ans. Lose $1\frac{3}{7}$ per cent.

Section XLVIII.

PRACTICE.

Practice is an expeditious way of performing questions in Compound Multiplication and Proportion.

Rule. — *Assume the price at some unit higher than the given price; that is, if the price be pence, or pence and farthings, assume the price at a shilling a yard, or pound, &c.; if the price be in shillings, or shillings and pence, &c., assume the price at a pound a yard, &c.; then take the aliquot parts of a pound.*

TABLE,

Showing the aliquot parts of Money and Weights.

Parts of a £.

s.	d.		
10		is	$\frac{1}{2}$
6	8	"	$\frac{1}{3}$
5		"	$\frac{1}{4}$
4		"	$\frac{1}{5}$
3	4	"	$\frac{1}{6}$
2	6	"	$\frac{1}{8}$
2		"	$\frac{1}{10}$
1	8	"	$\frac{1}{12}$
1		"	$\frac{1}{20}$

Parts of a shilling.

d.		
6	is	$\frac{1}{2}$
4	"	$\frac{1}{3}$
3	"	$\frac{1}{4}$
2	"	$\frac{1}{6}$
$1\frac{1}{2}$	"	$\frac{1}{8}$
1	"	$\frac{1}{12}$

Parts of a ton.

cwt.	qr.		
10		is	$\frac{1}{2}$
5		"	$\frac{1}{4}$
4		"	$\frac{1}{5}$
2	2	"	$\frac{1}{8}$
2		"	$\frac{1}{10}$
1		"	$\frac{1}{20}$

Parts of a cwt.

qr.	lb.		
2		is	$\frac{1}{2}$
1		"	$\frac{1}{4}$
	16	"	$\frac{1}{7}$
	14	"	$\frac{1}{8}$
	8	"	$\frac{1}{14}$
	7	"	$\frac{1}{16}$
	4	"	$\frac{1}{28}$
	2	"	$\frac{1}{56}$

Parts of a half-cwt.

lb.		
28	is	$\frac{1}{2}$
14	"	$\frac{1}{4}$
8	"	$\frac{1}{7}$
7	"	$\frac{1}{8}$
4	"	$\frac{1}{14}$
$3\frac{1}{2}$	"	$\frac{1}{16}$
2	"	$\frac{1}{28}$

Parts of a quarter-cwt.

lb.		
14	is	$\frac{1}{2}$
7	"	$\frac{1}{4}$
4	"	$\frac{1}{7}$
$3\frac{1}{2}$	"	$\frac{1}{8}$
2	"	$\frac{1}{14}$
1	"	$\frac{1}{28}$

EXAMPLES.

1. What will 368 yards of ribbon cost, at 6 pence a yard? Ans. 9£. 4s.

OPERATION.

$$\begin{array}{l} 6d. = \frac{1}{2})368s. \\ \quad 20)\overline{184} \\ \qquad 9£.\ 4s. \end{array}$$

We assume the price at a shilling a yard, and then say, if 368 shillings be the price at a shilling a yard, at 6 pence it must be half as much, viz. 184 shillings. We then reduce the shillings to pounds.

2. What will 4785 yards of cotton cost, at 8 pence a yard? Ans. 159£. 10s.

OPERATION.

$$\begin{array}{l} 6d. = \frac{1}{2})4785s. = \text{price at 1 shilling.} \\ 2d. = \frac{1}{3})\overline{2392s.\ 6d.} = \text{price at 6d.} \\ \qquad\quad 797s.\ 6d. = \text{price at 2d.} \\ \quad 20)\overline{3190s.\ 0d.} \\ \qquad\quad 159£.\ 10s. = \text{price at 8d.} \end{array}$$

Having found the price at 6d. as before, we find it for the 2d. by saying that 2d. is $\frac{1}{3}$ of 6d.

3. What is the interest of \$ 368, at 15 per cent.? Ans. \$ 55.20.

OPERATION.

$$\begin{array}{r} 10 \text{ per cent.} = \frac{1}{10})368 \\ 5 = \frac{1}{2})\overline{36.80} \\ 18.40 \\ \hline \$\ 55.20 \end{array}$$

10 per cent. is $\frac{1}{10}$ of the principal. and 5 per cent. is $\frac{1}{2}$ of 10 per cent.

4. What is the value of 17 acres 3 roods 35 rods of land, at \$ 80 per acre? Ans. \$ 1437.50.

OPERATION.

$$\begin{array}{rl} & \$\ 80 \\ & 17 \\ \hline & 560 \\ & 80 \\ \hline & \$1360 = \text{price of 17A.} \\ 2R. = \frac{1}{2}) & 40 = \text{do. of 2R.} \\ 1R. = \frac{1}{2}) & 20 = \text{do. of 1R.} \\ 20rd. = \frac{1}{2}) & 10 = \text{do. of 20rd.} \\ 10rd. = \frac{1}{2}) & 5 = \text{do. of 10rd.} \\ 5rd. = \frac{1}{2}) & 2.50 = \text{do. of 5rd.} \\ \hline & \$\ 1437.50 = \text{price of 17A. 3R. 35rd.} \end{array}$$

By dividing the price of 1 acre by 2, we obtain the price of 2R.; and by halving this, we find the price of 1R.; and as 20 rods is half of a rood, its value will be one half; and in the same manner 10 rods will be half the price of 20 rods, and 5 rods will be half the price of 10 rods.

5. What cost 14 tons 15cwt. 3qr. 21lb. of iron, at \$ 120 per ton? Ans. \$1775.62,5.

OPERATION.

	\$120	
	14	
	\$1680.00	= price of 14T.
10cwt. = $\frac{1}{2}$)	60.00	= do. of 10cwt.
	\$1740.00	
5cwt. = $\frac{1}{2}$)	30.00	= do. of 5cwt.
2qr. = $\frac{1}{10}$)	3.00	= do. of 2qr.
1qr. = $\frac{1}{2}$)	1.50	= do. of 1qr.
14lb. = $\frac{1}{2}$)	.75	= do. of 14lb.
7lb. = $\frac{1}{2}$)	.375	= do. of 7lb.
	\$1775.62,5	= do. of 14T. 15cwt. 3qr. 21lb.

6. What cost 387lb. of sugar, at 9 pence a pound? Ans. 14£. 10s. 3d.

7. What cost 498lb. of green tea, at 2 shillings and 6 pence per pound? Ans. 62£. 5s. 0d.

8. What cost 384 yards of cloth, at 4 shillings and 9 pence a yard? Ans. 91£. 4s. 0d.

9. What cost 714 yards of broadcloth, at 15 shillings and 6 pence per yard? Ans. 553£. 7s. 0d.

10. What cost 16cwt. 3qr. 10lb. of copperas, at \$2.50 per cwt.? Ans. \$42.09,8.

11. What cost 27cwt. 1qr. 21lb. of coffee, at \$14 per cwt.? Ans. \$384.12$\frac{1}{2}$.

12. What cost 7 tons 13cwt. 2qr. 7lb. of hay, at \$24.60 per ton? Ans. \$188.88,1$\frac{7}{8}$.

13. If 1 acre of land cost \$80.50, what will 25 acres 2 roods 35 rods cost? Ans. \$2070.35,9$\frac{3}{8}$.

14. If 1 acre cost \$32.32, what will 51A. 0R. 15rd. cost? Ans. \$1651.35.

15. If 1 yard of cloth cost \$5.60, what will 7yd. 3qr. 2na. cost? Ans. \$44.10.

16. What is the premium on \$6780, at 12$\frac{1}{2}$ per cent.? Ans. \$847.50.

17. What is the interest of \$1728 for 5 years 7 months and 20 days? Ans. \$584.64.

18. What will 19 tons 19cwt. 3qr. 27$\frac{1}{2}$lb. of copperas cost, at 19£. 19s. 11$\frac{3}{4}$d. per ton? Ans. 399£. 19s. 5$\frac{16641}{17920}$d.

19. The estimated distance of a certain railroad is 14m. 3fur. 35rd. 10ft.; what would be the expense of constructing it, at \$18675 per mile? Ans. \$270531.07+.

SECTION XLIX.

EQUATION OF PAYMENTS.

WHEN several sums of money, to be paid at different times, are reduced to a *mean* time for the payment of the whole, without gain or loss to the debtor or creditor, it is called Equation of Payments.

EXAMPLES.

1. A owes B \$19, \$5 of which is to be paid in 6 months, \$6 in 7 months, and \$8 in 10 months. What is the medium time for the payment of the whole?

OPERATION.

$$
\begin{array}{l}
\$5 \times 6 = 30 \\
\$6 \times 7 = 42 \\
\$8 \times 10 = 80 \\
\overline{19} \qquad 19)\overline{152}(8 \text{ months.} \\
\qquad\qquad 152
\end{array}
$$

By analysis. \$5 for 6 months is the same as \$1 for 30 months; and \$6 for 7 months is the same as \$1 for 42 months; and \$8 for 10 months is the same as \$1 for 80 months; therefore \$1 for $30 + 42 + 80 = 152$ months is the same as \$5 for 6 months, \$6 for 7 months, and \$8 for 10 months; but \$5, \$6, and \$8 are \$19; therefore \$1 for 152 months is the same as \$19 for $\frac{1}{19}$ of 152 months, which is 8 months, as before. Hence the propriety of the following

RULE.* — *Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the true time required.*

2. A owes B \$300, of which \$50 is to be paid in 2 months, \$100 in 5 months, and the remainder in 8 months. What is the equated time for the whole sum? Ans. 6 months.

* This is the rule usually adopted by merchants, but it is not *perfectly* correct; for if I owe a man \$200, \$100 of which I was to pay *down*, and the other \$100 in two years, the equated time for the payment of both sums would be one year. It is evident, that, for deferring the payment of the first \$100 for 1 year, I ought to pay the amount of \$100 for that time, which is \$106; but for the other \$100, which I pay a year *before* it is due, I ought to pay the *present worth* of \$100, which is \$94.33$\frac{51}{53}$, whereas, by Equation of Payments, I only pay \$200. *Strict justice* would therefore demand that *interest* should be required on all sums from the time they become due until the time of payment, and the *present worth* of all sums paid *before* they are due. The better rule would be, to find the present worth on each of the sums due, and then find in what time the sum of these present worths would amount to the payments.

3. There is owing to a merchant $1000; $200 of it is to be paid in 3 months, $300 in 5 months, and the remainder in 10 months. What is the equated time for the payment of the whole sum? Ans. 7 months 3 days.

4. A owes B $150, $50 to be paid in 4 months, and $100 in 8 months. B owes A $250 to be paid in 10 months. It is agreed between them that A shall make present payment of his whole debt, and that B shall pay his so much sooner as to balance the favor. I demand the time at which B must pay the $250. Ans. 6 months.

5. A merchant has $144 due him, to be paid in 7 months, but the debtor agrees to pay one half ready money, and one third in 4 months. What time should be allowed him to pay the remainder? Ans. 2 years 10 months.

6. There is due to a merchant $800, one sixth of which is to be paid in 2 months, one third in 3 months, and the remainder in six months; but the debtor agrees to pay one half *down*. How long may the debtor retain the other half so that neither party may sustain loss? Ans. 8⅔ months.

7. I have purchased goods of A. B. at sundry times and on various terms of credit, as by the statement annexed. When is the *medium* time of payment?

Jan.	1,	a bill amounting to	$375.50	on 4 months' credit.
"	20,	" "	168.75	on 5 months' credit.
Feb.	4,	" "	386.25	on 4 months' credit.
March	11,	" "	144.60	on 5 months' credit.
April	7,	" "	386.90	on 3 months' credit.

FORM OF STATEMENT.

Due May 1,	$375.50			
June 20,	168.75	×	50 =	843750
June 4,	386.25	×	34 =	1313250
Aug. 11,	144.60	×	102 =	1474920
July 7,	386.90	×	67 =	2592230

$1462.00 . . . . . .)6224150(42$\frac{1675}{2924}$da. Ans.
584800
376150
292400
83750

The medium time of payment will therefore be 42$\frac{1675}{2924}$ days, that is, 43 days from May 1, which will be June 12.

8. I have sold to C. D. several parcels of goods, at sundry times, and on various terms of credit, as by the statement annexed.

Jan.	1, a bill	amounting to	$ 600	on 4 months' credit.
Feb.	7, "	"	370	on 5 months' credit.
March	15, "	"	560	on 4 months' credit.
April	20, "	"	420	on 6 months' credit.

When is the equated time for the payment of all the bills?

Ans. July 11.

9. Purchased goods of John Brown, at sundry times, and on various terms of credit, as by the statement annexed.

March	1, 1845,	a bill	amounting to	$ 675.25	on 3 months.
July	4,	"	"	376.18	on 4 months.
Sept.	25,	"	"	821.75	on 2 months.
Oct.	1,	"	"	961.25	on 8 months.
Jan.	1, 1846,	"	"	144.50	on 3 months.
Feb.	10,	"	"	811.30	on 6 months.
March	12,	"	"	567.70	on 5 months.
April	15,	"	"	369.80	on 4 months.

What is the equated time for the payment of the above bills?

Ans. March 16, 1846.

SECTION L.

CUSTOM-HOUSE BUSINESS.

In every port of the United States where merchandise is either exported or imported, there is an establishment called a Custom-house. Connected with this are certain officers, appointed by government, called custom-house officers, whose business is to collect the duties on various kinds of merchandise, &c., imported into the United States.

The following article on Allowances, &c., was very politely furnished the author by the officers of the Boston custom-house, and may therefore be relied on as perfectly correct.

Allowances.

Draft is an allowance made by the officers of the United States government in the collection of duties on merchandise liable to a specific duty, and ascertained by weight, and is also

given by the usage of merchants in buying and selling. It is a deduction from the actual gross weight of the article paying duty by the pound or sold by weight.

For example, a box of sugar *actually* weighs 500 pounds. The draft upon this weight is 4 pounds.

500 gross.
4 draft.
496 difference. Upon this difference is made a further allowance of fifteen per cent. as *tare*, or as the actual weight of the box before the sugar was put into it. This tare is allowed by the government in the collection of the duty, and by the merchant in buying and selling. Take, then, the box of sugar, say

500lb. gross.
4 draft.
496 difference.
74 tare.
422 net weight, upon which a duty is paid to the government, or price is paid to the merchant in his sale. This tare of 74 pounds, or 15 per cent., is usually more than the *actual tare*, but is assumed as the *probable or actual tare*, by reason of the impossibility of "starting" every box to ascertain the *actual* weight of the sugar, and the *actual* weight of the box which contains it. This *tare* is sufficiently correct for the collector of the duty, and the merchant who deals in the article. It is intended to be a liberal allowance, and varies but little from the *actual* tare.

Drafts allowed at the custom-house in the collection of duties, and by the merchants in their purchases and sales, are as follow : —

Allowance for Draft.

Draft is another name for *Tret*, which is an allowance in weight for waste.

	lb.			lb.	lb.
On	112				1
Above	112	and not exceeding		224	2
"	224	"	"	336	3
"	336	"	"	1120	4
"	1120	"	"	2016	7
"	2016				9

EXPLANATION. — Many articles of merchandise are weighed separately; for example, boxes and casks of sugar, chests of

tea and indigo. Upon each box or cask, or chest, an allowance should be made for draft, according to its weight, as by the above rule. Bags of sugar and coffee, or bars of iron and bundles of steel, might be weighed together; say, 10 bags of coffee at one draft might weigh 1121 pounds; from this gross weight must be deducted 7 pounds as draft; 35 bars of Russia iron might be weighed at one draft, — weight 2250 pounds, upon which would be an allowance of 9 pounds draft, and by law and usage there can be no greater allowance than 9 pounds for draft. A greater or less number of bags of coffee, or bars of iron, or any other article of merchandise, is weighed, and the deduction is according to the weight of each draft. An old rule, and probably a better one, among merchants was the allowance of ½ per cent. on the gross weight of all merchandise weighed, *as draft.*

Allowance for Leakage.

Two per cent. is allowed on the gauge of ale, beer, porter, brandy, gin, molasses, oil, wine, and rum, and other liquors in casks, besides the *real* wants of the cask; for example, a cask of molasses may gauge 140 gallons, gross gauge; from this first deduct 5 gallons, the *actual* wants, or the quantity necessary to fill the cask, — we have

140	gross.
5	out.
135	difference.
3	two per cent. for leakage.
132	gallons net.

Tare is an allowance made for the *actual or supposed weight* of the cask, box, case, or bag, which contains the article of merchandise.

The usage of merchants is in conformity with the law and usage of the officers of the customs in their allowance for tare, directed by law, or found to be correct by their examination and experience.

The tariff of the United States being in its details so unsettled, it is deemed advisable not to insert any table.

EXAMPLES.

1. Find the net weight of a hogshead of sugar, weighing gross 1228lb., tare 12 per cent. Ans. 1075lb.

OPERATION.

1228lb. gross weight.
7lb. draft.
1221
12 per cent. of 1221lb. = 146lb. tare.
1075lb. net weight.

NOTE. — For draft let the pupil examine page 202.

2. Required the net weight of 6 boxes of sugar, weighing gross as follows, the tare being 15 per cent. : —

No. 1, 450 lbs.
No. 2, 470 do.
No. 3, 510 do.
No. 4, 496 do.
No. 5, 468 do.
No. 6, 520 do.
Gross, 2914

OPERATION.

2914lb. gross.
$6 \times 4 =$ 24lb. draft.
2890
Tare 15 per cent. 433
2457lb. net weight.

3. What is the net weight of 4 chests of tea, which weigh as follows, tare 22 per cent.? Ans. 384lb.

No. 1, 120 lbs.
No. 2, 116 do.
No. 3, 126 do.
No. 4, 118 do.
480 do.

OPERATION.

480lb. gross.
8lb. draft.
472
$22 \times 4 =$ 88lb. tare.
384lb. net.

4. What is the net weight of 5 bags of pepper, weighing as follows: 108lb., 112lb., 100lb., 120lb., and 98lb., tare 2 per cent.? Ans. 524.

OPERATION.

538lb. gross.
3lb. draft.
535
2 per cent. 11lb. tare.
524lb. net.

5. What is the net weight of 4 kegs of mace, weighing as follows: 112lb., 120lb., 118lb., and 110lb., tare 33 per cent.? Ans. 305lb.

OPERATION.

460lb. gross.
5lb. draft.
455
33 per cent. 150lb. tare.
305lb. net.

NOTE. — In making allowances, if there be a fraction of more than half a pound, 1 pound is added to the tare

AMERICAN DUTIES.

The duties on merchandise imported into the United States are either specific or ad valorem duties.

Specific duty is a certain sum paid on a ton, hundred weight, pound, square yard, gallon, &c.; but when the duty is a certain per cent. on the actual cost of the goods in the country from which they are imported, it is called an ***ad valorem duty***, that is, a duty according to the value of the article.

6. What is the duty on 6 hogsheads of sugar, weighing gross as follows: No. 1, 1276lb., No. 2, 1280lb., No. 3, 1178lb., No. 4, 1378lb., No. 5, 1570lb., No. 6, 1338lb.; duty $2\frac{1}{2}$ cents per lb., tare 12 per cent.? Ans. $175.52,5.

7. What is the duty on an invoice of woollen goods, which cost in London £986 sterling, at 44 per cent. ad valorem, the pound sterling being $4.84? Ans. $2099.78+.

8. Required the duty on 5 pipes of Port wine, gross gauge as follows: No. 1, 176 gallons, No. 2, 145 gallons, No. 3, 128 gallons, No 4, 148 gallons, No. 5, 150 gallons; *wants* of each pipe, 4 gallons; duty 15 cents per gallon. Ans. $106.80.

9. Required the duty on a cargo of iron, weighing 270 tons, at $30 per ton? Ans. $8100.

10. Compute the duty on 7890 pounds of tarred cordage, at 4 cents per pound; duty $1\frac{3}{4}$ per cent. Ans. $310.08.

11. What duty should be paid on 10 casks of nails, weighing each 450lb. gross, at 4 cents per lb.? Ans. $164.12.

SECTION LI.

RATIO.

RATIO is the relation which one quantity bears to another of the same kind with respect to magnitude; and the comparison is made by considering how often the one is contained in the other, or how often the one contains the other. Thus, the ratio of 12 to 3 is expressed by dividing 12 by $3 = \frac{12}{3} = 4$, ratio; or it may be expressed by dividing 3 by $12 = \frac{3}{12} = \frac{1}{4}$, ratio. The former is the method by which the English mathematicians express ratio, and the latter is the French method.

The former of these quantities is called the *antecedent*, and the latter the *consequent*.

When the antecedent is equal to the consequent, it is called a ratio of equality; thus the ratio of 6 to $6 = \frac{6}{6} = 1$. But if the antecedent be *larger* than the consequent, it is a ratio of *greater inequality;* and if the antecedent be less than the consequent, it is a ratio of *less inequality.*

The antecedent and consequent are called the *terms* of the ratio; and the quotient of the two terms is the *index* or *exponent* of the ratio.

Compound ratio is made up of two or more ratios, by multiplying their terms and exponents together.

The ratio of 8 to 6 and of 4 to 2 may be compounded; thus,

$$8 \text{ to } 6 = \tfrac{8}{6};\ 4 \text{ to } 2 = \tfrac{4}{2};$$
$$8 \times 4 \text{ to } 6 \times 2 = \tfrac{8 \times 4}{6 \times 2};\ 32 \text{ to } 12 = \tfrac{32}{12}.$$

If a ratio be compounded of two equal ratios, it is called a *duplicate* ratio; of three ratios, it is called a *triplicate* ratio, &c.

Thus, if the ratio of 4 to 2 be 2, and the ratio of 6 to 3 be 2, the ratio of 4×6 to 2×6 will be 2×2, that is, the ratio of 24 to 12 will be 2^2, &c.

If the terms of a ratio be *prime* to each other, no quantities can be found in the same ratio but what would be multiples thereof.

Numbers that are prime to each other are the least of all numbers in the same ratio.

If, therefore, we wish to ascertain whether the ratio of 3 to 7 is greater or less than the ratio of 4 to 9, since these ratios are represented by the fractions $\frac{3}{7}$ and $\frac{4}{9}$, we reduce them to a common denominator, $\frac{27}{63}$ and $\frac{28}{63}$; and, since the latter of these is greater than the former, it is evident that the ratio of 3 to 7 is less than the ratio of 4 to 9.

If we have the terms of a ratio given in large numbers, that are prime to each other, and we wish to find a ratio nearly equivalent, whose terms are expressed by smaller numbers, we adopt the following

RULE. — *Divide the greater term by the less, and that divisor by the remainder, as in Sect.* XVI., *Case* I., *of Vulgar Fractions. Then, if the antecedent be greater than the consequent, the first quotient divided by* 1 *gives the first ratio; if less, a unit divided by the first quotient will express the first ratio.*

Multiply the terms of the first ratio by the second quotient, and add a unit to the numerator or denominator, according as the antecedent of the original terms is greater or less than its consequent, and we have the second ratio.

Then, as a general principle, we multiply the terms of the ratio last

found by the next succeeding quotient, and to the product we add the corresponding terms of the preceding ratio, and we have the next succeeding ratio; and thus we proceed until there is no remainder, or until we have arrived at a sufficient approximation.

1. Let it be required to find a series of ratios in less numbers, constantly approaching to the ratio of 314159 to 100000, which is nearly the ratio of the circumference of a circle to its diameter.

OPERATION.

```
100000)314159(3
       300000
       ------
        14159)100000(7
               99113
               -----
                 887)14159(15
                     13305
                     -----
                       854)887(1
                           854
                           ---
                            33, &c.
```

$3 = \frac{3}{1}$, the first ratio.

$\frac{(3\times7)+1}{1\times7} = \frac{22}{7}$, the second ratio, being the approximation of Ar- [chimedes.

$\frac{22\times15+3}{7\times15+1} = \frac{333}{106}$, the third ratio.

$\frac{333\times1+22}{106\times1+7} = \frac{355}{113}$, the fourth ratio, the approximation of Metius.

Hence $\frac{314159}{100000} = 3 + \frac{1}{7} + \frac{1}{15} + \frac{1}{1}$, &c., in a continued fraction.

SECTION LII.

PROPORTION.

PROPORTION is the likeness or equalities of ratios. Thus, because 5 has the same relation or ratio to 10 that 8 has to 16, we say such numbers are in *proportion* to each other, and are therefore called *proportionals.*

If any four numbers whatever be taken, the first is said to have the same ratio or relation to the second, that the third has to the fourth, when the first number or term contains the second

as many times as the third contains the fourth, or when the second contains the first as many times as the fourth does the third. Thus, 8 has the same ratio to 4 that 12 has to 6, because 8 contains 4 as many times as 12 does 6. And 3 has the same relation to 9 that 4 has to 12, because 9 contains 3 as many times as 12 does 4. Ratios are represented by colons, and the equalities of ratios by double colons.

3 : 9 :: 8 : 24 is read thus: — 3 has the same ratio or relation to 9 as 8 to 24. The first and third numbers of a proportion are called *antecedents*, and the second and fourth are called *consequents;* also, the first and fourth are called *extremes*, and the second and third are called *means*.

Whatever four numbers are proportionals, if their antecedents or consequents be multiplied or divided by the same numbers, they are still proportionals; and if the terms of one proportion be multiplied or divided by the corresponding term of another proportion, their products and quotients are still proportionals.

This will appear evident from the various changes that the following example admits.

$4 : 8 :: 3 : 6$ Directly.
$8 : 4 :: 6 : 3$ By inversion.
$4 : 3 :: 8 : 6$ By permutation.
$4+8 : 8 :: 3+6 : 6$ By composition.
$4 : 4+8 :: 3 : 8+6$ By composition.
$4 : 8-4 :: 3 : 6-3$ By division.
$8-4 : 8 :: 6-3 : 6$ By division.
$4 \times 4 : 8 \times 8 :: 3 \times 3 : 6 \times 6$ By compound ratios.
$\frac{4}{3} : \frac{8}{6} :: \frac{3}{4} : \frac{6}{8}$ By division.

That the product of the extremes is equal to that of the means is evident from the following consideration. Let the following proportionals be taken. 12 : 3 :: 8 : 2. From the definition of proportion, the first term contains the second as many times as the third does the fourth; therefore, $\frac{12}{3} = \frac{8}{2}$; but $\frac{12}{3} = \frac{24}{6}$, and $\frac{8}{2} = \frac{24}{6}$; and if 24, the numerator of the first fraction, which is a substitute for the first term, be multiplied by 6, the denominator of the second fraction, and a substitute for the fourth term, the product will be the same as if 6, the denominator of the first fraction, and a substitute for the second term, be multiplied by 24, the numerator of the second fraction, and a substitute for the third term. Thus $24 \times 6 = 6 \times 24$. Therefore the product of the extremes is, in all cases, equal to that of the means.

If, then, one of the extremes be wanting, divide the product of the means by the extreme given; or, if one of the means be wanting, divide the product of the extremes by the means given, and the result will be the term sought.

To apply this, we will take the following question. If 5 yards of cloth cost $15, what will 7 yards cost? It is evident that twice the quantity of cloth would cost twice the sum, and that three times the quantity, three times the sum, &c.; that is, the price will be in *proportion* to the quantity purchased. We then have three terms of a proportion given, one of the extremes and the two means, to find the other extreme.

Thus, 5 : 7 : : 15. Therefore, to find the other extreme by the rule above stated, we multiply the two means, 7 and 15, and divide their product by the extreme given, and the quotient is the extreme required. $7 \times 15 = 105$. $105 \div 5 = 21$ dollars, the answer required.

To perform this question by analysis, we reason thus. If 5 yards cost 15 dollars, 1 yard will cost one fifth as much, which is 3 dollars; and if 1 yard cost 3 dollars, 7 yards will cost 7 times as much, which is 21 dollars.

RULE.* — *State the question by making that number which is of the same name or quality of the answer required the third term; then, if the answer required is to be greater than the third term, make the second term greater than the first; but if the answer is to be less than the third term, make the second less than the first.*

Reduce the first and second terms to the lowest denomination mentioned in either, and the third term to the lowest denomination mentioned in it.

Multiply the second and third terms together, and divide their product

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* This rule was formerly divided into the Rule of Three Direct, and the Rule of Three Inverse. The Rule of Three Direct included those questions where *more required more and less required less*; thus, — If 5lb. of coffee cost 60 cents, what would be the value of 10lb.? would be a question in the Rule of Three Direct, because the more coffee there was the more money it would take to purchase it.

But if the question were thus: — If 4 men can mow a certain field in 12 days, how long would it take 8 men? — it would be in Inverse, because the more men the less would be the time to perform the labor, that is, *more would require less.*

The method for stating questions was this: — To make that number which is the demand of the question the third term, that which is of the same name the first, and that which is of the same name as the answer required, the second term.

If the question was direct, the second and third terms must be multiplied together, and their product divided by the first; but if it was inverse, the first and second terms must be multiplied together, and their product divided by the third.

by the first, and the quotient is the answer, in the same denomination to which the third is reduced.

If any thing remains after division, reduce it to the next lower denomination, and divide as before.

If either of the terms consists of fractions, state the question as in whole numbers, and reduce the mixed numbers to improper fractions, compound fractions to simple ones, and invert the first term, and then multiply the three terms continually together, and the product is the answer to the question. Or the fractions may be reduced to a common denominator; and their numerators may be used as whole numbers. For when fractions are reduced to a common denominator, their relative value is as their numerators.

NOTE 1. — In the Rule of Three, the second term is the quantity whose price is wanted; the third term is the value of the first term; when, therefore, the second term is multiplied by the third, the answer is as much more than it should be, as the first term is greater than unity; therefore, by dividing by the first term, we have the value of the quantity required. Or, multiplying the third by the number of times which the second contains the first will produce the answer.

NOTE 2. — The pupil should perform every question by analysis, previous to his performing it by Proportion.

EXAMPLES.

1. If a man travel 243 miles in 9 days, how far will he travel in 24 days? Ans. 648 miles.

```
da.  da.    m.
 9 : 24 : : 243
             24
            ---
            972
           486
        9)5832
   Ans.   648 miles.
```

As the answer to the question must be in miles, we make the third term miles (243); and from the nature of the question we know that he will travel farther in 24 days than in 9 days; we therefore place 24, as the larger of the two remaining terms, in the *second place*, and the remaining number, 9 days, in the *first place*.

CANCELLING.

$$\frac{24 \times \overset{27}{\cancel{243}}}{\underset{1}{\cancel{9}}} = 648 \text{ miles, Ans.}$$

To perform this question by *analysis*, we proceed thus: — If he travel 243 miles in 9 days, he will in one day travel $\frac{1}{9}$ of 243 miles, which is 27 miles; then if he travel 27 miles in

one day, in 24 days he will travel 24 times as far, which is 648 miles, the answer, as before.

2. If 17 yards of broadcloth cost \$102, what will 7 yards cost? Ans. \$42.

$$\begin{array}{l} \text{yd.}\quad \text{yd.}\qquad \$ \\ 17 : 7 :: 102 \\ \qquad\qquad\ \ \underline{7} \\ \quad 17)714(\$42 \\ \qquad\ \ \underline{68} \\ \qquad\quad 34 \\ \qquad\quad \underline{34} \end{array}$$

As the answer is to be in dollars, we make the third term dollars (102); and as 7 yards will not cost so much as 17 yards, we make the second term (7) less than the first (17).

CANCELLING.

$$\frac{7\times \overset{6}{\cancel{102}}}{\underset{1}{\cancel{17}}} = \$42 \text{ Ans.}$$

To perform this question by *analysis*, we reason thus. If 17 yards cost \$102, one yard will cost $\frac{1}{17}$ of \$102, which is \$6; and if one yard cost \$6, 7 yards will cost 7 times as much, which is \$42, the answer, as before.

3. If 3 men drink a barrel of beer in 24 days, how long would it last 9 men? Ans. 8 days.

$$\begin{array}{l} \text{Men.}\ \ \text{m.}\quad \text{da.} \\ 9 : 3 :: 24 \\ \qquad\quad\ \ \underline{3} \\ \qquad 9)72 \\ \qquad\quad\ \ 8 \text{ days.} \end{array}$$

As the answer is to be in days, we make 24 days the third term, and because 9 men will drink the beer in less time than 3 men, the second term will be less than the first.

To perform this question *analytically*, we would say, that if 3 men drink a barrel of beer in 24 days, it would take one man 3 times 24 days, which is 72 days; and if one man drink a barrel of beer in 72 days, 9 men would drink it in $\frac{1}{9}$ of 72 days, which is 8 days, as before.

4. If 12 yards of cloth cost \$48, what will 15 yards cost? Ans. \$60.

CANCELLING.

$$\frac{15\times \overset{4}{\cancel{48}}}{\underset{1}{\cancel{12}}} = \$60 \text{ Ans.}$$

5. If 17 pounds of sugar cost $1.19, what will 365 pounds cost? Ans. $ 25.55.

6. If 16 acres of land cost $ 720, what will 197 acres cost? Ans. $ 8865.

7. If $ 8865 buy 197 acres, how many acres may be bought for $ 720? Ans. 16.

8. What will 84hhd. of molasses cost, if 15hhd. can be purchased for $175.95? Ans. $ 985.32.

9. If $100 gain $ 6 in 12 months, how much would it gain in 40 months? Ans. $ 20.

10. If a certain vessel has provisions sufficient to last a crew of 10 men 45 days, how long would the provisions last if the vessel were to ship 5 new hands? Ans. 30 days.

11. If 7 and 9 were 12, what, on the same supposition, would 8 and 4 be? Ans. 9.

12. If 9 men can perform a certain piece of labor in 17 days, how long would it take 3 men to do it? Ans. 51 days.

13. If 3 men can perform a piece of labor in 51 days, how many must be added to the number to perform the labor in 17 days? Ans. 6.

14. How much in length, that is $5\frac{1}{2}$ rods in breadth, is sufficient for an acre? Ans. $29\frac{1}{11}$ rods.

15. If 2 barrels of flour cost $12, what will 24 barrels cost? Ans. $144.

16. If 5 quintals of fish cost $16.25, what is the value of 75 quintals? Ans. $ 243.75.

17. If 2 cords of wood cost $11.50, what will 17 cords cost? Ans. $ 97.75.

18. If 7cwt. of iron cost $ 56.85, what will 49cwt. cost? Ans. $ 397.95.

19. If 5 acres of land be valued at $ 375.75, what would be the value of 35 acres? Ans. $ 2630.25.

20. If 7 pairs of shoes cost $10.50, how many pairs will $ 52.50 buy? Ans. 35.

21. If $ 4.75 be paid for 19lb. of salmon, how many pounds will $ 25.50 buy? Ans. 102lb.

22. If a man travels 48 miles in 6 hours, how far will he travel in 24 hours? Ans. 192 miles.

23. If 8 men eat a barrel of flour in 24 days, how long would it last 3 men? Ans. 64 days.

24. If 7 ounces of silver are sufficient to make 17 teaspoons, how many may be made from 42 ounces? Ans. 102.

25. If $ 100 gain $ 6 in a year, how much will $ 850 gain? Ans. $ 51.

26. If $100 gain $ 6 in a year, how much would be sufficient to gain $ 32 in a year? Ans. $ 533.33$\frac{1}{3}$.

27. If 20 gallons of water weigh 167lb., what will 180 gallons weigh? Ans. 1503lb.

28. If a staff 3 feet long cast a shadow 2 feet, how high is that steeple whose shadow is 75 feet? Ans. 112$\frac{1}{2}$ feet.

29. If 5$\frac{3}{5}$cwt. be carried 36 miles for $ 4.75, how far might it be carried for $160? Ans. 1212$\frac{12}{19}$ miles.

30. If 100 workmen can finish a piece of work in 12 days, how many men are sufficient to finish the work in 8 days? Ans. 150.

31. If $\frac{7}{12}$ of a yard cost $\frac{7}{20}$ of a dollar, what will $\frac{4}{5}$ of a yard cost? Ans. $ 0.48.

$$\overset{\text{yd.}}{\tfrac{7}{12}} : \overset{\text{yd.}}{\tfrac{4}{5}} :: \overset{\$}{\tfrac{7}{20}} \qquad \tfrac{12}{7} \times \tfrac{4}{5} \times \tfrac{7}{20} = \tfrac{336}{700} = \tfrac{48}{100} = \$\,0.48 \text{ Ans.}$$

CANCELLING.

$$\frac{12}{\cancel{7}} \times \frac{\overset{1}{\cancel{4}}}{5} \times \frac{\cancel{7}}{\underset{5}{\cancel{20}}} = \tfrac{12}{25} = \$\,0.48 \text{ Ans.}$$

To perform this question by analysis, we would proceed thus: — If $\frac{7}{12}$ of a yard cost $\frac{7}{20}$ of a dollar, $\frac{1}{12}$ would cost $\frac{1}{7}$ of $ $\frac{7}{20}$, which is $\frac{1}{20}$; and if $\frac{1}{12}$ cost $\frac{1}{20}$, $\frac{12}{12}$ will cost 12 times $\frac{1}{20}$, which is $\frac{12}{20} = \frac{3}{5}$ of a dollar. And if one yard cost $\frac{3}{5}$ of a dollar, $\frac{1}{5}$ of a yard will cost $\frac{3}{25}$ of a dollar, and $\frac{4}{5}$ will cost 4 times as much, that is, it will cost 4 times $\frac{3}{25}$, which is $\frac{12}{25}$ = $ 0.48, as before.

32. If $\frac{1}{2}$ of a yard cost $\frac{3}{5}$ of a £., what will $\frac{7}{8}$ of a yard cost? Ans. 1£. 1s. 0d.

33. If 4$\frac{1}{2}$ yards cost $ 9.75, what will 13$\frac{1}{2}$ yards cost? Ans. $ 29.25.

34. How much in length, that is 2$\frac{1}{2}$ inches wide, will make a square foot? Ans. 57$\frac{3}{5}$ inches.

35. If $\frac{7}{16}$ of a ship cost 51£., what are $\frac{3}{32}$ of her worth? Ans. 10£. 18s. 6$\frac{6}{7}$d.

36. A merchant bought a number of bales of velvet, each containing 129$\frac{17}{27}$ yards, at the rate of $ 7 for 5 yards, and sold them out again at the rate of $11 for 7 yards, and gained $ 200 by the bargain; how many bales were there? Ans. 9 bales.

37. If the moon moves 13° 10′ 35″ in one day, in what time does she perform one revolution? Ans. 27da. 7h. 43m.+

38. If 7lb. of sugar cost $\frac{3}{4}$ of a dollar, what are 12lb. worth? Ans. \$1.28$\frac{4}{7}$.

39. If \$1.75 will buy 7lb. of loaf-sugar, how much will \$213.50 buy? Ans. 7cwt. 2qr. 14lb.

40. If 7 ounces of gold are worth 30£., what is the value of 7lb. 11oz.? Ans. 407£. 2s. 10$\frac{2}{7}$d.

41. My friend borrowed of me \$500 for 6 months, promising me like favor; soon after, I had occasion for \$600; how long should I keep it to receive full compensation for the kindness? Ans. 5 months.

42. If the penny loaf weighs 7oz. when flour is \$8 per barrel, how much should it weigh when flour is \$7.50 per barrel? Ans. 7$\frac{7}{15}$ ounces.

43. If a regiment of soldiers consisting of 1000 men were to be clothed, each suit containing 3$\frac{3}{4}$ yards of cloth, that is 1$\frac{7}{8}$ yards wide, and to be lined with flannel 1$\frac{1}{4}$ yards wide, how many yards will it take to line the whole? Ans. 5625yd.

44. If 9$\frac{5}{7}$ yards of broadcloth cost \$11$\frac{3}{5}$, what will 16$\frac{16}{203}$ ells English cost? Ans. \$24.

45. A merchant failing in trade owes in all \$17280; his effects are sold for \$15120; what does he pay on a dollar, and what does A receive, to whom he owes \$5670?

Ans. He pays \$0.87$\frac{1}{2}$ on the dollar; A receives \$4961.25.

46. Bought in London 57 yards of broadcloth for 49 guineas, 28 shillings each; what did it cost per ell English?

Ans. 1£. 10s. 1$\frac{1}{19}$d.

47. Bought a cask of wine, at \$1.15 per gallon, for \$100; how much did it contain? Ans. 86gal. 3qt. 1$\frac{15}{23}$pt.

48. A merchant bought 9 packages of cloth for \$34560, each package containing 8 parcels, each parcel 12 pieces, and each piece 20 yards; what was the price per yard?

Ans. \$2.00.

49. If 75 gallons of water fall into a cistern containing 500 gallons, and by a pipe in the cistern 40 gallons run out in an hour, in what time will it be filled? Ans. 14h. 17m. 8$\frac{4}{7}$sec.

50. How many dozen pairs of gloves, at \$0.56 per pair, may be bought for \$120.96? Ans. 18doz.

51. A certain cistern has three pipes; the first will empty it in 20 minutes, the second in 40 minutes, and the third in 75 minutes; in what time would they all empty it?

Ans. 11m. 19$\frac{13}{53}$sec.

52. A can mow a certain field in 5 days, and B can mow it in 6 days; in what time would they both mow it?

Ans. $2\frac{8}{11}$ days.

53. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to finish the wall in 6 days? Ans. 36 men.

54. A can build a boat in 20 days, but with the assistance of C he can do it in 12 days; in what time would C do it himself? Ans. 30 days.

55. In a fort there are 700 men provided with 184000lb. of provisions, of which each man consumes 5lb. a week; how long can they subsist? Ans. 52 weeks 4 days.

56. If 25 men have $\frac{3}{4}$ of a pound of beef each three times in a week, how long will 3150lb. last them? Ans. 56 weeks.

57. How many tiles 8 inches square will lay a floor 20 feet long, and 16 feet wide? Ans. 720.

58. How many stones 10 inches long, 9 inches broad, and 4 inches thick, would it require to build a wall 80 feet long, 20 feet high, and $2\frac{1}{4}$ feet thick? Ans. 17280 stones.

59. Bought threescore pieces of Hollands for three times as many dollars, and sold them again for four times as many dollars; but if they had cost me as much as I sold them for, for what should I have sold them to gain at the same rate?

Ans. $320.

60. A sets out on a journey, and travels 27 miles a day; 7 days after, B sets out and travels the same road 36 miles a day; in how many days will B overtake A? Ans. 21 days.

61. If I sell coffee at 2s. 3d. per lb. and gain 35 per cent., what did I give per lb.? Ans. 1s. 8d.

62. A detachment of 2000 soldiers were supplied with bread sufficient to last them 12 weeks, allowing each man 14 ounces a day; but on examination find 105 barrels, containing 200lb. each, wholly spoiled; how much a day may each man eat, that the remainder may supply them 12 weeks? Ans. 12oz.

63. In consequence of having a seventh part of their bread spoiled, 2000 soldiers were put on an allowance of 12 ounces of bread per day for 12 weeks; what was the whole weight of their bread (good and bad), and how much was spoiled?

Ans. The whole weight, 147000lb.; spoiled, 21000lb.

64. Two thousand soldiers, having lost 105 barrels of bread, weighing 200lb. each, were obliged to subsist on 12 ounces a day for 12 weeks; but had none been lost, they might have had 14 ounces a day for the same time. What was the whole

weight, including what was lost, and how much had they left to subsist on?

Ans. The whole weight, 147000lb.; left to subsist on, 126000lb.

65. If 2000 soldiers, after losing one seventh part of their bread, had each 12 ounces a day for 12 weeks, what was the whole weight of their bread, including that lost, and how much might they have had per day, each man, if none had been lost?

Ans. The whole weight was 147000lb.; the loss, 21000lb.; had *none* been lost, they might have had 14 ounces per day.

66. If .85 of a gallon of wine cost \$2.72, how much will .25 of a gallon cost? Ans. \$0.80.

67. If 61.3 pounds of tea cost \$44.9942, what is the price per lb.? Ans. \$0.73,4.

68. What is the value of .15 of a hogshead of lime, at \$2.39 per hhd.? Ans. \$0.35,85.

69. If .75 of a ton of hay cost \$15, what is it per ton? Ans. \$20.

70. How many yards of carpeting that is half a yard wide will cover a room that is 30 feet long and 18 feet wide? Ans. 120 yards.

71. If a man perform a journey in 15 days when the day is 12 hours long, in how many days will he do it when the day is but 10 hours long? Ans. 18 days.

72. If 450 men are in a garrison, and their provisions will last them but 5 months, how many must leave the garrison that the same provisions may be sufficient for those who remain 9 months? Ans. 200 men.

73. The hour and minute hands of a watch are together at 12 o'clock; when will they next be together? Ans. 1h. 5m. $27\frac{3}{11}$sec.

74. A and B can perform a piece of work in $5\frac{5}{11}$ days, B and C in $6\frac{2}{3}$ days, and A and C in 6 days; in what time would each of them perform the work alone, and how long would it take them to do the work together?

Ans. A would do the work in 10 days; B, in 12 days; C, in 15 days; A, B, and C, together, in 4 days.

75. A, B, and C can perform a piece of work in 4 days, B can do it in 12 days, C can do it in 15 days; in what time would A and B perform the labor? Ans. $5\frac{5}{11}$ days.

76. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will it require to build the walls of a house which is 46 feet long, 28 feet wide, and 25 feet high, and the walls to be 18 inches thick? Ans. 143,775 bricks.

77. Lent a friend \$ 200 for 12 months, on condition of his returning the favor; how long ought he to lend me \$150 to requite my kindness? Ans. 16 months.

78. If 5 oxen or 7 cows eat $3\frac{4}{11}$ tons of hay in 87 days, in what time will 2 oxen and 3 cows eat the same quantity of hay? Ans. 105 days.

79. If 360 men be placed in a garrison, and have provisions for 6 months, how many men must be sent away at the end of 4 months that the remaining provision may last them 8 months longer? Ans. 270 men.

80. My tailor informs me it will take $10\frac{1}{4}$ square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{7}{8}$ yards wide, and on sponging it will shrink 5 per cent. in width and length. How many yards of the above cloth must I purchase for my "new suit"? Ans. $6\frac{62}{1083}$ yd.

SECTION LIII.

COMPOUND PROPORTION,

OR

DOUBLE RULE OF THREE.

COMPOUND PROPORTION is the method of performing such operations in Proportion as require two or more statements.

EXAMPLES.

1. If a man travel 117 miles in 30 days, employing only 9 hours a day, how far would he go in 20 days, travelling 12 hours a day?

The distance to be travelled depends on two circumstances, — the number of *days* the man travels, and the number of *hours* he travels in each day.

We will first suppose the hours to be the same in each case; the question will then be, — If a man travel 117 miles in 30 days, how far will he travel in 20 days?

This will lead to the following proportion.

$$30 \text{ days} : 20 :: 117 \text{ miles} : \frac{117 \times 20}{30} = 78 \text{ miles}.$$

That is, if we multiply 117 by 20, and divide the product by 30, we obtain the number of miles he will travel in 20 days, which is 78.

Now, if we take into consideration the number of hours, we must say, — If a man, travelling 9 hours a day for a certain number of days, has travelled 78 miles, how far will he go in the same time, if he travel 12 hours a day? This will furnish the following proportion.

9 hours : 12 hours : : 78 miles : $\frac{12 \times 78}{9}$ = 104 miles, the answer to the question.

By this mode of resolving the question, we see that 117 miles have, to the answer 104 miles, the proportion that 30 days have to 20 days, and that 9 hours have to 12 hours. Stating this in Compound Proportion, we have

$$\left.\begin{matrix} 30 : 20 \\ 9 : 12 \end{matrix}\right\} : : 117 : 104 \text{ miles, the answer.}$$

Thus it appears that if 117 be multiplied by both 20 and 12, and the product be divided by 30 times 9, the quotient will be 104 miles; or if we multiply 117 by 20, and divide the product by 30, and then multiply this quotient by 12 and divide by 9, it will produce the same answer as before.

This question may be performed by analysis thus: — If he travel 117 miles in 30 days, in one day he will travel $\frac{1}{30}$ of 117 miles, which is $\frac{117}{30}$ miles; and, travelling 9 hours a day, he will in one hour travel $\frac{1}{9}$ of $\frac{117}{30}$ miles, which is $\frac{13}{30}$ miles; and in a day of 12 hours he will travel 12 times $\frac{13}{30}$ miles, which is $\frac{156}{30}$ miles; and in 20 days he will travel 20 times $\frac{156}{30}$ miles, which is 104 miles, the answer, as before.

The answer to the above question might have been obtained by dividing the third term by the product of the two ratios which the first two terms have to the second terms; that is, by the ratio of 30 to 20, which is $\frac{30}{20} = \frac{3}{2}$; and of 9 to 12, which is $\frac{9}{12} = \frac{3}{4}$. Thus,

$$117 \div \tfrac{3}{2} \times \tfrac{3}{4} = 117 \div \tfrac{9}{8} = \tfrac{117}{1} \times \tfrac{8}{9} = \tfrac{936}{9} = 104 \text{ Ans.}$$

2. If 6 men in 16 days of 9 hours each build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days of 8 hours each will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

In stating this question, there are several circumstances to be taken into consideration; the number of men employed,

the length of the days, length of the wall, and its height and breadth.

As the answer to the question is to be in days, we make the days the third term.

Were all the circumstances of the question alike, except the number of men and the number of days, the question would consist in finding in how many days 24 men would perform the same labor that 6 men had done in 16 days; that is, if 6 men had built a certain wall in 16 days, how many days would it take 24 men to perform the same labor? This would furnish the following proportion.

$$24 \text{ men} : 6 \text{ men} :: 16 \text{ days} : \frac{6\times16}{24} = 4 \text{ days}.$$

Or, if this were the question, — If a certain number of men, by laboring 9 hours a day, perform a piece of work in 16 days, how many days would it take the same men to do the labor by working 8 hours a day? — the following would be the proportion.

$$8 \text{ hours} : 9 \text{ hours} :: 16 \text{ days} : \frac{9\times16}{8} = 18 \text{ days}.$$

Or, if this were the question, — If a certain number of men build a wall 20 feet long in 16 days, how long would it take the same men to build a wall 200 feet long? — the following would be the statement.

$$20 \text{ feet} : 200 \text{ feet} :: 16 \text{ days} : \frac{16\times200}{20} = 160 \text{ days}.$$

Or, if only the days and height of the wall were considered, this would be the statement.

$$6 \text{ feet} : 8 \text{ feet} :: 16 \text{ days} : \frac{8\times16}{6} = 21\tfrac{1}{3} \text{ days}.$$

Lastly, were we to consider only the days and the thickness of the wall, it would furnish the following statement.

$$4 \text{ feet} : 6 \text{ feet} :: 16 \text{ days} : \frac{6\times16}{4} = 24 \text{ days}.$$

We see, by this mode of resolving the question, that 16 days must have to the *true answer* the ratio compounded of the ratios

That 24 men have to 6 men;
That 8 hours have to 9 hours;
That 20 feet have to 200 feet;
That 6 feet have to 8 feet; and
That 4 feet have to 6 feet.

Stating the above in Compound Proportion, we have

$$\left.\begin{array}{rl} 24 \text{ men} & : 6 \text{ men} \\ 8 \text{ hours} & : 9 \text{ hours} \\ 20 \text{ feet} & : 200 \text{ feet} \\ 6 \text{ feet} & : 8 \text{ feet} \\ 4 \text{ feet} & : 6 \text{ feet} \end{array}\right\} :: 16 \text{ days} : 90 \text{ days} = \text{Answer.}$$

The continued product of all the second terms by the third term, and this divided by the continued product of the first terms, will produce the answer.

Thus $\frac{6\times9\times200\times8\times6\times16}{24\times8\times20\times6\times4} = \frac{829440}{92160} = 90$ days, Answer.

OPERATION BY CANCELLING.

$$\frac{\cancel{6}\times 9\times \overset{10}{\cancel{200}}\times\cancel{8}\times\cancel{6}\times\overset{2}{\cancel{16}}}{\underset{3}{\cancel{24}}\times\cancel{8}\times\cancel{20}\times\cancel{6}\times\cancel{4}} = 90 \text{ days, Ans.}$$

3. If 5 compositors in 16 days, 11 hours long, can compose 25 sheets of 24 pages in each sheet, and 44 lines in a page, and 40 letters in a line, in how many days 10 hours long may 9 compositors compose a volume, to be printed on the same letter, consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters in a line? Ans. 12 days.

STATEMENT.

$$\left.\begin{array}{rl} 9 \text{ comp.} & : 5 \text{ comp.} \\ 10 \text{ hours} & : 11 \text{ hours} \\ 25 \text{ sheets} & : 36 \text{ sheets} \\ 24 \text{ pages} & : 16 \text{ pages} \\ 44 \text{ lines} & : 50 \text{ lines} \\ 40 \text{ letters} & : 45 \text{ letters} \end{array}\right\} :: 16 \text{ days} : 12 \text{ days, Ans.}$$

OPERATION BY CANCELLING.

$$\frac{\cancel{5}\times\cancel{11}\times\overset{3}{\cancel{36}}\times\overset{2}{\cancel{16}}\times\overset{5}{\cancel{50}}\times\overset{5}{\cancel{45}}\times\overset{4}{\cancel{16}}}{\cancel{9}\times\cancel{10}\times\underset{5}{\cancel{25}}\times\underset{2}{\cancel{24}}\times\underset{4}{\cancel{44}}\times\underset{4}{\cancel{40}}} = 12 \text{ days, Ans.}$$

RULE. — *Make that number which is of the same kind as the answer required the third term; and of the remaining numbers, take any two that are of the same kind, and consider whether an answer depending upon these alone would be greater or less than the third term, and place them as directed in Simple Proportion. Then take any other two, and consider whether an answer depending only upon them would be greater or less than the third term, and arrange them accordingly; and so on,*

until all are used. Multiply the continued product of the second terms by the third, and divide by the continued product of the first, and you produce the answer.

NOTE. — All the following questions are to be performed not only by the *Rule*, but by analysis. The pupil should also apply the cancelling rule.

4. If \$100 gain \$6 in one year, how much would \$500 gain in four months? Ans. \$10.

5. If \$100 gain \$6 in one year, what must be the sum to gain \$10 in 4 months? Ans. \$500.

6. How long will it take \$500 to gain \$10, if \$100 gain \$6 in one year? Ans. 4 months.

7. If \$500 gain \$10 in 4 months, what is the rate per cent.? Ans. 6 per cent.

8. If 8 men spend \$32 in 13 weeks, what will 24 men spend in 52 weeks? Ans. \$384.

9. If 12 men can build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days, when the days are 12 hours long, in what time will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when they work only 8 hours a day? Ans. 120 days.

10. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days? Ans. 336 bushels.

11. If the carriage of 5cwt. 3qr. 150 miles cost \$24.58, what must be paid for the carriage of 7cwt. 2qr. 25lb. 64 miles, at the same rate? Ans. \$14.08,6.

12. If $7\frac{1}{4}$oz. of bread be bought for $4\frac{3}{4}$d. when corn is 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price per bushel is 5s. 6d.? Ans. $16\frac{118}{627}$oz.

13. If 496 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness 465 feet long, $3\frac{2}{3}$ wide, $2\frac{1}{3}$ deep, in how many days of 9 hours long will 24 men dig a trench of 4 degrees of hardness $337\frac{1}{2}$ feet long, $5\frac{2}{3}$ wide, and $3\frac{1}{2}$ deep? Ans. 132 days.

SECTION LIV.

CHAIN RULE.

THE CHAIN RULE consists in joining many proportions together, and by the relation which the several antecedents have

to their consequents the proportion between the first antecedent and the last consequent is discovered.

This rule may often be abridged by cancelling equal quantities on both sides, and abbreviating commensurables.

NOTE. — The first numbers in each part of the question are called *antecedents*, and the following *consequents*.

1. If 20lb. at Boston make 23lb. at Antwerp, and 155lb. at Antwerp make 180lb. at Leghorn, how many pounds at Boston are equal to 144lb. at Leghorn?

OPERATION.

20lb. of Boston = 23 of Antwerp,
155lb. of Antwerp = 180 of Leghorn,
144lb of Leghorn?

$$\frac{20 \times 155 \times 144}{23 \times 180} = \frac{446400}{4140} = 107\tfrac{19}{23}\text{lb. Ans.}$$

It will be perceived in the operation that the continued product of the antecedents is divided by the continued product of the consequents.

CANCELLING.

$$\frac{\cancel{20} \times 155 \times \overset{16}{\cancel{144}}}{23 \times \underset{\cancel{9}}{\cancel{180}}} = \frac{2480}{23} = 107\tfrac{19}{23}\text{lb. Ans.}$$

RULE. — *Write the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right; and, if the last number stands at the left hand, multiply the numbers of the left-hand column continually together for a dividend, and those at the right for a divisor; but, if the last number stands at the right hand, multiply the numbers of the right-hand column continually together for a dividend, and those at the left for a divisor; and the quotient will be the answer.*

NOTE. — The demonstration for this rule is the same as for Compound Proportion.

2. If 12lb. at Boston make 10lb. at Amsterdam, and 10lb. at Amsterdam make 12lb. at Paris, how many pounds at Boston are equal to 80lb. at Paris? Ans. 80lb.

CANCELLING.

$$\frac{\cancel{12} \times \cancel{10} \times 80}{\cancel{10} \times \cancel{12}} = 80\text{lb. Ans.}$$

3. If 25lb. at Boston are equal to 22lb. at Nuremburg, and 88lb. at Nuremburg are equal to 92lb. at Hamburg, and 46lb. at Hamburg are equal to 49lb. at Lyons, how many pounds at Boston are equal to 98lb. at Lyons? Ans. 100lb.

CANCELLING.

$$\frac{25 \times \overset{4}{\cancel{88}} \times \cancel{46} \times \overset{2}{\cancel{98}}}{\cancel{22} \times \underset{2}{\cancel{92}} \times \cancel{49}} = 100\text{lb. Ans.}$$

NOTE. — The pupil may cancel all the following questions in a similar manner.

4. If 24 shillings in Massachusetts are equal to 32 shillings in New York, and if 48 shillings in New York are equal to 45 shillings in Pennsylvania, and if 15 shillings in Pennsylvania are equal to 10 shillings in Canada, how many shillings in Canada are equal to 100 shillings in Massachusetts? Ans. $83\frac{1}{3}$ shillings.

5. If 17 men can do as much work as 25 women, and 5 women do as much as 7 boys, how many men would it take to do the work of 75 boys? Ans. $36\frac{3}{7}$ men.

6. If 10 barrels of apples will pay for 5 cords of wood, and 20 cords of wood for 4 tons of hay, how many barrels of apples will it take to purchase 50 tons of hay? Ans. 500bbl.

7. If 100 acres in Bradford be worth 120 in Haverhill, and 50 in Haverhill be worth 65 in Methuen, how many acres in Bradford are equal to 150 in Methuen? Ans. $96\frac{2}{13}$ acres.

8. If 10lb. of cheese are equal in value to 7lb. of butter, and 11lb. of butter to 2 bushels of corn, and 11 bushels of corn to 8 bushels of rye, and 4 bushels of rye to one cord of wood, how many pounds of cheese are equal in value to 10 cords of wood? Ans. $432\frac{1}{7}$lb.

SECTION LV.

PARTNERSHIP, OR COMPANY BUSINESS.

PARTNERSHIP is the association of two or more persons in business, with an agreement to share the profits and losses in proportion to the amount of the capital stock contributed by each.

EXAMPLES.

1. Three men, A, B, and C, enter into partnership for two years, with a capital of \$1080. A puts in \$240, B \$360, and

C $480. They gain $54. What is each man's share of the gain?

As the whole stock in trade is $1080, of which $240 belongs to A, A's share of the stock, therefore, will be $\frac{240}{1080}=\frac{2}{9}$, and as each man's gain is in proportion to his stock, $\frac{2}{9}$ of $54, which is $12, is A's gain. B's stock in trade is $360; therefore $\frac{360}{1080}=\frac{1}{3}$ of $54, which is $18, is B's gain. C's stock is $480; therefore his part of the whole stock is $\frac{480}{1080}=\frac{4}{9}$; consequently C's share of the gain is $\frac{4}{9}$ of $54, which is $24. Hence, to find any man's gain or loss in trade we have the following

Rule. — *Multiply the whole gain or loss by each man's fractional part of the stock.*

Note. — The pupil who may be desirous of performing the questions of this rule in the "*old way*" will adopt the following

Rule. — *As the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss.*

The following is the statement of the first question, with the answers and proof.

As the stock $1080 : $54 :: $240 : $12 A's gain.
$1080 : $54 :: $360 : $18 B's gain.
$1080 : $54 :: $480 : $24 C's gain.
Proof, $12 + $18 + $24 = $54 whole gain.

2. A, B, and C enter into partnership, with a capital of $1100, of which A put in $250, B put in $300, and C $550; they lost by trading 5 per cent. on their capital. What was each man's share of the loss? A's loss, $12.50.
B's loss, $15.00.
C's loss, $27.50.

3. Two merchants, C and D, engaged in trade; C put in $6780, and D put in $12,000; they gain $1000. What is each man's share? C's share, $361.02,2$\frac{14}{313}$.
D's share, $638.97,7$\frac{129}{313}$.

4. M, P, and Q trade in company, with a capital of $10,000 M put in $3000, P $2000, and Q $5000; they gain $500. What is each man's share of the gain? M's gain, $150.
P's gain, $100.
Q's gain, $250

5. A, B, and C enter into partnership; A put in $500, B $250, and C put in 320 yards of broadcloth; they gain $332.50,

of which C's share is $120. What were A's and B's shares of the gain, and what was the value of C's cloth per yard?

A's gain, $125.00.
B's gain, $ 87.50.
C's cloth per yd. $1.50.

6. A, B, and C trade in company; A put in $ 5000, B put in $ 6500, and C put in $ 7500; they gain 40 per cent. on their capital, but receive the whole amount of their gains in bills, for which they are obliged to allow a discount of 10 per cent. How much was each man's net gain?

A's gain, $1800.
B's gain, $ 2340.
C's gain, $ 2700.

7. A merchant, failing in trade, owes A $ 600, B $ 760, C $ 840, and D $ 800. His effects are sold for $ 2275. What will each receive of the dividend?

A, $ 455.00.
B, $ 576.33⅓.
C, $ 637.00.
D, $ 606.66⅔.

8. A bankrupt owes $ 5000. His effects, sold at auction, amount to $ 4000. What will his creditors receive on a dollar?

Ans. $ 0.80.

9. A merchant, having sustained many losses, is obliged to become a bankrupt. His effects are valued at $1728, with which he can pay only fifteen cents on the dollar. What did he owe?

Ans. $11,520.

SECTION LVI.

PARTNERSHIP ON TIME.

WHEN merchants in partnership employ their stock for unequal times, it is called Partnership on Time.

EXAMPLES.

1. Two men, A and B, trade in company. A puts in $ 420 for 5 months, and B puts in $ 350 for 8 months; they gain $ 84. What is each man's share of the gain?

METHOD OF OPERATION BY ANALYSIS.

$ 420 for 5 months is the same as $5 \times \$ 420 = \$ 2100$ for 1

month; and \$350 for 8 months is the same as $8 \times \$350 = \$2800$ for 1 month. The question, therefore, is the same as if A had put in \$2100 and B \$2800 for 1 month each. The whole stock would then be \$4900. A's share of the gains, therefore, will be $\frac{2100}{4900} = \frac{3}{7}$ of $\$84 = \$36$. And B's share will be $\frac{2800}{4900} = \frac{4}{7}$ of $\$84 = \$48$.

RULE. — *Multiply each man's stock by the time it continued in trade, and consider each product a numerator to be written over the sum of all the numerators, as a common denominator; then multiply the whole gain or loss by each fraction, and the several products will be the gain or loss of each man.*

NOTE. — It might be well for the student to be acquainted with the "*old way*" of performing these questions. The following is the

RULE. — *Multiply each man's stock by the time it was continued in trade; then, as the sum of all the products is to the whole gain or loss, so is each man's particular product to his share of the gain or loss.*

The first question would be performed thus:

$\$420 \times 5 = 2100$ $4900 : 84 :: 2100 : \$36$ A's gain.
$\$350 \times 8 = \underline{2800}$ $4900 : 84 :: 2800 : \$48$ B's gain.
4900

Proof, $\$36 + \$48 = \$84 =$ A and B's gain.

2. A commenced business, January 1, with a capital of \$3200; May 1, he took B into partnership, with a capital of \$4200; and at the end of the year they had gained \$240. What was each man's share of the gain? \$128, A's gain.
\$112, B's gain.

3. A, B, and C traded in company. A put in \$300 for 5 months, B put in \$400 for 8 months, and C put in \$500 for 3 months; they gain \$100. What is the gain of each?
$\$24.19\frac{11}{31}$, A's gain.
$\$51.61\frac{9}{31}$, B's gain.
$\$24.19\frac{11}{31}$, C's gain.

4. Three men hire a pasture in common, for which they are to pay \$26.40. A put in 24 oxen for 8 weeks, B put in 18 oxen for 12 weeks, and C put in 12 oxen for 10 weeks. What ought each to pay? A should pay \$ 9.60.
B should pay \$10.80.
C should pay \$ 6.00.

5. Two men in Boston hire a carriage for \$25, to go to Concord, N. H., the distance being 72 miles, with the privilege of taking in three more persons. Having gone 20 miles, they

take in A; at Concord, they take in B; and when within 30 miles of the city, they take in C. How much shall each man pay?

First man pays	\$7.60,9$\frac{103}{108}$.
Second man pays	\$7.60,9$\frac{103}{108}$.
A pays	\$5.87,3$\frac{91}{108}$.
B pays	\$2.86,4$\frac{7}{12}$.
C pays	\$1.04,1$\frac{8}{12}$.
	\$25.00,0

6. Three men engage in partnership for 20 months. A at first put into the firm \$4000, and at the end of four months he put in \$500 more, but at the end of 16 months he took out \$1000; B at first put in \$3000, but at the end of 10 months he took out \$1500, and at the end of 14 months he put in \$3000; C at first put in \$2000, and at the end of 6 months he put in \$2000 more, and at the end of 14 months he put in \$2000 more, but at the end of 16 months he took out \$1500; they had gained by trade \$4420. What is each man's share of the gain?

A's gain, \$1680.
B's gain, \$1260.
C's gain, \$1480.

7. John Jones, Samuel Eaton, and Joseph Brown formed a partnership, under the firm of Jones, Eaton, & Co., with a capital of \$10,000; of which Jones put in \$4000, Eaton put in \$3500, and Brown put in \$2500; but at the end of 6 months Jones withdrew \$2000 of his stock, and at the end of 8 months Eaton withdrew \$1500 from the firm; but at the end of 10 months Brown added \$2000 to his stock. At the end of 2 years they found their gains to be \$1041.80. What was the share of each man?

Jones's gain, \$300.51$\frac{12}{13}$.
Eaton's gain, \$300.51$\frac{12}{13}$.
Brown's gain, \$440.76$\frac{2}{13}$.

Section LVII.

GENERAL AVERAGE.

General Average is in mercantile law whatever damage or loss is incurred by any part of a ship and cargo for the preservation of the rest.

When real damage occurs, the several persons interested in the ship, freight, and cargo contribute their respective proportions to indemnify the owners of the part in question against the

damage or necessary expense which has been incurred for the good of all.

No general average takes place unless the danger was imminent, and absolutely necessary for the safety of the ship.

Particular Average signifies a partial loss of the ship and cargo, arising from accidents at sea, and the loss must be borne by the owners of the property, who have sustained damage. Underwriters must pay such proportion of the prime cost as corresponds with the proportion of diminution in value occasioned by the damage.

Goods, whether lost, injured, or saved, are to be valued at the price they would have brought in ready money on the ship's arriving at her destined port. The value of the vessel is also estimated in the same manner.

It is a general custom to allow in the general average two thirds of the expense incurred in procuring new masts, cables, and other furniture of the ship, the new materials being much better than the old. A deduction in the general average of from one third to one half is made from the seamen's wages while the vessel is detained in port. Goods are valued at the *invoice* price when the average claim is settled at the port of *lading*.

To find the General Average.

RULE. — *As the value of all the articles subject to contribution is to the whole loss, so is each person's share of that value to his* average *proportion of the loss; or so is* $100 *to the loss per cent.*

The ship General Taylor, in her voyage from Boston to Mobile, on the night of January 10, 1847, was wrecked on Nantucket Shoals; in consequence of which the captain was obliged to cut away her masts, and heave overboard some of the furniture of the ship and part of the cargo. The vessel was finally towed into New York by the steamboat Ohio. A statement of the loss and expenses is as follows: —

Amount of the Loss.

Goods of R. S. Davis cast overboard, . . .	$1728
Damage of J. Smith's goods, ...	772
" " C. Dana's goods, ...	866
Amount of the freight of goods thrown overboard,	334
" " two thirds of the expense in procuring new masts, cable, anchors, &c., ...	875
Pilotage and port duties, ...	400
Miscellaneous expenses, ...	25
	$5000

Contributary Articles.

Value of the ship, ...	$ 9272
Do. of R. S. Davis's goods thrown overboard, . .	1728
Do. of J. Smith's goods at Mobile, deducting freight,	2000
Do. of C. Dana's goods do. do.	7000
Do. of two thirds the ship's freight, ...	5000
	$ 25,000

A Statement of what each Contributary Article pays in the General Average.

The ship,	$ 9272 × .20	pays	$1854.40
Ship's freight,	5000 × .20	"	1000.00
Davis's goods,	1728 × .20	"	345.60
Smith's goods,	2000 × .20	"	400.00
Dana's goods,	7000 × .20	"	1400.00
	$ 25,000		$ 5000.00

A Statement of what each Party receives.

Davis receives	$1728 × .80 =	$1382.40	
Smith do.	772 × .80 =	617.60	and his goods.
Dana do.	866 × .80 =	692.80	do. do.
Owners receive		2307.20	
		$ 5000.00	

SECTION LVIII.

PROFIT AND LOSS.

By this rule merchants estimate their *profit* and *loss* in buying and selling goods.

The questions are to be performed by Proportion and the other preceding rules. The pupil should give an analysis of each question.

One of four things is generally required by this rule : —

1. To know what per cent. is gained or lost, either in purchasing or selling goods.

2. To ascertain at what price to sell an article to gain or lose a certain per cent.

3. To ascertain the price of an article, when we know wha per cent. is gained or lost.

4. If goods be sold at a certain price, and there be gained

or lost a certain per cent., to ascertain what would be gained or lost at some other price.

The eight following examples will illustrate the above problems.

EXAMPLES.

1. If I buy cloth at $ 4 per yard, and sell it at $ 5 per yard, what per cent. do I gain? Ans. 25 per cent.

OPERATION BY PROPORTION.

$ 5 — $ 4 = $1; $ 4 : $1 :: $100 : $ 25, that is, 25 per cent.

By analysis. If $ 4 gain $1, it is evident that $1 will gain $\frac{1}{4}$ of $1 = $ 0.25; and $100 will gain 100 times $ 0.25 = $ 25, that is, 25 per cent., answer as before.

2. When cloth is purchased at $ 5 per yard, and sold at $ 4 per yard, what per cent. is lost? Ans. 20 per cent.

OPERATION BY PROPORTION.

$ 5 — $ 4 = $1. $5 : $1 :: $100 : $ 20, that is, 20 per cent.

By analysis. If $1 be lost on $ 5, it is certain that on $1 there will be lost $\frac{1}{5}$ of $1 = $ 0.20; and if on $1 there be lost $ 0.20, on $100 there will be lost 100 times $ 0.20 = $20, that is, 20 per cent., answer as before; therefore, when we wish to know what per cent. is gained or lost, either in purchasing or disposing of goods, we adopt the following

RULE. — *As the price of the goods is to the gain or loss, so is $ 100 to the per cent. gained or lost.*

3. If I buy cloth at $ 4 per yard, for how much must I sell it to gain 25 per cent.? Ans. $ 5.

OPERATION BY PROPORTION.

$100 + $ 25 = $125; $100 : $125 :: $ 4 : $ 5 Ans.

By analysis. If for $100 I receive $125, it is evident that for $1 I shall receive only $\frac{1}{100}$ of $125 = $1.25, and for $ 4 I shall have 4 times $1.25 = $ 5, answer as before.

4. If I buy cloth at $ 5 per yard, for what must I dispose of it per yard to lose 20 per cent.? Ans. $ 4.

OPERATION BY PROPORTION.

$100 — $ 20 = $ 80; $100 : $ 80 :: $ 5 : $ 4 Ans.

By analysis. If $ 80 are to be received for $100, it is certain that for $ 1 there will be paid only $\frac{80}{100} = \frac{4}{5}$ of a dollar =

\$ 0.80, and for \$ 5 I can receive only 5 times \$ 0.80 = \$ 4, answer as before; therefore, to ascertain at what price to sell an article to gain or lose a certain per cent. we adopt the following

RULE. — *As \$ 100 is to \$ 100 with the profit added or loss subtracted, so is the price given to the price required.*

5. If I sell cloth at \$ 5 per yard, and thereby make 25 per cent., what was the prime cost of the goods?

OPERATION BY PROPORTION.

\$100 + \$ 25 = \$125; \$125 : \$100 :: \$ 5 : \$ 4 Ans.

By analysis. As \$ 125 are received for \$ 100, it is evident that for \$ 1 there will be received only $\frac{100}{125} = \frac{4}{5}$ of a dollar = \$ 0.80; and for \$ 5, 5 times \$ 0.80 = \$ 4.00 Ans.

6. If I dispose of cloth at \$ 4 per yard, and by so doing lose 20 per cent., required the prime cost of the goods. Ans. \$ 5.

OPERATION BY PROPORTION.

\$100 — \$ 20 = \$ 80; \$ 80 : 100 :: \$ 4 : \$ 5 Ans.

By analysis. As \$100 are received for \$ 80, it is certain that for $\frac{100}{80} = \frac{5}{4}$ of a dollar = \$1.25 there would be received only \$1; therefore, for \$ 4 which I receive I should have had 4 times \$1.25 = \$ 5, answer as before; therefore, when we wish to ascertain the price of an article, when we know what per cent. is gained or lost, we adopt the following

RULE. — *As \$ 100 with the gain per cent. added or loss per cent. subtracted is to \$ 100, so is the price to the prime cost.*

7. If I sell cloth at \$ 5 per yard, and thereby gain 25 per cent., what would be my gain if I were to obtain \$ 7 per yard?

Ans. 75 per cent.

OPERATION BY PROPORTION.

\$100 + \$25 = \$125; \$ 5 : \$ 7 :: \$125 : \$175;
\$175 — \$100 = \$ 75, that is, 75 per cent.

By analysis. As \$ 5 amounts to \$125, it is evident that \$ 7 will amount to $\frac{7}{5}$ of \$125 = \$175; and if \$ 7 amount to \$175 it is certain that \$175 — \$100 = \$ 75 are gained on \$ 100, that is, 75 per cent.

8. If I purchase cloth at \$ 7 per yard, and thereby gain 75 per cent., do I gain or lose if I sell the same at \$ 3 per yard?

Ans. lose 25 per cent.

OPERATION BY PROPORTION.

$$\$100 + \$75 = \$175;\ \$7 : \$3 :: \$175 : \$75.$$
$$\$100 - \$75 = \$25;\ \text{the loss is therefore 25 per cent.}$$

By analysis. If \$7 give \$175, \$3 will give $\frac{3}{7}$ of \$175 = \$75. Therefore, for \$100 there are received only \$75; therefore there is \$100 — \$75 = \$25, or 25 per cent. loss, answer.

If, therefore, goods be sold at a certain price, and there be gained or lost a certain per cent., and we wish to ascertain what would be gained or lost per cent. at some other price, we deduce the following

RULE. — *As the first price is to the proposed price, so is \$100 with the profit per cent. added or the loss per cent. subtracted to the gain or loss per cent. at the assumed price.*

NOTE. — If the answer exceeds \$100, the *excess* is the gain per cent.; but if it be less than \$100, the *deficiency* is the loss per cent.

EXAMPLES TO EXERCISE THE PRECEDING RULES.

9. Sold broadcloth at \$6.12½ per yard, and by so doing lost 12½ per cent. What was the original cost per yard?

Ans. \$7.00.

By analysis. If 12½ per cent. be lost, 87½ per cent. will remain. It is now required to find of what number \$6.12½ is $\frac{87\frac{1}{2}}{100}$. This is done by multiplying \$6.12½ by 100, and dividing by 87½, and it produces the answer, \$7.00.

10. Bought cloth at \$7.00 per yard, and sold it at \$6.12½. What per cent. did I lose? Ans. 12½ per cent.

11. Bought cloth at \$7.00 per yard, and sold it for 12½ per cent. less than what it cost. What did I receive?

Ans. \$6.12½.

12. Bought cloth at \$3.60 per yard. For how much must I sell it to gain 12½ per cent.? Ans. \$4.05.

13. Sold cloth at \$4.05 per yard, and by so doing I gained 12½ per cent. What did it cost? Ans. \$3.60.

14. Bargained for cheese at \$8.50 per cwt. For how much must I sell it to gain 10 per cent.? Ans. \$9.35 per cwt.

15. Sold cheese at \$9.35 per cwt. and gained 10 per cent. What did I give for it? Ans. \$8.50 per cwt.

16. Sold cloth at \$1.25 per yard, and by so doing lost 15 per cent. For what should I have sold it to gain 12 per cent.?

Ans. \$1.64,7$\frac{1}{17}$ per yard.

17. Sold cloth at \$1.25 per yard, and lost 15 per cent. What

per cent. should I have gained had I sold it for \$1.64,7$\frac{1}{17}$ per yard? Ans. 12 per cent.

18. Sold cloth for \$1.64,7$\frac{1}{17}$ per yard, and gained 12 per cent. For what should I have sold it to lose 15 per cent.? Ans. \$1.25 per yard.

19. Sold cloth for \$1.64,7$\frac{1}{17}$ per yard, and gained 12 per cent. What should I have lost had I sold it for \$1.25 per yard? Ans. 15 per cent.

20. A buys corn for \$0.90 per bushel, and sells it for \$1.20. B buys for \$1.12$\frac{1}{2}$, and sells for \$1.50. Who gains the most per cent.? Ans. both gain alike.

21. If I buy cotton cloth at 13 cents per yard, on 8 months' credit, and sell it again at 12 cents cash, do I gain or lose, and how much per cent.? Ans. lose 4 per cent.

22. If 24 yards of cloth are sold at \$2.50 per yard, and there is 7$\frac{1}{2}$ per cent. loss in the sale, what is the prime cost of the whole? Ans. \$64.86,4$\frac{32}{37}$.

23. Bought 24 yards of cloth for \$64.86,4$\frac{32}{37}$. For what must I sell it per yard to lost 7$\frac{1}{2}$ per cent.? Ans. \$2.50.

24. Bought a certain quantity of cloth for \$64.86,4$\frac{32}{37}$, and by selling it at \$2.50 per yard, I lost 7$\frac{1}{2}$ per cent. How many yards were bought? Ans. 24 yards.

25. Bought 24 yards of cloth for \$64.86,4$\frac{32}{37}$, and sold it at \$2.50 per yard; what per cent. is lost? Ans. 7$\frac{1}{2}$ per cent.

26. If 27$\frac{1}{2}$cwt. of sugar be sold at \$12.50 per cwt., and there is gained 17 per cent., what was the first cost? Ans. \$10.68,3$\frac{89}{117}$.

27. Sold a horse for \$75, and by so doing I lost 25 per cent.; whereas, I ought to have gained 30 per cent. How much was he sold under his real value? Ans. \$55.00.

28. Bought a horse which was worth 30 per cent. more than I gave for him; but having been injured, I sold him for 25 per cent. less than what he cost, and thereby lost \$55 on his original value. What was received for the horse? Ans. \$75.00.

29. Bought molasses at 42 cents per gallon, but not proving so good as I expected, I am willing to lose 5 per cent. For what must I sell it per gallon? Ans. \$0.39,9.

30. Bought a hogshead of molasses for \$112, but 15 gallons having leaked out, I am willing to lose 5 per cent. on the cost. For how much per gallon must I sell it? Ans. \$2.21,6$\frac{2}{3}$.

31. Bought a hogshead of molasses for \$112, but a number of gallons having leaked out, I sell the remainder for \$2.21,6$\frac{2}{3}$

per gallon, and by so doing I lose 5 per cent. on the cost. How many gallons leaked out? Ans. 15 gallons.

32. Bought a hogshead of molasses for a certain sum; but 15 gallons having leaked out, I sell the remainder for \$ $2.21,6\frac{2}{3}$ per gallon, and thereby lose 5 per cent. on the cost. What was the cost? Ans. \$112.00.

33. Bought a hogshead of molasses for \$112.00; but 15 gallons having leaked out, I sell the remainder at \$ $2.21,6\frac{2}{3}$ per gallon. What per cent. is my loss? Ans. 5 per cent.

34. If I sell cloth at \$ 5.60 per yard, and thereby lose 7 per cent., should I gain or lose, and how much per cent., by selling it at \$ 6.25 per yard? Ans. $3\frac{89}{112}$ per cent. gain.

35. Sold a watch which cost me \$ 30 for \$ 35, on a credit of 8 months. What did I gain by the bargain? Ans. \$ $3.65,3\frac{1}{13}$.

36. When tea is sold at \$1.25 per lb. there is lost 25 per cent.; what would be the gain or loss per cent. if it should be sold at \$1.40 per lb.? Ans. 16 per cent. loss.

37. A exchanges with B 50lbs. of indigo at \$1.00 per lb. cash, and in barter \$1.35; but he is willing to lose 12 per cent. to have one third ready money. What is the cash price of 1 yard of cloth delivered by B at \$ 5.00 per yard to equal A's bartering price reduced 12 per cent., and how many yards were delivered? Ans. \$ $4.20\frac{260}{297}$, cash price of 1 yard;
$7\frac{23}{25}$ yards delivered by B.

Section LIX.

DUODECIMALS.

Duodecimals are so called because they decrease by twelves from the place of feet towards the right.

Inches are called *primes*, and are marked thus $'$; the next division after is called *seconds*, marked thus $''$; the next is *thirds*, marked thus $'''$; and so on.

Duodecimals are commonly used by workmen and artificers in finding the contents of their work.

EXAMPLES.

1. Multiply 6 feet 8 inches by 4 feet 5 inches.

OPERATION.

$$\begin{array}{rrr} 6 & 8' & \\ 4 & 5 & \\ \hline 26 & 8' & \\ 2 & 9 & 4'' \\ \hline 29 & 5' & 4'' \end{array}$$

As feet are the integers or units, it is evident that feet multiplied by feet will produce feet; and as inches are twelfths of a foot, the product of inches by feet will be twelfths of a foot. For the same reason, inches multiplied by inches will produce twelfths of an inch, or one hundred and forty-fourths of a foot.

RULE. — *Under the multiplicand write the same names or denominations of the multiplier; that is, feet under feet, inches under inches, &c. Multiply each term in the multiplicand, beginning at the lowest, by the feet of the multiplier, and write each result under its respective term, observing to carry a unit for every 12 from each denomination to its next superior. In the same manner multiply the multiplicand by the inches of the multiplier, and write the result of each term one place further to wards the right than the corresponding terms in the preceding product. Proceed in the same manner with the seconds and all the rest of the denominations, and the sum of the several products will be the product required.*

The denomination of the particular products will be as follows.

Feet multiplied by feet give feet.
Feet multiplied by primes give primes.
Feet multiplied by seconds give seconds.
Primes multiplied by primes give seconds.
Primes multiplied by seconds give thirds.
Primes multiplied by thirds give fourths.
Seconds multiplied by seconds give fourths.
Seconds multiplied by thirds give fifths.
Seconds multiplied by fourths give sixths.
Thirds multiplied by thirds give sixths.
Thirds multiplied by fourths give sevenths.
Thirds multiplied by fifths give eighths, &c.

2. Multiply 4ft. 7′ by 6ft. 4′. Ans. 29ft. 0′ 4″.
3. Multiply 14ft. 9′ by 4ft. 6′. Ans. 66ft. 4′ 6″.
4. Multiply 4ft. 7′ 8″ by 9ft. 6′. Ans. 44ft. 0′ 10″.
5. Multiply 10ft. 4′ 5″ by 7ft. 8′ 6″.
Ans. 79ft. 11′ 0″ 6‴ 6⁗.
6. Multiply 39ft. 10′ 7″ by 18ft. 8′ 4″.
Ans. 745ft. 6′ 10″ 2‴ 4⁗.

7. How many square feet in a floor 48 feet 6 inches long, 24 feet 3 inches broad? Ans. 1176ft. 1′ 6″.

8. What are the contents of a marble slab, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches?
Ans. 10ft. 2′ 10″.

9. The length of a room being 20 feet, its breadth 14 feet 6 inches, and height 10 feet 4 inches, how many yards of painting are in it, deducting a surplus of 4 feet by 4 feet 4 inches, and 2 windows, each 6 feet by 3 feet 2 inches ?
Ans. 73$\frac{2}{27}$ yards.

10. Required the solid contents of a wall 53 feet 6 inches long, 10 feet 3 inches high, and 2 feet thick.
Ans. 1096ft. 9′.

11. There is a house with four tiers of windows, and 4 windows in a tier; the height of the first is 6 feet 8 inches; the second, 5 feet 9 inches; the third, 4 feet 6 inches; the fourth, 3 feet 10 inches; and the breadth is 3 feet 5 inches; how many square feet do they contain in the whole? Ans. 283ft. 7in.

12. How many feet of boards would it require to make 15 boxes, each of which is 7 feet 9 inches long, 3 feet 4 inches wide, and 2 feet 10 inches high; and how many cubic yards would they contain? Ans. 1717ft. 1in. 40$\frac{215}{324}$ cubic yds.

13. A mason has plastered 3 rooms; the ceiling of each is 20 feet by 16 feet 6 inches, the walls of each are 9 feet 6 inches high, and there are to be 90 yards deducted for doors, windows, &c. How many yards must he be paid for?
Ans. 251yd. 1ft. 6in.

14. How many feet in a board which is 17 feet 6 inches long, and 1 foot 7 inches wide? Ans. 27ft. 8′ 6″.

15. How many feet in a board 27 feet 9 inches long, 29 inches wide? Ans. 67ft. 0′ 9″.

16. How many feet of boards will it take to cover the side of a building 47 feet long, 17 feet 9 inches high?
Ans. 834ft. 3′.

NOTE.— A board to be merchantable should be 1 inch thick; therefore to reduce a plank to board measure, the superficial contents of the plank should be multiplied by its thickness.

17. How many feet, board measure, are in a plank 18 feet 9 inches long, 1 foot 6 inches wide, and 3 inches thick?
Ans. 84ft. 4′ 6″.

18. How many feet, board measure, are in a plank 20 feet long, 1 foot 6 inches wide, and 2½ inches thick? Ans. 75ft.

19. How many feet in a plank 40 feet 6 inches long, 30 inches wide, and 2¾ inches thick? Ans. 278ft. 5′ 3″.

NOTE.—A pile of wood that is 8 feet long, 4 feet high, and 4 feet wide, contains 128 cubic feet, or a cord, and every cord contains 8 *cord-feet;* and as 8 is $\frac{1}{16}$ of 128, every *cord-foot* contains 16 cubic feet; therefore, dividing the cubic feet in a pile of wood by 16, the quotient is the cord-feet; and if cord-feet be divided by 8, the quotient is cords.

20. How many cords of wood in a pile 18 feet long, 6 feet high, and 4 feet wide? Ans. $3\frac{3}{8}$ cords.

21. How many cords in a pile 10 feet long, 5 feet high, 7 feet wide? Ans. 2 cords, 94 cubic feet.

22. How many cords in a pile 35 feet long, 4 feet wide, 4 feet high? Ans. $4\frac{3}{8}$ cords.

23. How many cords in a pile that measures 8 feet on each side? Ans. 4 cords.

24. How many cords in a pile that is 10 feet on each side? Ans. $7\frac{13}{16}$ cords.

NOTE. — When wood is "corded" in a pile, 4 feet wide, by multiplying its length by its height, and dividing the product by 4, the quotient is the cord-feet; and if a load of wood be 8 feet long, and its height be multiplied by its width, and the product divided by 2, the quotient is the cord-feet.

25. How many cords of wood in a pile 4 feet wide, 70 feet 6 inches long, and 5 feet 3 inches high? Ans. $11\frac{9}{16}$ cords.

NOTE. — Small fractions are rejected.

26. How many cords in a pile of wood 97 feet 9 inches long, 4 feet wide, and 3 feet 6 inches high? Ans. $10\frac{177}{256}$ cords.

27. Required the number of cords of wood in a pile 100 feet long, 4 feet wide, and 6 feet 11 inches high. Ans. $21\frac{59}{96}$.

28. Agreed with a man for 10 cords of wood, at $ 5.00 a cord; it was to be cut 4 feet long, but by *mistake* it was cut only 46 inches long. How much *in justice* should be deducted from the *stipulated* price? Ans. $ $2.08\frac{1}{3}$.

29. If a load of wood be 8 feet long, 3 feet 8 inches wide, and 5 feet high, how much does it contain? Ans. $9\frac{1}{6}$ cord-feet.

30. If a load of wood be 8 feet long, 3 feet 10 inches wide, and 6 feet 6 inches high, how much does it contain? Ans. $12\frac{11}{24}$ cord-feet.

31. If a load of wood be 8 feet long, 3 feet 6 inches wide, how high should it be to contain 1 cord? Ans. 4ft. 6′ $10\frac{2}{7}$″.

32. If a load of wood be 12 feet long, and 3 feet 9 inches wide, how high should it be to contain 2 cords? Ans. 5ft. 8′ $3\frac{1}{5}$″.

33. D. H. Sanborn's parlour is 17ft. 9in. long, 14ft. 8in. wide, and 8ft. 9in. high. There are two doors 3ft. 4in. wide, and 7ft. high, and four windows 5ft. 3in. high, and 3ft. 4in. wide; the mop-boards are 9in. high. B. Gordon, a first-rate mason, will charge 10 cents per square yard for plastering the

room. The paper for the room is 20 inches wide, and costs $6\frac{1}{4}$ cents per yard. E. Eaton will "paper" the room for 4 cents per square yard. Each window has 12 lights of 10in. by 14in. glass, the price of which is $12\frac{1}{2}$ cents per square foot. The painter's bill for setting the glass is 8 cents per light, and for painting the floor, mop-boards, and doors is 25 cents per square yard. What is the amount of Mr. Sanborn's bill?

Ans. \$ $33.72\frac{89}{216}$.

SECTION LX.

INVOLUTION.

INVOLUTION is the raising of powers from any given number, as a root.

A power is a quantity produced by multiplying any given number, called a *root*, a certain number of times continually by itself; thus,

$2 = 2$ is the root, or 1st power of $2 = 2^1$.
$2 \times 2 = 4$ is the 2d power, or square of $2 = 2^2$.
$2 \times 2 \times 2 = 8$ is the 3d power, or cube of $2 = 2^3$.
$2 \times 2 \times 2 \times 2 = 16$ is the 4th power, or biquadrate of $2 = 2^4$.

The number denoting the power is called the *index* or *exponent* of the power. Thus, the fourth power of 3, $= 81$, is expressed by 3^4, and 4 is the index or exponent; and the second power of 7, $= 49$, is expressed by 7^2.

To raise a number to any power required.

RULE. — *Multiply the given number continually by itself, till the number of multiplications be one less than the index of the power to be found, and the last product will be the power required.*

EXAMPLES.

1. What is the 5th power of 4?
   $4 \times 4 \times 4 \times 4 \times 4 = 1024$ Ans.
2. What is the 3d power of 8? Ans. 512.
3. What is the 10th power of 7? Ans. 282475249.
4. What is the 6th power of 5? Ans. 15625.
5. What is the 3d power of $\frac{3}{4}$? Ans. $\frac{27}{64}$.
6. What is the 5th power of $\frac{1}{3}$? Ans. $\frac{1}{243}$.
7. What is the 4th power of $2\frac{2}{3}$? Ans. $50\frac{46}{81}$.

8. What is the 6th power of $1\frac{3}{5}$? Ans. $16\frac{12144}{15625}$.
9. What is the 4th power of .045? Ans. .000004100625.
10. What is the 0 power of 1728? Ans. 1.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

Evolution is the reverse of Involution, and teaches to find the roots of any given powers.

The *root* is a number whose continual multiplication into itself produces the power, which is denominated the 2d, 3d, 4th, &c., power, according to the number of times which the root is multiplied into itself. Thus, 4 is the square root of 16, because $4 \times 4 = 16$; and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$; and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers of which precise roots can never be determined; but, by the help of decimals, we can approximate towards the root to any assigned degree of exactness.

The roots which approximate are called *surd roots;* and those which are perfectly accurate are called *rational roots.*

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the root over it; thus, the 3d root of 36 is expressed $\sqrt[3]{36}$, and the second root of 36 is $\sqrt{36}$, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers with the sign $+$ or $-$ between them, a line is drawn from the top of the sign over all the parts of it; thus, the 3d root of $\overline{42+22}$ is $\sqrt[3]{42+22}$, and the second root of $\overline{59-17}$ is $\sqrt{59-17}$, &c.

Sometimes roots are designated like powers with fractional indices. Thus the square root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and the 4th root of $\overline{37-20}$ is $\overline{37-20}^{\frac{1}{4}}$, &c.

It sometimes will happen that one root is involved in another, thus:

$$\sqrt[3]{\overline{125-5}+\sqrt{19+6}}, \text{ or } \sqrt{161-\sqrt[3]{147}}.$$

$$\sqrt[5]{\sqrt[4]{\overline{178+7}\ \sqrt[2]{33-8}}+\sqrt[7]{84-5}-\sqrt[3]{87+16}}.$$

A TABLE OF POWERS.

1st Power	1	2	3	4	5	6	7	8	9
2d Power	1	4	9	16	25	36	49	64	81
3d Power	1	8	27	64	125	216	343	512	729
4th Power	1	16	81	256	625	1296	2401	4096	6561
5th Power	1	32	243	1024	3125	7776	16807	32768	59049
6th Power	1	64	729	4096	15625	46656	117649	262144	531441
7th Power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th Power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th Power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th Power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401

SECTION LXI.

EXTRACTION OF THE SQUARE ROOT.

1. LET it be required to find what number multiplied into itself will produce 1296.

OPERATION.

```
            1296(30 + 6 = 36 Ans.
             900
60 + 6 = 66)396
             396
```

In contemplating this problem, we perceive that the root or number sought must consist of *two* figures, since the product of any two numbers can have *at most* but as many figures as there are in both factors, and *at least* but one less. We perceive, also, that the first figure of the root multiplied by itself must give a number not exceeding 12, and as 12 is not the second power of any number, and the second power of 4 is more than 12, we take 3 for the first figure of the root, which multiplied into itself gives 9. Now, the second power of 3, *considered as occupying the place of tens*, is 900, of which we have the root 30. Taking 900 from 1296, we have a remainder, 396; and having found the root of 900, we are now to seek a number, which, being added to this root (30) and multiplied into itself *once*, and into 30 *twice*,* will produce 396. This number is found by dividing 396 by twice 30 plus the number sought. Q. E. D.

NOTE. — Owing to the fact that the number of figures in the product of any two numbers is always limited as above stated, we ascertain the number of figures in the root of any given second power by putting a dot over the place of units, then over the place of hundreds, and so on. The number of dots gives the number of figures in the root. Thus the square root of 133225 consists of three figures.

2. What is the square root of 576?

OPERATION.

```
    576(24 Ans.
    400
44)176
    176
```

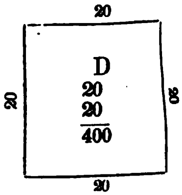
To illustrate this question in a different way from the first, we will suppose that we have 576 tiles, each of which is one foot square, and we wish to know the side of a square room whose floor they will pave or cover.

* By adding 6 to 30 and multiplying the sum (36) into itself, we can easily see that we multiply 6 by itself *once*, and 30 by 6 *twice*, since 30 is contained in both factors, and in the operation is multiplied by the 6 in each.

If we find a number which multiplied into itself will produce 576, that number will give the side of the room required. We perceive that as our number (576) consists of three figures, there will be two figures in its root, since the square of no number expressed by a single figure can be so large as 576; and, if the root were supposed to have more than two figures, its square would exceed 576. Dividing the number into periods thus, 576, we now find by trial, or by the table of powers, that the greatest square number or second power in the left-hand period, 5 (hundred), is 4 (hundred), and that its root is 2, which we write in the quotient. (See operation.) As this 2 is in the place of tens, its value must be 20, and its square or second power 400.

Let this be represented by a square whose sides measure 20 feet each, and whose contents will therefore be 400 square feet. See figure D. We now subtract 400 from 576 and there remain 176 square feet to be arranged on two sides of the figure D, in order that its form may remain square. We therefore double the root 20, one of the sides, and it gives the length of the two sides to be enlarged, viz. 40. We then inquire how many times 40 as a divisor is contained in the dividend (except the right-hand figure) and find it to be 4 times: this we write in the root, and also in the divisor.

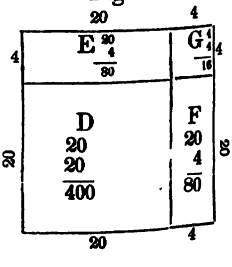
Fig. 1.



This 4 is the breadth of the addition to our square. (See figure 2.) And this breadth, multiplied by the length of the two additions (40), gives the contents of the two figures E and F, 160 square feet, which is 80 feet for each.

There now remains the space G, to complete the square, each side of which is 4 feet; it being equal to the breadth of the additions E and F. Therefore, if we square 4 we have the contents of the last addition G = 16. It is on account of this last addition that the last figure of the root is placed in the divisor. If we now multiply the divisor, 44, by the last figure in the root (4), the product will be

Fig. 2.



176, which is equal to the remaining feet after we had formed our first square, and equal to the additions E, F, and G, in figure 2. We therefore perceive that figure 2 may represent a floor 24 feet square, containing 576 square feet.

D	contains	400	square	feet.
E	"	80	"	"
F	"	80	"	"
G	"	16	"	"
	Proof,	576		

or

$24 \times 24 = 576$

RULE. — 1. *Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points show the number of figures the root will consist of.*

2. *Find the greatest square number in the first or left-hand period, placing the root of it at the right hand of the given number (after the manner of a quotient in division), for the first figure of the root, and the square number under the period, subtracting it therefrom; and to the remainder bring down the next period for a dividend, always, however, omitting the right-hand figure of this dividend in dividing.*

3. *Place the double of the root already found on the left hand of the dividend for a divisor.*

4. *Find how often the divisor is contained in the dividend (omitting the right-hand figure), placing the answer in the root for the second figure of it, and likewise on the right hand of the divisor.** *Multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend. To the remainder join the next period for a new dividend.*

5. *Double the figures already found in the root for a new divisor (or bring down the last divisor for a new one, doubling the right-hand figure of it), and from these find the next figure in the root as last directed, and continue the operation in the same manner, till you have brought down all the periods.*

NOTE 1. — If, when the given power is pointed off as the case requires, the left-hand period should be deficient, it must nevertheless stand as the first period.

NOTE 2. — If there be decimals in the given number, it must be pointed both ways from the place of units. If, when there are integers, the first period in the decimals be deficient, it may be completed by annexing so many ciphers as the power requires. And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the given numbers are exhausted, the operation may be continued at pleasure by annexing ciphers.

---

* One or two units are generally to be allowed on account of other deficiencies in enlarging the square.

3. What is the square root of 278784?

$$
\begin{array}{r l}
 & 2\dot{7}8\dot{7}8\dot{4}(528 \text{ Answer.} \\
 & 25 \\
102) & \overline{287} \\
 & 204 \\
1048) & \overline{8384} \\
 & 8384 \\
\end{array}
$$

4. What is the square root of 776161? Ans. 881.
5. What is the square root of 998001? Ans. 999.
6. What is the square root of 10342656? Ans. 3216.
7. What is the square root of 9645192360241? Ans. 3105671.
8. Extract the square root of 234.09. Ans. 15.3.
9. Extract the square root of .000729. Ans. .027.
10. Required the square root of 17.3056. Ans. 4.16.
11. Required the square root of 373. Ans. 19.3132079+.
12. Required the square root of 8.93. Ans. 2.98831055+.
13. What is the square root of 1.96? Ans. 1.4.
14. Extract the square root of 3.15. Ans. 1.77482393+.
15. What is the square root of 572199960721? Ans. 756439.

If it be required to extract the square root of a vulgar fraction, reduce the fraction to its lowest terms; then extract the square root of the numerator for a new numerator, and of the denominator for a new denominator; or reduce the vulgar fraction to a decimal, and extract its root.

16. What is the square root of $\frac{144}{15129}$? Ans. $\frac{4}{41}$.
17. What is the square root of $\frac{2793}{8208}$? Ans. $\frac{7}{12}$.
18. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.
19. What is the square root of $52\frac{9}{16}$? Ans. $7\frac{1}{4}$.
20. What is the square root of $95\frac{1}{16}$? Ans. $9\frac{3}{4}$.
21. What is the square root of $363\frac{1}{361}$? Ans. $19\frac{1}{19}$.
22. Extract the square root of $6\frac{2}{5}$. Ans. 2.5298+.
23. Extract the square root of $8\frac{5}{7}$. Ans. 2.9519+.
24. Required the square root of 2. Ans. 1.41421+.

APPLICATION OF THE SQUARE ROOT.

DEFINITIONS.

1. A *circle* is a figure bounded by a line equally distant from a point called the centre.

2. A *triangle* is a figure of three sides.

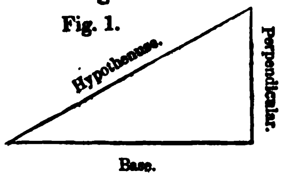
3. An equilateral triangle is that whose three sides are equal.

4. An isosceles triangle is that which has two sides equal, and a perpendicular from the angle where the equal sides meet bisects the base.

5. A right-angled triangle is a figure of three sides and three angles, one of which is a right angle. See figure 1.

The longest side is called the hypothenuse, the horizontal side the base, and the other side is called the perpendicular.

Fig. 1.

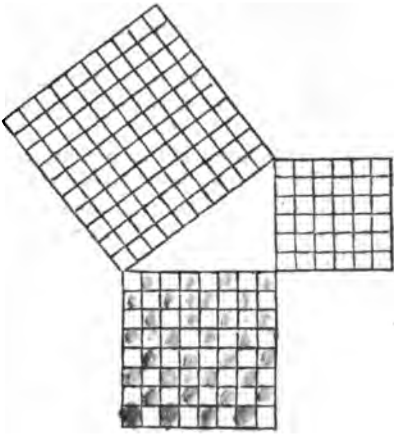


The following propositions are demonstrably true.

In a right-angled triangle, the square of the longest side is equal to the sum of the squares of the other two sides.

In all similar triangles, that is, in all triangles whose corresponding angles are equal, the sides about the equal angles are in direct proportion to each other; that is, as the longest side of one triangle is to the longest side of the other, so is either of the other sides of the former triangle to the corresponding side of the latter.

Fig. 2.



Let A B C and D E F be two similar triangles, and let A B be 6 feet, B C, 8 feet, and A C, 10 feet. Again, let D E be 12 feet, E F, 16 feet, and D F will be 20 feet. That is, B C will be to E F as A B to D E. Then 8 feet : 16

Fig. 3.

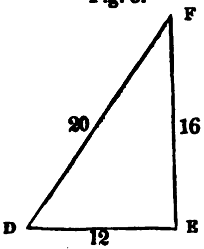
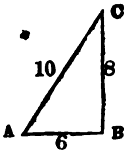


Fig. 4.



feet : : 6 feet : 12 feet. Again, A B will be to D E as A C to D F. That is, 6 feet : 12 feet : : 10 feet : 20 feet.

Circles are to each other as the squares of their diameters.

If the diameter of a circle be multiplied by 3.14159, the product is the circumference.

If the square of the diameter of a circle be multiplied by .785398, the product is the area.

If the square root of half the square of the diameter of a circle be extracted, it is the side of an inscribed square.

If the area of a circle be divided by .785398, the quotient is the square of the diameter.

EXAMPLES.

25. A certain general has an army of 141376 men. How many must he place in rank and file to form them into a square? Ans. 376.

26. If the area of a circle be 1760 yards, how many feet must the side of a square measure to contain that quantity? Ans. 125.857+ feet.

27. If the diameter of a round stick of timber be 24 inches, how large a square stick may be hewn from it? Ans. 16.97+ inches.

28. I wish to set out an orchard of 2400 mulberry-trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards. How many trees must there be in the length, and how many in the breadth; and on how many square yards of ground will they stand?

Ans. 60 in length; 40 in breadth; 112749 square yards.

29. If a lead pipe $\frac{3}{4}$ of an inch in diameter will fill a cistern in 3 hours, what should be its diameter to fill it in 2 hours? Ans. .918+ inches.

30. If a pipe $1\frac{1}{2}$ inches in diameter will fill a cistern in 50 minutes, how long would it require a pipe that is 2 inches in diameter to fill the same cistern? Ans. 28m. $7\frac{1}{2}$sec.

31. If a pipe 6 inches in diameter will draw off a certain quantity of water in 4 hours, in what time would it take 3 pipes of four inches in diameter to draw off twice the quantity? Ans. 6 hours.

32. If a line 144 feet long will reach from the top of a fort to the opposite side of a river that is 64 feet wide, what is the height of the fort? Ans. 128.99+.

33. A certain room is 20 feet long, 16 feet wide, and 12 feet high; how long must a line be to extend from one of the lower corners to an opposite upper corner? Ans. 28.28 feet.

34. Two ships sail from the same port; one goes due north 128 miles, the other due east 72 miles; how far are the ships from each other? Ans. 146.86+.

35. There are two columns in the ruins of Persepolis left standing upright; one is 70 feet above the plane, and the other 50; in a straight line between these stands a small statue, 5 feet in height, the head of which is 100 feet from the summit of the higher, and 80 feet from the top of the lower column. Required the distance between the tops of the two columns.

Ans. 143.537+ feet.

36. The height of a tree, growing in the centre of a circular island, 100 feet in diameter, is 160 feet; and a line extending from the top of it to the farther shore is 400 feet. What is the breadth of the stream, provided the land on each side of the water be level? Ans. 316.6+ feet.

37. A ladder 70 feet long is so planted as to reach a window 40 feet from the ground, on one side of the street, and without moving it at the foot it will reach a window 30 feet high on the other side; what is the breadth of the street?

Ans. 120.69+ feet.

38. If an iron wire $\frac{1}{10}$ inch in diameter will sustain a weight of 450 pounds, what weight might be sustained by a wire an inch in diameter? Ans. 45,000lbs.

39. A tree 80 feet in height stands on a horizontal plane; at what height from the ground must it be cut off, so that the top of it may fall on a point 40 feet from the bottom of the tree, the end where it was cut off resting on the stump?

Ans. 30 feet.

40. Four men, A, B, C, D, bought a grindstone, the diameter of which was 4 feet; they agreed that A should grind off his share first, and that each man should have it alternately until he had worn off his share; how much will each man grind off?

Ans. A 3.22+, B 3.81+, C 4.97+, D 12 inches.

41. What is the length of a rope that must be tied to a horse's neck, that he may feed over an acre? Ans. 7.136+ rods.

42. Required the greatest possible number of hills of corn that can be planted on a square acre, the hills to occupy only a mathematical point, and no two hills to be within three and a half feet of each other. Ans. 4165.

43. James Page has a circular garden, 10 rods in diameter; how many trees can be set upon it, so that no two shall be within ten feet of each other, and no tree within two and a half feet of the fence inclosing the garden? Ans. 241.

44. I have a board whose surface contains $49\frac{5}{8}$ square feet; the board is $1\frac{1}{2}$ inches thick, and I wish to make a cubical box of it. Required the length of one of its equal sides.

Ans. 36 inches.*

45. A carpenter has a plank 1 foot wide, $22\frac{31}{72}$ feet long, and $2\frac{1}{2}$ inches thick; and he wishes to make a box whose width shall be twice its height, and whose length shall be twice its width. Required the contents of the box.

Ans. 5719 cubic inches.*

* The operation may be found on page 203 of the last edition of the Key.

SECTION LXII.

EXTRACTION OF THE CUBE ROOT.

A CUBE is a square prism, being bounded by six equal sides, which are perpendicular to each other.

A number is said to be cubed when it is multiplied into its square.

To extract the cube root is to find a number, which, multiplied into its square, will produce the given number.

RULE. — 1. *Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.*

2. *Find by the table the greatest cube in the left-hand period, and put its root in the quotient.*

3. *Subtract the cube thus found from this period, and to the remainder bring down the next period; call this the dividend.*

4. *Multiply the square of the quotient by 300, calling it the triple square; multiply also the quotient by 30, calling it the triple quotient; the sum of these call the divisor.*

5. *Find how many times the divisor is contained in the dividend, and place the result in the quotient.*

NOTE. — One or two units, and sometimes three, must be allowed.

6. *Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the subtrahend.*

7. *Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on, till the whole is completed.*

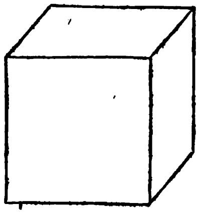
Note. — The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root.

ILLUSTRATION.

We suppose we have 46,656 cubic blocks of granite, which measure one foot on each side. With these we wish to erect a cubical monument. It is required to ascertain how many blocks, or feet, will be the length of one side of the monument.

It is evident that the number of blocks will be equal to the cube root of 46,656. As the given number consists of five figures, its cube root will contain two places; for the cube of any number can never contain more than three times that number, and at least but two less. We therefore separate the given number into periods of three figures each, putting a point over the unit figure, and every third figure beyond the place of units; thus, $4\dot{6},65\dot{6}$. We find by the table of powers, or by trial, the greatest power in the left-hand period, 46 (thousand), is 27 (thousand), the root of which is 3. This root we write in the quotient; and, as it will occupy the place of tens, its real value is 30. If this be considered the side of a cube, it will contain 27,000 cubic feet, $30 \times 30 \times 30 = 27,000$ feet.

Fig. 1.



Let this cube be represented by figure 1, each of whose sides measures 30 feet; therefore its contents will be $30 \times 30 \times 30 = 27,000$ feet, as above. We subtract the contents of this cube from 46,656, and there remain 19656 cubic feet.

OPERATION.

$$
\begin{array}{rl}
 & 4\dot{6},65\dot{6}(36 \text{ Ans.} \\
 & \underline{27,000} \\
2700) & 19,656 \\
 & \overline{16,200} \\
 & 3,240 \\
 & \underline{216} \\
 & 19,656
\end{array}
$$

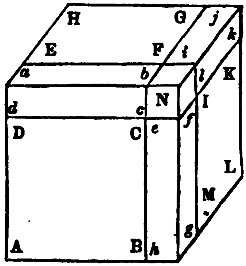
Or we might have subtracted the cube of $3, = 27$, from the first period, and to the remainder have brought down the next period, and the result would have been the same. (See operation.) The cubic blocks that remain must be applied to the three sides of figure 1. For, unless a cube

be equally increased on *three* sides, it ceases to be a cube. To effect this, we must find the superficial contents of three sides of the cube, and with these we must divide the remaining number of cubic feet or blocks, and the quotient will show the thickness of the additions. As the length of a side is 30 feet, the superficial contents will be $30 \times 30 = 900$ square feet, and this multiplied by 3, the number of sides, will be $900 \times 3 = 2700$ feet. With this as a divisor, we inquire how many times it is contained in 19,656, and find it to be 6 times (one or two units, and sometimes three, must be allowed on account of the other deficiencies in enlarging the cube). This 6 is the thickness of the additions to be made to the three sides of the cube, and by multiplying their superficial contents by it, we have the solid contents of the additions to be made $2700 \times 6 = 16200$; that is, we multiply the triple square by the last quotient figure, and this may be represented by the three superficies A B C D, E F G H, and I K L M. (See figure 2.)

Having applied these additions to our cube, we find there are three other deficiencies, *a b c d*, *e f g h*, *i j k l*, the length of which is equal to that of the additions, 30 feet, and the height and breadth of each are equal to the thickness of the additions, 6 feet. To find the contents of these, we multiply the product of their length, breadth, and thickness by their number; thus, $6 \times 6 \times 30 \times 3 = 3240$; or, which is the same thing, we multiply the triple quotient by the square of the last quotient figure; thus, $90 \times 6 \times 6 = 3240$. See rule.

Having made these additions to the cube, we still find one other deficiency, N. (See figure 2.) The length, breadth, and thickness of which are equal to the thickness of the former additions, viz. 6 feet. The contents of this are found by multiplying its length, breadth, and thickness together; that is, cubing the last quotient figure; thus, $6 \times 6 \times 6 = 216$. By making this last addition, we find that our cubical monument is finished, and that the first figure together with the several additions is equal to the cubical blocks, 46,656.

Fig. 2.



Proof.

27000 = contents of fig. 1.
16200 = " " first additions.
3240 = " " second additions.
216 = " " third addition.
46656 = contents of the whole monument.

1. Required the cube root of 77308776.

OPERATION.

77308776(426 root.	$4 \times 4 \times 300 = 4800$
64	$4 \times 30 = 120$
4920)13308 = 1st dividend.	1st divisor = 4920
9600	$4800 \times 2 = 9600$
480	$120 \times 2 \times 2 = 480$
8	$2 \times 2 \times 2 = 8$
10088 = 1st subtrahend.	1st subtrahend = 10088
530460) 3220776 = 2d dividend.	$42 \times 42 \times 300 = 529200$
3175200	$42 \times 30 = 1260$
45360	2d divisor = 530460
216	$529200 \times 6 = 3175200$
3220776 = 2d subtrahend.	$1260 \times 6 \times 6 = 45360$
	$6 \times 6 \times 6 = 216$
	2d subtrahend = 3220776

2. What is the cube root of 34965783? Ans. 327.
3. What is the cube root of 436036824287? Ans. 7583.
4. What is the cube root of 84.604519? Ans. 4.39.
5. Required the cube root of 54439939. Ans. 379.
6. Extract the cube root of 60236288. Ans. 392.
7. Extract the cube root of 109215352. Ans. 478.
8. What is the cube root of 116.930169? Ans. 4.89.
9. What is the cube root of .726572699? Ans. .899.
10. Required the cube root of 2. Ans. 1.2599+.
11. Find the cube root of 11. Ans. 2.2239+.
12. What is the cube root of 122615327232? Ans. 4968.
13. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
14. What is the cube root of $\frac{1331}{4913}$? Ans. $\frac{11}{17}$.
15. What is the cube root of $\frac{512}{729}$? Ans. $\frac{8}{9}$.
16. What is the cube root of $\frac{59319}{68921}$? Ans. $\frac{39}{41}$.

To find the cube root of any number *mentally*, less than 1,000,000, when the number has an exact root.

RULE. — *As there will be two figures in the root, the first may easily be found mentally, or by the table of powers; and if the unit figure of the power be* 1, *the unit figure in the root will be* 1; *and if it be* 8, *the root will be* 2; *and if* 7 *it will be* 3; *and if the unit of the power be* 6, *the unit of the root will be* 6; *and if* 5, *it will be* 5; *if* 3, *it will be* 7; *if* 2, *it will be* 8; *and if the unit of the power be* 9, *the unit of the root will be* 9. *This will appear evident by inspecting the table of powers.*

17. What is the cube root of 97336? Ans. 46.

Explanation. By examining the left-hand period, we find the root of 97 is 4, and the cube of 4 is 64. The root cannot be 5, because the cube of 5 is 125. The unit of the power is 6; therefore, by the above rule, the unit figure in the root is 6. The answer, therefore, is 46.

18. What is the cube root of 132651? Ans. 51.
19. What is the cube root of 148877? Ans.
20. What is the cube root of 175616? Ans.
21. What is the cube root of 185193? Ans.
22. What is the cube root of 238328? Ans.
23. What is the cube root of 262144? Ans.
24. What is the cube root of 389017? Ans.
25. What is the cube root of 405224? Ans.
26. What is the cube root of 531441? Ans.
27. What is the cube root of 24389? Ans.
28. What is the cube root of 42875? Ans.

SECOND METHOD OF EXTRACTING THE CUBE ROOT.

The following rule for the extraction of the root of the third power, though it is essentially the same with the former, may yet serve to make the *reasons* for the several steps of the operation more intelligible to the learner.

RULE. — *Separate the number whose root is to be found into periods, as under the former rule, and find by trial the greatest root in the left-hand period, and put it in the place of the quotient.*

Subtract the third power of this root from the period to which it belongs, and to the remainder bring down the next period for a dividend.

*Then, to find a divisor, annex a cipher to the root already found, and multiply twice the number thus formed by the number itself, and to the product add the second power of this number.**

* This is the same as multiplying the square of the radical figure by 300, as in the former rule.

*Ascertain how many times this divisor is contained in the dividend, and write the result in the quotient.**

Then, to find the subtrahend, multiply this divisor by its quotient, and write the product under the dividend. To this add three times the preceding radical figure with a cipher annexed,† multiplied by the second power of the figure last obtained, and also the third power of this last figure. Subtract the sum of their several products from the dividend above them, and to the remainder bring down the next period for a new dividend. With the parts of the root already found proceed to find a divisor and subtract as above, and so on, till the successive figures of the root are all obtained.

The rationale of the above rule may be made to appear by the solution of the following question.

Let it be required to find the cube root of 17576.

OPERATION.

$$\begin{array}{rr}
20 \times 2 = & 40 \\
 & 20 \\
\hline
 & 800 \\
20 \times 20 = & 400 \\
\hline
\text{Divisor} & 1200
\end{array}
\qquad
\begin{array}{rl}
\dot{1}757\dot{6} & (20 + 6 = 26 \text{ Ans.} \\
8000 & \\
\hline
)\ 9576 & \text{dividend.} \\
\hline
7200 & \\
2160 & \\
216 & \\
\hline
9576 & \text{subtrahend.}
\end{array}$$

We now raise the quantity 20 + 6 to the third power.

$$\begin{array}{rrrr}
20 + 6 & & & \\
20 + 6 & & & \\
\hline
400 + 120 & & & \\
 & 120 + 36 & & \\
\hline
400 + 240 & + 36 & & \\
20 + 6 & & & \\
\hline
8000 + 4800 & + 720 & & \\
 & 2400 + 1440 & + 216 & \\
\hline
8000 + 7200 & + 2160 & + 216 & = 17576.
\end{array}$$

* This quotient figure must sometimes be less than the one indicated by the divisor, and in extreme cases the divisor may give a quotient too large by several units. The quotient required can, of course, never exceed 9.

† This accounts for multiplying by 30, in the foregoing rule, which is a factor in the triple quotient in finding the subtrahend.

Now, by observing this operation, and remarking what would be lost in the course of it by omitting the second figure of the root, 6, taking 20 instead of 26, we see that when we have found the 20 the next inquiry is, what number must be added to 20, so that, if we multiply it into itself once and into 20 twice, and the sum of these products, together with the second power of twenty, by twenty plus this number, the result will be 17576, or $8000 + 9576$. Then, in order to obtain this number, or an approximation to it, we take twice the part of the root found, 20, and multiply the result, 40, by 20, as we should do in raising it to the third power, and make this, which is 800, a part of the divisor, and the product of which by 6 was lost in the operation for want of the 6 added to 20. But this is not all the loss. There is also the second power of 20 by 6, and therefore 400 to be added to the 800 for a divisor. There still remains the further loss of the third power of 6 (216), and also of 6 times 240 and 20 times 36; but these we neglect in the formation of the divisor. The divisor is contained in the dividend 7 times; but, making the allowance of a unit for the neglect of the numbers above named, we take 6 for the quotient figure, and proceed to find the subtrahend, which, according to the rule and the foregoing operation of raising $20 + 6$ to the third power, must be $1200 \times 6 + 1440 + 720 + 216 = 17576$.

APPLICATION OF THE CUBE ROOT.

PRINCIPLES ASSUMED.

Spheres are to each other as the cubes of their diameter.

Cubes, and all solids whose corresponding parts are similar and proportional to each other, are to each other as the cubes of their diameters, or of their homologous sides.

29. If a ball, 3 inches in diameter, weigh 4 pounds, what will be the weight of a ball that is 6 inches in diameter?

Ans. 32lbs.

30. If a globe of gold, one inch in diameter, be worth \$120, what is the value of a globe $3\frac{1}{2}$ inches in diameter?

Ans. \$5145.

31. If the weight of a well-proportioned man, 5 feet 10 inches in height, be 180 pounds, what must have been the weight of Goliath of Gath, who was 10 feet $4\frac{3}{8}$ inches in height?

Ans. 1015.1+lbs.

32. If a bell, 4 inches in height, 3 inches in width, and $\frac{1}{4}$ of

an inch in thickness, weigh 2 pounds, what should be the dimensions of a bell that would weigh 2000 pounds?

Ans. 3ft. 4in. high, 2ft. 6in. wide, and $2\frac{1}{2}$in. thick.

33. Having a small stack of hay, 5 feet in height, weighing 1cwt., I wish to know the weight of a similar stack that is 20 feet in height. Ans. 64cwt.

34. If a man dig a small square cellar, which will measure 6 feet each way, in one day, how long would it take him to dig a similar one that measured 10 feet each way?

Ans. 4.629+ days.

35. If an ox, whose girth is 6 feet, weighs 600lbs., what is the weight of an ox whose girth is 8 feet? Ans. 1422.2+lbs.

36. Four women own a ball of butter, 5 inches in diameter. It is agreed that each shall take her share separately from the surface of the ball. How many inches of its diameter shall each take?

Ans. First, .45+ inches; second, .57+ inches; third, .82+ inches; fourth, 3.149+ inches.

37. John Jones has a stack of hay in the form of a pyramid. It is 16 feet in height, and 12 feet wide at its base. It contains 5 tons of hay, worth $ 17.50 per ton. Mr. Jones has sold this hay to Messrs. Pierce, Rowe, Wells, and Northend. As the upper part of the stack has been injured, it is agreed that Mr. Pierce, who takes the upper part, shall have 10 per cent. more of the hay than Mr. Rowe; and Mr. Rowe, who takes his share next, shall have 8 per cent. more than Mr. Wells; and Mr. Northend, who has the bottom of the stack, that has been much injured, shall have 10 per cent. more than Mr. Wells. Required the quantity of hay, and how many feet of the height of the stack, beginning at the top, each receives.

Ans. Pierce receives $27\frac{54}{273}$cwt. and 10.366+ feet in height; Rowe, $24\frac{198}{273}$cwt. and 2.503 feet; Wells, $22\frac{244}{273}$cwt. and 1.666 feet; Northend, $25\frac{50}{273}$cwt. and 1.474 feet.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

RULE. — 1. *Prepare the given number for extraction, by pointing off from the unit's place, as the required root directs.*

2. *Find the first figure of the root by trial, or by inspection, in the table of powers, and subtract its power from the left-hand period.*

3. *To the remainder, bring down the first figure in the next period, and call it the dividend.*

4. *Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power for a divisor.*

5. *Find how many times the divisor is contained in the dividend, and the quotient will be another figure of the root. An allowance of two or three units is generally made, and for the higher powers a still greater allowance is necessary.*

6. *Involve the whole root to the given power, and subtract it from the given number, as before.*

7. *Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before; and in like manner proceed till the whole is finished.*

1. What is the cube root of 20346417?

OPERATION.

	$\dot{2}034\dot{6}41\dot{7}$(273	
$2^3 =$	8	= 1st subtrahend.
$2^2 \times 3 = 12)$	123	= 1st dividend.
$27^3 =$	19683	= 2d subtrahend.
$27^2 \times 3 = 2187)$	6634	= 2d dividend
$273^3 =$	20346417	= 3d subtrahend.

2. What is the fourth root of 34828517376?

OPERATION.

	$3\dot{4}8285\dot{1}737\dot{6}$(432 Ans.	
$4^4 =$	256	= 1st subtrahend.
$4^3 \times 4 = 256)$	922	= 1st dividend.
$43^4 =$	3418801	= 2d subtrahend.
$43^3 \times 4 = 318028)$	640507	= 2d dividend.
$432^4 =$	34828517376	= 3d subtrahend.

3. What is the 5th root of 281950621875? Ans. 195.

4. Required the sixth root of 1178420166015625. Ans. 325.

5. Required the seventh root of 1283918464548864. Ans. 144.

6. Required the eighth root of 218340105584896. Ans. 62.

Section LXIII.

ARITHMETICAL PROGRESSION.

When a series of quantities or numbers increases or decreases by a constant difference, it is called arithmetical progression, or progression by difference. The constant difference is called the *common difference* or the *ratio* of the progression.

Thus, let there be the two following series: —

1, 5, 9, 13, 17, 21, 25, 29, 33,
25, 22, 19, 16, 13, 10, 7, 4, 1.

The first is called an ascending series or progression. The second is called a descending series or progression.

The numbers which form the series are called the *terms* of the progression.

The first and last terms of the progression are called the *extremes*, and the other terms, the *means*.

Any three of the five following things being given, the other two may be found: —

1st, the first term,
2d, the last term,
3d, the number of terms,
4th, the common difference,
5th, the sum of the terms.

PROBLEM I.

The first term, last term, and number of terms being given, to find the common difference.

To illustrate this problem, we will examine the following series, —

3, 5, 7, 9, 11, 13, 15, 17, 19.

It will be perceived that in this series 3 and 19 are the extremes, 2 the common difference, 9 the number of terms, and 99 the sum of the series.

It is evident, that the number of common differences in any number of terms will be *one* less than the number of terms. Hence, if there be 9 terms, the number of common differences will be 8, and the sum of these common differences will be equal to the difference of the extremes; therefore if the difference of the extremes, $19 - 3 = 16$, be divided by the number of common differences, the quotient will be the common difference. Thus $16 \div 8 = 2$ is the common difference.

RULE. — *Divide the difference of the extremes by the number of terms less one, and the quotient is the common difference.*

1. The extremes are 3 and 45, and the number of terms is 22. What is the common difference?

OPERATION.

$$\frac{45 - 3}{22 - 1} = 2 \text{ Answer.}$$

2. A man is to travel from Albany to a certain place in 11 days, and to go but 5 miles the first day, increasing the distance equally each day, so that the last day's journey may be 45 miles. Required the daily increase. Ans. 4 miles.

3. A man had 10 sons, whose several ages differed alike; the youngest was 3 years, and the oldest 48. What was the common difference of their ages? Ans. 5 years.

4. A certain school consists of 19 scholars; the youngest is 3 years old, and the oldest 39. What is the common difference of their ages? Ans. 2 years.

PROBLEM II.

The first term, last term, and number of terms being given, to find the sum of all the terms.

Illustration.

Let 3, 5, 7, 9, 11, 13, 15, 17, 19, be the series,
and 19, 17, 15, 13, 11, 9, 7, 5, 3, the same series inverted.
22, 22, 22, 22, 22, 22, 22, 22, 22, sum of both series.

From the arrangement of the above series, we see that, by adding the two as they stand, we have the same number for the sum of the successive terms, and that the sum of both series is double the sum of either series.

It is evident that if, in the above series, 22 be multiplied by 9, the number of terms, the product will be the sum of both series, $22 \times 9 = 198$, and therefore the sum of either series will be $198 \div 2 = 99$. But 22 is also the sum of the extremes in either series, $3 + 19 = 22$. Therefore, *if the sum of the extremes be multiplied by the number of terms, the product will be double the sum of the series.*

RULE. — *Multiply the sum of the extremes by the number of terms, and half the product will be the sum of the series.*

5. The extremes of an arithmetical series are 3 and 45, and the number of terms 22. Required the sum of the series.

OPERATION.

$$\frac{45+3\times 22}{2}=528 \text{ Answer.}$$

6. A man going a journey travelled the first day 7 miles, the last day 51 miles, and he continued his journey 12 days. How far did he travel? Ans. 348 miles.

7. In a certain school there are 19 scholars; the youngest is 3 years old, and the oldest 39. What is the sum of their ages? Ans. 399 years.

8. Suppose a number of stones were laid a rod distant from each other, for thirty miles, and the first stone a rod from a basket. What length of ground will that man travel over who gathers them up singly, returning with them one by one to the basket? Ans. 288090 miles 2 rods.

PROBLEM III.

The extremes and the common difference being given, to find the number of terms.

Illustration. — Let the extremes be 3 and 19, and the common difference 2. The difference of the extremes will be $19-3=16$; and it is evident, that, if the difference of the extremes be divided by the common difference, the quotient is the number of common differences; thus $16\div 2=8$. We have demonstrated in Problem I. that the number of terms is *one* more than the number of differences; therefore $8+1=9$, the number of terms.

RULE. — *Divide the difference of the extremes by the common difference, and the quotient increased by one will be the number of terms required.*

9. If the extremes are 3 and 45, and the common difference 2, what is the number of terms?

OPERATION.

$$\frac{45-3}{2}+1=22 \text{ Answer.}$$

10. In a certain school the ages of all the scholars differ alike; the oldest is 39 years, the youngest is 3 years, and the difference between the ages of each is 2 years. Required the number of scholars. Ans. 19.

11. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles. How many days did he travel? Ans. 12.

PROBLEM IV.

The extremes and common difference being given, to find the sum of the series.

Illustration. — Let the extremes be 3 and 19, and the common difference 2. The difference of the extremes will be $19 - 3 = 16$; and it has been shown in the last problem, that, if the difference of the extremes be divided by the common difference, the quotient will be the number of terms less one; therefore the number of terms less one will be $16 \div 2 = 8$, and the number of terms $8 + 1 = 9$. It was demonstrated in Problem II. that if the number of terms was multiplied by the sum of the extremes, and the product divided by 2, the quotient would be the sum of the series.

RULE. — *Divide the difference of the extremes by the common difference, and add* 1 *to the quotient; multiply this quotient by the sum of the extremes, and half the product is the sum of the series.*

12. If the extremes are 3 and 45, and the common difference 2, what is the sum of the series? Ans. 528.

13. A owes B a certain sum, to be discharged in a year, by paying 6 cents the first week, 18 cents the second week, and thus to increase every week by 12 cents, till the last payment should be \$6.18. What is the debt? Ans. \$162.24.

PROBLEM V.

The extremes and sum of the series being given, to find the common difference.

Illustration. — Let the extremes be 3 and 19, and the sum of the series 99, to find the common difference. We have before shown, that, if the extremes be multiplied by the number of terms, the product would be twice the sum of the series; therefore, if twice the sum of the series be divided by the extremes, the quotient will be the number of terms. Thus, $99 \times 2 = 198$; $3 + 19 = 22$; $198 \div 22 = 9$ is the number of terms. And we have before shown, that, if the difference of the extremes be divided by the number of terms less one, the quotient will be the common difference; therefore, $19 - 3 = 16$; $9 - 1 = 8$; $16 \div 8 = 2$ is the common difference.

RULE. — *Divide twice the sum of the series by the sum of the extremes, and from the quotient subtract* 1; *and with this remainder divide the difference of the extremes, and the quotient is the common difference.*

14. The extremes are 3 and 45, and the sum of the series 528. What is the common difference? Ans. 2.

PROBLEM VI.

The first term, number of terms, and the sum of the series being given, to find the last term.

Illustration. — Let 3 be the first term, 9 the number of terms, and 99 the sum of the series. By Problem II. it was shown, that, if the sum of the extremes were multiplied by the number of terms, the product was twice the sum of the series; therefore, if twice the sum of the series be divided by the number of terms, the quotient is the sum of the extremes. If from this we subtract the first term, the remainder is the last term; thus $99 \times 2 = 198$; $198 \div 9 = 22$; $22 - 3 = 19$, last term.

RULE. — *Divide twice the sum of the series by the number of terms; from the quotient take the first term, and the remainder will be the last term.*

15. A merchant being indebted to 22 creditors \$ 528, ordered his clerk to pay the first \$ 3, and the rest increasing in arithmetical progression. What is the difference of the payments, and the last payment?

Ans. Difference 2; last payment, \$ 45.

SECTION LXIV.

GEOMETRICAL SERIES,

OR SERIES BY QUOTIENT.

IF there be three or more numbers, and if there be the same quotient when the second is divided by the first, and the third divided by the second, and the fourth divided by the third, &c., those numbers are in *geometrical progression*. If the series *increase*, the quotient is more than unity; if it *decrease*, it is less than unity.

The following series are examples of this kind: —

2,	6,	18,	54,	162,	486.
64,	32,	16,	8,	4,	2.

The former is called an *ascending* series, and the latter a *descending* series.

In the first, the quotient is 3, and is called the *ratio*; in the second, it is $\frac{1}{2}$.

The first and last terms of a series are called *extremes*, and the other terms *means*.

PROBLEM I.

One of the extremes, the ratio, and the number of terms being given, to find the other extreme.

Let the first term be 3, the ratio 2, and the number of terms 8, to find the last term.

Illustration. — It is evident, that, if we multiply the first term by the ratio, the product will be the second term; and, if we multiply the second term by the ratio, the product will be the third term; and, in this manner, we may carry the series to any desirable extent. By examining the following series, we find that 3, carried to the 8th term, is 384; thus,

(1.) (2.) (3.) (4.) (5.) (6.) (7.) (8.)
3, 6, 12, 24, 48, 96, 192, 384, ascending series.

But the factors of 384 are 3, 2, 2, 2, 2, 2, 2, and 2; therefore the continued product of these numbers will produce 384. But, by multiplying 2 by itself six times, and that product by 3, is the same as raising the ratio, 2, to the seventh power, and then multiplying that power by the first term. Hence the following

RULE. — *Raise the ratio to a power whose index is equal to the number of terms less one; then multiply this power by the first term, and the product is the last term, or other extreme.*

Illustration. — In the above question the number of terms is 8; we therefore raise 2, the ratio, to the seventh power, it being *one less* than 8; thus, $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$. We then multiply this number by 3, the first term, and the product is the last term; thus, $128 \times 3 = 384$, last term.

The above rule will apply in a descending series. Let the following numbers be a geometrical descending series: —

384, 192, 96, 48, 24, 12, 6, 3, descending series.

Let the first term be 384, the number of terms 8, and the ratio $\frac{1}{2}$, to find the other extreme. By the above rule, we raise the ratio, $\frac{1}{2}$, to the seventh power, it being one less than the number of terms, 8. Thus, $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{128}$. We then multiply this power by the first term; thus, $\frac{1}{128} \times \frac{384}{1} = \frac{384}{128} = 3$, the extreme required.

But as the last term, or any term near the last, is very tedious to be found by continual multiplication, it will often be necessary, in order to ascertain it, to have a series of numbers

in arithmetical proportion, called *indices* or *exponents*, beginning either with a cipher or a unit, whose common difference is *one*. When the *first* term of the series and *ratio* are *equal*, the *indices* must begin with a unit, and in this case the product of any two terms is equal to that term signified by the *sum* of their *indices*.

Thus, $\left\{\begin{array}{l} 1, 2, 3, 4, 5, 6, \text{\&c., indices or arithmetical series.} \\ 2, 4, 8, 16, 32, 64, \text{\&c., geometrical series.} \end{array}\right.$

Now $6 + 6 = 12 =$ the index of the twelfth term, and $64 \times 64 = 4096 =$ the twelfth term.

But when the *first* term of the series and the *ratio* are *different*, the *indices* must begin with a cipher, and the sum of the *indices* made choice of must be *one less* than the *number of terms* given in the question; because 1 in the *indices* stands over the *second term*, and 2 in the *indices* over the *third* term, &c. And, in this case, the *product* of any *two* terms, divided by the *first*, is equal to that term *beyond* the first signified by the sum of their *indices*.

Thus, $\left\{\begin{array}{l} 0, 1, 2, 3, 4, 5, 6, \text{\&c., indices.} \\ 1, 3, 9, 27, 81, 243, 729, \text{\&c., geometrical series.} \end{array}\right.$

Here, $6 + 5 = 11$, the index of the 12th term.

$729 \times 243 = 177147$, the 12th term, because the first term of the series and ratio are different, by which means a cipher stands over the first term.

Thus, by the help of these *indices*, and a few of the first terms in any geometrical series, any term whose distance from the first term is assigned, though it were ever so remote, may be obtained without producing all the terms.

1. If the first term be 4, the ratio 4, and the number of terms 9, what is the last term?

OPERATION.

1. 2. 3. $4 + 4 = 8$

4. 16. 64. $256 \times 256 = 65536 =$ power of the ratio, whose exponent is less by 1 than the number of terms. 65536×4, the first term $= 262144 =$ last term.

Or, $4 \times 4^8 = 262144 =$ last term, as before.

2 If the first term be 262144, the ratio $\frac{1}{4}$, and the number of terms 9, what is the last term? Ans. 4.

$(\frac{1}{4})^8 = \frac{1}{65536}$; $\frac{1}{65536} \times \frac{262144}{1} = \frac{262144}{65536} = 4$, the last term.

3. If the first term be 72, the ratio $\frac{1}{3}$, and the number of terms 6, what is the last term? Ans. $\frac{8}{27}$.

4. If I were to buy 30 oxen, giving 2 cents for the first ox, 4 cents for the second, 8 cents for the third, &c., what would be the price of the last ox? Ans. $10737418.24.

5. If the first term be 5, and the ratio 3, what is the seventh term? Ans. 3645.

6. If the first term be 50, the ratio 1.06, and the number of terms 5, what is the last term? Ans. 63.123848.

7. What is the amount of $160.00 at compound interest for 6 years? Ans. $ 226.96,305796096.

8. What is the amount of $ 300.00 at compound interest at 5 per cent. for 8 years? Ans. $ 443.23,6+.

9. What is the amount of $100.00 at 6 per cent. for 30 years?

Ans. $ 574.34,9117291325011626410633231080264584635-7252196069357387776.

PROBLEM II.

The first term, the ratio, and the number of terms being given, to find the sum of all the terms.

In order that the pupil may understand the following rule, we will examine a question *analytically*.

Let the following be a geometrical series, and we wish to obtain its sum: —

1, 3, 9, 27, 81.

Illustration. — By examining this series, we find the first term to be 1, the last term 81, the ratio 3. If we multiply each term of the following series, 1, 3, 9, 27, 81, by 3, the ratio, their product will be 3, 9, 27, 81, 243, and the sum of this last series will be three times as much as the first series. The *difference*, therefore, between these series will be *twice* as much as the *first* series.

$$\begin{array}{rrrrrrl} & 3, & 9, & 27, & 81, & 243 & = \text{second series.} \\ 1, & 3, & 9, & 27, & 81, & & = \text{first series.} \\ \hline & 0, & 0, & 0, & 0, & 243 - 1 & = 242, \text{ difference of the series.} \end{array}$$

As this difference must be twice the sum of the first series, therefore the sum of the first series must be $242 \div 2 = 121$.

By examining the above series, we find the terms in both the same, with the exception of the *first* term in the first series, and the *last* term in the second series. We have only, then, to

subtract the first term in the first series from the last term in the second series, and the remainder is twice the sum of the first series; and half of this being taken gives the sum of the series required.

RULE. — *Find the other extreme, as before, multiply it by the ratio, and from the product subtract the given extreme. Divide the remainder by the ratio less* 1 (*unless the ratio be less than a unit, in which case the ratio must be subtracted from* 1), *and the quotient will be the sum of the series required.* See operation, question 10. *Or, raise the ratio to a power whose index is equal to the number of terms; from which subtract* 1, *divide the remainder by the ratio less* 1, *and the quotient, multiplied by the given extreme, will give the sum of the series.* See operation, question 11.

But if the ratio be a fraction less than a unit, raise the ratio to a power whose index shall be equal to the number of terms; subtract this power from 1, *divide the remainder by the difference between* 1 *and the ratio, and the quotient, multiplied by the given extreme, will give the sum of the series required.* See operation, question 12.

10. If the first term be 10, the ratio 3, and the number of terms 7, what is the sum of the series? Ans. 10930.

OPERATION.

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 10 = 7290, \text{ last term.}$$
$$7290 \times 3 = 21870;\ 21870 - 10 = 21860;\ 21860 \div \overline{3 - 1} = 10930 \text{ Ans.}$$

11. If the first term be 4, the ratio 3, and the number of terms 5, what is the sum of the series? Ans. 484.

OPERATION.

$$3 \times 3 \times 3 \times 3 \times 3 = 243;\ 243 - 1 = 242;$$
$$242 \div \overline{3 - 1} = 121;\ 121 \times 4 = 484 \text{ Ans.}$$

12. If the first term be 6, the ratio $\frac{2}{5}$, and the number of terms 4, what is the sum of the series? Ans. $9\frac{93}{125}$.

OPERATION.

$$\tfrac{2}{5} \times \tfrac{2}{5} \times \tfrac{2}{5} \times \tfrac{2}{5} = \tfrac{16}{625};\ 1 = \tfrac{625}{625};\ \tfrac{625}{625} - \tfrac{16}{625} = \tfrac{609}{625}.$$
$$1 = \tfrac{5}{5};\ \tfrac{5}{5} - \tfrac{2}{5} = \tfrac{3}{5};\ \tfrac{609}{625} \div \tfrac{3}{5} = \tfrac{609}{625} \times \tfrac{5}{3} = \tfrac{3045}{1875} = \tfrac{203}{125};\ \tfrac{203}{125} \times \tfrac{6}{1} = \tfrac{1218}{125} = 9\tfrac{93}{125} \text{ Ans.}$$

13. How large a debt may be discharged in a year, by paying \$1 the first month, \$10 the second, and so on, in a tenfold proportion, each month? Ans. \$111111111111.

14. A gentleman offered a house for sale, on the following terms; that for the first door he should charge 10 cents, for the second 20 cents, for the third 40 cents, and so on in a geo-

metrical ratio, there being 40 doors. What was the price of the house? Ans. \$ 109951162777.50.

15. If the first term be 50, the ratio 1.06, and the number of terms 4, what is the sum of the series? Ans. 218.7308.

16. A gentleman deposited annually \$10 in a bank, from the time his son was born until he was 20 years of age. Required the amount of the deposits at 6 per cent., compound interest, when his son was 21 years old. Ans. \$ 423.92,2+.

17. If the first term be 7, the ratio $\frac{1}{4}$, and the number of terms 5, what is the sum of the series? Ans. $9\frac{83}{256}$.

18. If one mill had been put at interest at the commencement of the Christian era, what would it amount to at compound interest, supposing the principal to have doubled itself every 12 years, January 1, 1837?

Ans. \$ 114179815416476790484662877555959610910619-72.99,2.

If this sum was all in dollars, it would take the present inhabitants of the globe more than 1,000,000 years to count it. If it was reduced to its value in pure gold, and was formed into a globe, it would be many million times larger than all the bodies that compose the solar system.

PROBLEM III.

To find the sum of the second powers of any number of terms, whose roots differ by unity.

RULE. — *Add one to the number of terms, and multiply this sum by the number of terms; then add one to twice the number of terms, and multiply this sum by the former product, and the last product, divided by 6, will give the sum of all the terms.*

19. What is the sum of 10 terms of the series $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2$?

OPERATION.

$$\frac{10 \times \overline{10+1} \times \overline{20+1}}{6} = 385 \text{ Ans.}$$

20. What is the sum of 100 terms of the series $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2$, &c., to 100^2? Ans. 338350.

21. Purchased 50 lots of land; the first was one rod square, the second was two rods square, the third was three rods square, and so on, the last being 50 rods square. How many square rods were there in the 50 lots? Ans. 42925.

22. Let it be required to find the number of cannon shot in a square pile, whose side is 80. Ans. 173880.

NOTE. — A square pile is formed by continued horizontal courses of shot laid one above another, and these courses are squares, whose sides decrease by unity from the bottom of the pile to the top row, which is composed of only *one* shot.

PROBLEM IV.

To find the sum of the third power of any number of terms, whose roots differ by unity.

RULE. — *Add one to the number of terms, and multiply this sum by half the number of terms; the square of this product is the sum of all the series.*

23. Required the sum of the following series: — 1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, 9^3, 10^3, 11^3, 12^3.

OPERATION.

$12 + 1 = 13$; $12 \div 2 = 6$; $13 \times 6 = 78$; $78 \times 78 = 6084$ Ans.

24. I have 10 blocks of marble, each of which is an exact cube. A side of the first cube measures one foot, a side of the second 2 feet, a side of the third 3 feet, and so on to the 10th, whose side measures 10 feet. Required the number of cubical feet in the blocks? Ans. 3025 cubic feet.

25. What is the sum of 50 terms of the series 1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, &c., up to 50^3? Ans. 1625625.

SECTION LXV.

INFINITE SERIES.

AN INFINITE SERIES is such as, being continued, would run on *ad infinitum;* but the nature of its progression is such, that, by having a few of its terms given, the others to any extent may be known. Such are the following series: —

1, 2, 4, 8, 16, 32, 64, 128, &c., *ad infinitum.*
125, 25, 5, 1, $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$, $\frac{1}{625}$, &c., *ad infinitum.*

To find the sum of a decreasing series.

RULE. — *Multiply the first term by the ratio, and divide the product by the ratio less* 1, *and the quotient is the sum of an infinite decreasing series.*

1. What is the sum of the series 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c., continued to an infinite number of terms?

OPERATION.

$$\frac{4 \times 4}{3} = 5\tfrac{1}{3} \text{ Answer.}$$

2. What is the sum of the series 5, 1, $\frac{1}{5}$, $\frac{1}{25}$, &c., continued to infinity? Ans. $6\frac{1}{4}$.

3. If the following series, 8, $\frac{8}{7}$, $\frac{8}{49}$, $\frac{8}{343}$, &c., were carried to infinity, what would be its sum? Ans. $9\frac{1}{3}$.

4. What is the sum of the following series, carried to infinity: 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c.? Ans. $1\frac{1}{2}$.

5. What is the sum of the following series, carried to infinity: 11, $\frac{11}{7}$, $\frac{11}{49}$, &c.? Ans. $12\frac{5}{6}$.

6. If the series $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, &c., were carried to infinity, what would be its sum? Ans. $1\frac{1}{3}$.

SECTION LXVI.

DISCOUNT BY COMPOUND INTEREST.

1. What is the present worth of \$ 600.00, due 3 years hence, at 6 per cent. compound interest?

OPERATION.

$$1.06)^3 = 1.191016)600.00(\$\,503.77+ \text{ Ans.}$$

By analysis. — We find the amount of \$ 1 at compound interest for 3 years to be \$ 1.191016; therefore \$ 1 is the present worth of \$ 1.191016 due 3 years hence. And if \$ 1 is the present worth of \$ 1.191016, the present worth of

$$\$\,600 = \frac{600}{1.191016} = \$\,503.77,1+$$

RULE. — *Divide the debt by the amount of one dollar for the given time, and the quotient is the present worth, which, if subtracted from the debt, will leave the discount.*

2. What is the present worth of \$ 500.00, due 4 years hence, at 6 per cent. compound interest? Ans. \$ 396.04,6+.

3. What is the present worth of \$1000.00, due 10 years hence, at 5 per cent. compound interest? Ans. \$ 613.91,3+.

4. What is the discount on \$ 800.00, due 2 years hence, at 6 per cent. compound interest? Ans. \$ 88.00,3+.

5. What is the present worth of \$ 1728, due 5 years hence, at 6 per cent. compound interest? Ans. \$ 1291.26.

6. What is the discount on \$ 3700, due 10 years hence, at 5 per cent. discount, compound interest? Ans. \$ 1428.52.

7. What is the present worth of \$ 7000, due 2 years hence, at 5 per cent. compound interest? Ans. \$ 63492.10.

SECTION LXVII.

ANNUITIES AT COMPOUND INTEREST.

AN *annuity* is a certain sum of money to be paid at regular periods, either for a limited time or for ever.

The *present worth* or *value* of an annuity is that sum which, being improved at compound interest, will be sufficient to pay the annuity.

The *amount* of an annuity is the compound interest of all the payments added to their sum.

To find the amount of an annuity at compound interest.

RULE. — *Make $1.00 the first term of a geometrical series, and the amount of $1.00 at the given rate per cent. the ratio. Carry the series to so many terms as the number of years, and find its sum. Multiply the sum thus found by the given annuity, and the product will be the amount.*

EXAMPLES.

1. What will an annuity of $60 per annum, payable yearly, amount to in 4 years, at 6 per cent.?

$$1 + 1.06 + \overline{1.06}^{2} + \overline{1.06}^{3} = 4.374616.$$

$$4.374616 \times 60 = \$\,262.47,6+ \text{ Answer.}$$

Or, $$\frac{1.06^{4} - 1}{1.06 - 1} \times 60 = \$\,262.47,6+ \text{ Answer.}$$

2. What will an annuity of $500.00 amount to in 5 years, at 6 per cent.? Ans. $2818.54,6+.

3. What will an annuity of $1000.00, payable yearly, amount to in 10 years? Ans. $13180.79,4+.

4. What will an annuity of $30.00, payable yearly, amount to in 3 years? Ans. $95.50,8+.

To find the present worth of an annuity.

As the first payment is made at the end of the year, its present worth or value is a sum that will amount in one year to that payment; and as the second payment is made at the end of the second year, its value is a sum that will, at compound interest, amount in two years to that payment; and the same principle is adopted for the third year, fourth year, &c. This may be illustrated in the following question.

5. What is the present worth of an annuity of $1.00, to continue 5 years, at compound interest?

The present worth of $ 1.00 for 1 year = $ 0.943396
The present worth of $ 1.00 for 2 years = $ 0.889996
The present worth of $ 1.00 for 3 years = $ 0.839619
The present worth of $ 1.00 for 4 years = $ 0.792094
The present worth of $ 1.00 for 5 years = $ 0.747258

$ 4.212363

By the above illustration, we perceive that the present worth of an annuity of $ 1, to continue 5 years, is $ 4.21,2+. Hence, having found the present worth of an annuity of $ 1 for any given time by Section LXVI., the present worth of any other sum may be found by multiplying it by the present worth of $1 for that time.

RULE. — *Multiply the present worth of the annuity of one dollar for the given time by the given annuity, and the product is the present worth required. Or, find the amount of the annuity by the last rule, and then find its present worth.*

6. What is the present worth of an annuity of $ 60, to be continued 4 years, at compound interest?

First Method.

The present worth of $ 1.00 for 1 year = $ 0.943396
The present worth of $ 1.00 for 2 years = $ 0.889996
The present worth of $ 1.00 for 3 years = $ 0.839619
The present worth of $ 1.00 for 4 years = $ 0.792093

$ 3.465104

$ 3.465104 × 60 = $ 207.90,6+ Answer.

Second Method.

$$\frac{1.06^4 - 1}{1.06 - 1} = \$\,4.374616 \times .792093 = \$\,3.465102 \times 60 =$$
$$\$\,207.90,6+ \text{ Ans.}$$

7. A gentleman wishes to purchase an annuity, which shall afford him, at 6 per cent. compound interest, $ 500 a year for ten years. What sum must he deposit in the annuity office to produce it? Ans. $ 3680.04+.

8. What is the present worth of an annuity of $ 1000, to continue 10 years? Ans. $ 7360.08.

9. What is the present worth of an annuity of $ 1728, to continue 8 years? Ans. $ 4618.96.

By the assistance of the following tables, questions in annuities may be easily performed.

TABLE I.

Showing the amount of $1 *annuity* from 1 year to 40.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1.000000	1.000000	21	35.719252	39.992727
2	2.050000	2.060000	22	38.505214	43.392290
3	3.152500	3.183600	23	41.430475	46.995828
4	4.310125	4.374616	24	44.501999	50.815577
5	5.525631	5.637093	25	47.727099	54.864512
6	6.801913	6.975319	26	51.113454	59.156383
7	8.142008	8.393838	27	54.669126	63.705766
8	9.549109	9.897468	28	58.402583	68.528112
9	11.026564	11.491316	29	62.322712	73.639798
10	12.577893	13.180795	30	66.438847	79.058186
11	14.206787	14.971643	31	70.760790	84.801677
12	15,917127	16.869941	32	75.298829	90.889778
13	17.712983	18.882138	33	80.063771	97.343165
14	19.598632	21.015066	34	85.066959	104.183755
15	21.578564	23.275970	35	90.220307	111.434780
16	23.657492	25.672528	36	95.836323	119.120867
17	25.840366	28.212880	37	101.628139	127.268119
18	28.132385	30.905653	38	107.709546	135.904206
19	30.539004	33.759992	39	114.095023	145.058458
20	33.065954	36.785591	40	120.799774	154.761966

TABLE II.

Showing the present value of an *annuity* of $1 from 1 year to 40.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0 952381	0.943396	21	12.821153	11.764077
2	1.859410	1.833393	22	13.163003	12.041582
3	2.723248	2.673012	23	13.488574	12.303379
4	3.545950	3.465106	24	13.798642	12.550358
5	4.329477	4.212364	25	14.093945	12.783356
6	5.075692	4.917324	26	14.375185	13.003166
7	5.786373	5.582381	27	14.643034	13.210534
8	6.463213	6.209794	28	14.898127	13.406164
9	7.107822	6.801692	29	15.141074	13.590721
10	7.721735	7.360087	30	15.372451	13.764831
11	8 306414	7.886875	31	15.592810	13.929086
12	8.863252	8.388844	32	15.802677	14.084043
13	9.393573	8.852683	33	16.002549	14.230230
14	9.898641	9.294984	34	16.192904	14.368141
15	10.379658	9.712249	35	16.374194	14.498246
16	10.837770	10.105895	36	16.546852	14.620987
17	11.274066	10 477260	37	16.711287	14.736780
18	11.689587	10.827603	38	16.867893	14.846019
19	12.085321	11.158116	39	17.017041	14.949075
20	12.462216	11.469921	40	17.159086	15.046297

10. What is the present worth of an annuity of $ 200, at 5 per cent. compound interest, for 7 years? Ans. $1157.27+.

11. What is the present worth of an annuity of $ 300, to continue 8 years, at 6 per cent. compound interest?

Ans. $ 1862.93,8+.

12. What is the present value of an annuity of $ 100, at 6 per cent. for 9 years? Ans. $ 680.16,9+.

Questions to be performed by the preceding tables.

13. What will an annuity of $ 30 amount to in 11 years, at 6 per cent.?

By Table I., the amount of $ 1 for 11 years is $14.971643; therefore, $ 14.971643 × 30 = $ 449.14,9+ Answer.

14. What is the present worth of an annuity of $ 80 for 30 years, at 5 per cent.?

By Table II., the present worth of $ 1 for 30 years is $15.372451, therefore $15.372451 × 80 = $ 1229.79,6+ Ans.

15. What will an annuity of $ 800 amount to in 25 years, at 5 per cent.? Ans. $ 38181.67,9+.

16. What will an annuity of $ 40 amount to in 30 years, at 6 per cent.? Ans. $ 3162.32,7+.

17. Required the present worth of an annuity of $ 500, to continue 40 years, at 6 per cent. Ans. $ 7523.14,8+.

18. A certain parish in the town of B., having neglected for 6 years to pay their minister's salary of $ 700, what in justice, provided he has preached the truth, should he receive?

Ans. $ 4882.72,3.

SECTION LXVIII.

ASSESSMENT OF TAXES.

A TAX is a duty laid by government, for public purposes, on the *property* of the inhabitants of a town, county, or State, and also on the polls * of the male citizens liable by law to assessment.

A tax may be either general or particular; that is, it may affect all classes indiscriminately, or only one or more classes.

* *Poll* is said to be a Saxon word, meaning *head*. In the constitution it means a person; that is, a person who is liable to taxation.

Taxes may be either direct or indirect; that is, they may either be imposed on the incomes or property of individuals, or on the articles on which these incomes or property are expended.

The method of assessing town taxes is not precisely the same in all the States, yet the *principle* is virtually the same.

In some of the States the poll tax is more than in others.

The following is the law regulating taxation in Massachusetts (see *Revised Statutes*, page 79): —

"The assessors shall assess upon the *polls*, as nearly as the same can be conveniently done, one sixth part of the whole sum to be raised; provided the whole poll tax assessed in any one year upon any individual for town and county purposes, except highway taxes, shall not exceed one dollar and fifty cents; and the residue of said whole sum to be raised shall be apportioned upon property"; that is, on the real and personal estate of individuals which is taxable.

RULE FOR ASSESSING TAXES. — *First take an inventory of all the taxable property, real and personal, in the town or county, and then the number of polls liable to taxation. Multiply the sum assessed on each poll by the number of taxable polls in the town. Subtract this amount from the sum to be raised by the town. Then as the whole valuation of the town is to the sum to be raised, after having deducted the amount to be paid by the polls, so is the amount of each man's real and personal estate to his tax. Or, if the sum to be raised on property be divided by the valuation of the town, the quotient will be the sum to be paid on each dollar of an individual's real or personal estate. Multiply each man's property by this sum, and the product will be the amount of his taxes.*

The town of B. is to be taxed $4109. The real estate of the town is valued at $493,000, and the personal property at $177,000. There are 506 polls, each of which is taxed $1.50. What is John Smith's tax, whose real estate is valued at $3700 and his personal at $2300, he paying for 6 polls? And what will be the tax on $1.00?

OPERATION.

$1.50 × 506 = $759, amount assessed on the polls.
$493,000 + $177,000 = $670,000, amount of taxable property.
$4109 — $759 = $3350, amount to be assessed on property.
$670,000 : $3350 : : $1 : $.005, to be assessed on each dollar.
$3700 × .005 = $18.50, tax on Smith's real estate.
$2300 × .005 = $11.50, tax on Smith's personal estate.
$1.50 × 6 = $9.00, tax on 6 polls.
$18.50 + $11.50 + $9.00 = $39.00, amount of Smith's tax.

What will be the amount of taxation on each of the following individuals of the above town, their taxable property being as annexed to their names?

Persons.	Real Estate.	Personal Estate.	No. of Polls.
James Dow,	$ 4780	$ 1720	3
John Brown,	7500	2120	1
Samuel Foster,	1135	175	7
James Emerson,	8960	5000	2
A. C. Hasseltine,	7140	3720	0

Form of a tax-list committed to the collector, containing the answers to the above questions.

Names.	No. of Polls.	Poll Tax.	Tax on Real Estate.	Tax on Personal Estate.	Total.	Time when paid.
James Dow,	3	$ 4.50	$ 23.90	$ 8.60	$ 37.00	
John Brown,	1	1.50	37.50	10.60	49.60	
Samuel Foster,	7	10.50	5.67½	.87½	17.05	
James Emerson,	2	3.00	44.80	25.00	72.80	
A. C. Hasseltine,	0	0.00	35.70	18.60	54.30	
John Smith,	6	9.00	18.50	11.50	39.00	

Having found the amount to be raised on the dollar, the operation of assessing taxes will be much facilitated by the use of the following

TABLE.

$		$	$		$	$		$
1	gives	0.005	40	gives	0.20	700	gives	3.50
2	"	0.010	50	"	0.25	800	"	4.00
3	"	0.015	60	"	0.30	900	"	4.50
4	"	0.020	70	"	0.35	1000	"	5.00
5	"	0.025	80	"	0.40	2000	"	10.00
6	"	0.030	90	"	0.45	3000	"	15.00
7	"	0.035	100	"	0.50	4000	"	20.00
8	"	0.040	200	"	1.00	5000	"	25.00
9	"	0.045	300	"	1.50	6000	"	30.00
10	"	0.050	400	"	2.00	7000	"	35.00
20	"	0.100	500	"	2.50	8000	"	40.00
30	"	0.150	600	"	3.00	9000	"	45.00

By the aid of the above table the amount of any person's tax may be found.

Required the amount of James Dow's tax, his real estate being $ 4780, his personal $ 1720, and he paying for 3 polls.

1. To find the amount on his real estate.

OPERATION.

Dow's	tax	on	$ 4000	is	$ 20.00	
"	"		700	"	3.50	
"	"		80	"	.40	
			$ 4780		$ 23.90	Amount of Dow's tax on real [estate.

2. To find the amount on his personal estate.

OPERATION.

Dow's	tax	on	$ 1000	is	$ 5.00	
"	"		700	"	3.50	
"	"		20	"	.10	
			$ 1720		$ 8.60	Amount of Dow's tax on per- [sonal estate.

Tax on real estate,	$ 23.90
Tax on personal estate,	8.60
Tax on 3 polls,	4.50
Dow's whole tax,	$ 37.00

NOTE. — It will be necessary to construct a different table, although on the same principle, when a different per cent. is paid on the dollar.

SECTION LXIX.

ALLIGATION.

ALLIGATION teaches how to compound or mix together several simples of different qualities, so that the composition may be of some intermediate quality or rate. It is of two kinds, Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL.

Alligation Medial teaches how to find the mean price of several articles mixed, the quantity and value of each being given.

RULE. — *As the sum of the quantities to be mixed is to their value, so is any part of the composition to its mean price.*

EXAMPLES.

1. A grocer mixed 2cwt. of sugar at $ 9.00 per cwt., and 1cwt. at $ 7.00 per cwt., and 2cwt. at $ 10.00 per cwt.; what is the value of 1cwt. of this mixture?

2cwt. at $ 9.00 = $ 18.00
1 " 7.00 = 7.00
2 " 10.00 = 20.00

5 " : $ 45.00 : : 1cwt. : $ 9.00 Answer.

2. If 19 bushels of wheat at $ 1.00 per bushel should be mixed with 40 bushels of rye at $ 0.66 per bushel, and 11 bushels of barley at $ 0.50 per bushel, what would a bushel of the mixture be worth? Ans. $ 0.72,7$\frac{1}{7}$.

3. If 3 pounds of gold of 22 carats fine be mixed with 3 pounds of 20 carats fine, what is the fineness of the mixture? Ans. 21 carats.

4. If I mix 20 pounds of tea at 70 cents per pound with 15 pounds at 60 cents per pound, and 80 pounds at 40 cents per pound, what is the value of 1 pound of this mixture? Ans. $ 0.47$\frac{19}{23}$.

NOTE. — If an ounce, or any other quantity, of pure gold be divided into 24 equal parts, these parts are called *carats*. But gold is often mixed with some baser metal, which is called the *alloy*; and the mixture is said to be so many carats fine, according to the proportion of pure gold contained in it; thus, if 22 carats of pure gold and 2 of alloy be mixed together, it is said to be 22 carats fine.

ALLIGATION ALTERNATE.

This rule teaches us how, from the prices of several articles given, to find how much of each must be mixed to bear a certain price.

CASE I.

RULE. — *Place the prices under each other, in the order of their value; connect the price of each ingredient, which is less in value than the intended compound, with one which is of greater value than the compound. Place the difference between the price and that of each simple, opposite to the price with which they are connected.*

EXAMPLES.

5. A merchant has spices, some at 18 cents a pound, some at 24 cents, some at 48 cents, and some at 60 cents. How much of each sort must he mix that he may sell the mixture at 40 cents a pound?

$$\text{Mean rate } 40 \left\{ \begin{array}{c} \text{cts.} \\ 18 \\ 24 \\ 48 \\ 60 \end{array} \right. \quad \begin{array}{ccc} \text{lbs.} & & \text{cts.} \\ 20 & \text{at} & 18 \\ 8 & \text{"} & 24 \\ 16 & \text{"} & 48 \\ 22 & \text{"} & 60 \end{array} \left. \vphantom{\begin{array}{c} a \\ a \\ a \\ a \\ a \end{array}} \right\} \text{Answers.}$$

	cts.	lbs.		cts.	
Mean rate 40	18	8	at	18	Answers.
	24	20	"	24	
	48	22	"	48	
	60	16	"	60	

	cts.	lbs.	lbs.	lbs.		cts.	
Mean rate 40	18	20 +	8	28	at	18	Answers.
	24	20 +	8	28	"	24	
	48	22 +	16	38	"	48	
	60	22 +	16	38	"	60	

Explanation. — By connecting the less rate with the greater, and placing the differences between them and the mean rate alternately, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss upon the whole must be equal, and the compound will have the value of the proposed rate; the same will be true of any other two simples managed according to the rule. In like manner, let the number of simples be what they may, and with how many soever every one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious, from the rule, that questions of this sort admit of a great variety of answers; for having found one answer, we may find as many others as we please by only multiplying or dividing each of the quantities found by 2, 3, or 4, &c., the reason of which is evident; for if two quantities of two simples make a balance of loss and gain with respect to the mean price, so must also the double or treble, the half or third part, or any other equimultiples or parts of these quantities.

6. How much barley at 50 cents a bushel, and rye at 75 cents, and wheat at $ 1.00, must be mixed, that the composition may be worth 80 cents a bushel?

Ans. 20 bushels of rye, 20 of barley, and 35 of wheat.

7. A goldsmith would mix gold of 19 carats fine with some of 15, 23, and 24 carats fine, that the compound may be 20 carats fine. What quantity of each must he take?

Ans. 4oz. of 15 carats, 3oz. of 19, 1oz. of 23, and 5oz. of 24.

8. It is required to mix several sorts of wine at 60 cents, 80 cents, and $ 1.20, with water, that the mixture may be worth 75 cents per gallon; how much of each sort must be taken?

Ans. 45gals. of water, 5gals. of 60 cents, 15gals. of 80 cents, and 75gals. of $1.20.

CASE II.

When one of the ingredients is limited to a certain quantity.

RULE. — *Take the difference between each price and the mean rate, as before; then say, as the difference of that simple whose quantity is given is to the rest of the differences severally, so is the quantity given to the several quantities required.*

EXAMPLES.

9. How much wine at 5s., at 5s. 6d., and at 6s. a gallon, must be mixed with 3 gallons at 4s. per gallon, so that the mixture may be worth 5s. 4d. per gallon?

64	48	8 + 2 = 10	Then 10 : 10 :: 3 : 3
	60	8 + 2 = 10	10 : 20 :: 3 : 6
	66	16 + 4 = 20	10 : 20 :: 3 : 6
	72	16 + 4 = 20	

Ans. 3gals. at 5s., 6 at 5s. 6d., and 6 at 6s.

10. A grocer would mix teas at 12s., 10s., and 6s. per pound, with 20 pounds at 4s. per pound; how much of each sort must he take to make the composition worth 8s. per pound?

Ans. 20lbs. at 4s., 10lbs. at 6s., 10lbs. at 10s., and 20lbs. at 12s.

11. How much port wine at $ 1.75 per gallon, and temperance wine at $ 1.25 per gallon, must be mixed with 20 gallons of water, that the whole may be sold at $ 1.00 per gallon?

Ans. 20gals. port wine, and 20gals. temperance wine.

12. How much gold of 15, 17, and 22 carats fine must be mixed with 5 ounces of 18 carats fine, so that the composition may be 20 carats fine?

Ans. 5oz. of 15 carats, 5oz. of 17, and 25oz. of 22.

CASE III.*

When the sum and quality of the ingredients are given.

RULE. — *Find an answer as before, by linking; then say, as the sum of the quantities or differences, thus determined, is to the given quantity, so is each ingredient found by linking to the required quantity of each.*

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* To this case belongs the curious fact of King Hiero's crown.

Hiero, King of Syracuse, gave orders for a crown to be made of pure gold; but suspecting that the workmen had debased it, by mixing it with silver or copper, he recommended the discovery of the fraud to the famous Archimedes, and desired to know the exact quantity of alloy in the crown.

Archimedes, in order to detect the imposition, procured two other masses, the one of pure gold, the other of copper, and each of the same weight of the former; and by putting each separately into a vessel full of water, the quantity of water expelled by them determined their specific gravi-

EXAMPLES.

13. How many gallons of water must be mixed with wine at \$ 1.50 per gallon, so as to fill a vessel containing 100 gallons, that it may be sold at \$ 1.20 per gallon?

$$120 \left\{ \begin{array}{l} 0 \\ 150 \end{array} \right. \quad \begin{array}{r} 30 \\ \underline{120} \\ 150 \end{array} \qquad \begin{array}{l} \overset{\text{gals.}}{150} : \overset{\text{gals.}}{100} :: \overset{\text{gals.}}{30} : \overset{\text{gals.}}{20} \text{ water} \\ 150 : 100 :: 120 : 80 \text{ wine} \end{array} \left. \vphantom{\begin{array}{l} 0 \\ 0 \end{array}} \right\} \text{Answer.}$$

14. A merchant has sugar at 8 cents, 10 cents, 12 cents, and 20 cents per pound; with these he would fill a hogshead that would contain 200 pounds. How much of each kind must he take, that he may sell the mixture at 15 cents per pound?

Ans. $33\frac{1}{3}$lbs. of 8, 10, and 12 cts., and 100lbs. of 20 cts.

SECTION LXX.

PERMUTATIONS AND COMBINATIONS.

THE permutation of quantities is the showing how many different ways the order or position of any given number of things may be changed.

The combination of quantities is the showing how often a less number of things can be taken out of a greater, and combined together, without considering their places or the *order* they stand in.

CASE I.

To find the number of permutations or changes that can be made of any given number of things, all different from each other.

RULE. — *Multiply all the terms of the natural series of numbers, from*

---

ties; from which, and their given weights, the exact quantities of gold and alloy in the crown may be determined.

Suppose the weight of each crown to be 10 pounds, and that the water expelled by the copper was 92 pounds, by the gold 52 pounds, and by the compound crown 64 pounds: what will be the quantities of gold and alloy in the crown?

The rates of the simples are 92 and 52, and of the compound 64; therefore,

$$64 \left\{ \begin{array}{l} 92 \\ 52 \end{array} \right. \quad \begin{array}{l} 12 \text{ of copper,} \\ 28 \text{ of gold,} \end{array} \quad \begin{array}{l} 40 : 10 :: 12 : 3\text{lbs. of copper} \\ 40 : 10 :: 28 : 7\text{lbs. of gold} \end{array} \left. \vphantom{\begin{array}{l} 0 \\ 0 \end{array}} \right\} \text{Answer.}$$

1 up to the given number, continually together, and the last product will be the answer required.

This rule may be illustrated by inquiring how many different numbers may be formed from the figures of the following number, 789, making use of the three figures in each number.

OPERATION.

789, 798, 879, 897, 978, 987.

It will be perceived, that six are all the permutations the above number will admit of. By adopting the rule, we find the same answer.

$1 \times 2 \times 3 = 6$ Ans.

1. How many changes may be rung on 6 bells?

OPERATION.

$1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ changes. Ans.

2. For how many days can 10 persons be placed in a different position at dinner? Ans. 3628800.

3. How many changes may be rung on 12 bells, and how long would they be in ringing, supposing 10 changes to be rung in one minute, and the year to consist of 365 days, 5 hours, and 49 minutes? Ans. 479001600, and 91y. 26d. 22h. 41m.

4. How many changes or variations will the letters of the alphabet admit of? Ans. 403291461126605635584000000.

CASE II.

Any number of different things being given, to find how many changes can be made out of them, by taking any given number of quantities at a time.

RULE. — *Take a series of numbers, beginning at the number of things given, and decreasing by 1, to the number of quantities to be taken at a time; the product of all the terms will be the answer required.*

Illustration. — The above rule may be illustrated by inquiring how many different numbers can be made by a selection of any three figures from the following number, 1234.

OPERATION.

123, 124, 132, 134, 142, 143, 213, 214, 231, 234, 241, 243, 312, 314, 321, 324, 341, 342, 412, 413, 421, 423, 431, 432.

By examining the above, it will be perceived that there are 24 different numbers or permutations; and this number may be obtained by multiplying the number of things given, 4, by the next lower number, and that product by the next lower, and

so continuing the multiplications as many times as there are things to be taken at a time; and the things to be taken at a time are 3; we therefore find the continued product of the first three numbers, beginning at 4; thus,

$$4 \times 3 \times 2 = 24 \text{ Answer.}$$

NOTE. — The questions under this rule refer to *permutations* and not combinations.

5. How many changes can be rung with 4 bells out of 8?

OPERATION.

$$8 \times 7 \times 6 \times 5 = 1680 \text{ changes, Answer.}$$

6. How many words can be made with 6 letters out of 26 of the alphabet, admitting a word might be made from consonants?

Ans. 165765600.

CASE III.

To find the number of combinations of any given number of things, all different one from another, taking any given number at a time, without reference to their *arrangement*.

RULE. —*Take the series* 1, 2, 3, 4, *&c., up to the number to be taken at a time, and find the product of all the terms.*

Take a series of as many terms, decreasing by 1 *from the given number out of which the election is to be made, and find the product of all the terms.*

Divide the last product by the former, and the quotient will be the number required.

7. How many combinations can there be of any three letters of the alphabet out of any four letters, without reference to their *arrangement* or *permutations*?

Illustration. — By examining four letters, a, b, c, d, we find they will admit of only four combinations. Thus, abc, abd, acd, bcd. And this number can be ascertained in the following manner: — $1 \times 2 \times 3 = 6$; $4 \times 3 \times 2 = 24$; $24 \div 6 = 4$, the number of combinations.

8. How many combinations can be made of 7 letters out of 10, the letters all being different? Ans. 120.

OPERATION.

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 = 5040$; 7 being the number taken at a time.

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 604800 =$ same number from 10.

$604800 \div 5040 = 120$ Ans. Number of combinations.

CASE IV.

To find the compositions of any number of things, in an equal number of sets, the things being all different.

RULE.—*Multiply the number of things in every set continually together, and the product will be the answer required.*

9. Suppose there are 4 companies, in each of which there are 9 men; it is required to find how many ways 4 men may be chosen, one out of each company. $9 \times 9 \times 9 \times 9 = 6561$ Ans.

10. There are 4 companies, in one of which there are 6 men, in another 8, and in each of the other two 9 men. What are the choices, by a composition of 4 men, one out of each company? Ans. 3888.

11. How many changes are there in throwing 5 dice? Ans. 7776.

SECTION LXXI.

LIFE INSURANCE.

INSURANCE on life is a contract, which stipulates for the payment of a certain sum of money on the death of an individual, in consideration of the immediate payment of a specified sum, or more frequently of an annuity, or annual premium, to be continued during the existence of the life insured. Contracts of this kind are of immense importance to society. Every man whose income depends on his own life or exertions, and on whom others are dependent for support, must be sensible of the advantages of arrangements by means of which, at a small sacrifice of immediate comfort, he is enabled effectually to provide against the casualties of life. Though nothing can be more uncertain than the continuance of an individual life, yet nothing is more invariable than the duration of life in the mass; consequently, the exact value of life insurances can be calculated without any uncertainty whatever, and a man by effecting an insurance secures to his family, against risk of accident, the advantages they would have from enjoying his exact proportion of the average duration of life. Such transactions provide against destitution, and tend directly to the accumulation of capital. No wise man will therefore neglect to provide against contingencies.

The following table from Milne's Treatise on the Valuation of Annuities and Assurances (Vol. II. p. 565) shows the "expectation of life" at every age, according to the law of mortality at Carlisle.

Expectation of Life.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
0	38.72	26	37.14	52	19.68	78	6.12
1	44.68	27	36.41	53	18.97	79	5.80
2	47.55	28	35.69	54	18.28	80	5.51
3	49.82	29	35.00	55	17.58	81	5.21
4	50.76	30	34.34	56	16.89	82	4.93
5	51.25	31	33.68	57	16.21	83	4.65
6	51.17	32	33.03	58	15.55	84	4.39
7	50.80	33	32.36	59	14.92	85	4.12
8	50.24	34	31.68	60	14.34	86	3.90
9	45.57	35	31.00	61	13.82	87	3.71
10	48.82	36	30.32	62	13.31	88	3.59
11	48.04	37	29.64	63	12.81	89	3.47
12	47.27	38	28.96	64	12.30	90	3.28
13	46.51	39	28.28	65	11.79	91	3.26
14	45.75	40	27.61	66	11.27	92	3.37
15	45.00	41	26.97	67	10.75	93	3.48
16	44.27	42	26.34	68	10.23	94	3.53
17	43.57	43	25.71	69	9.70	95	3.52
18	42.87	44	25.09	70	9.19	96	3.46
19	42.17	45	24.46	71	8.65	97	3.28
20	41.46	46	23.82	72	8.16	98	3.07
21	40.75	47	23.17	73	7.72	99	2.77
22	40.04	48	22.50	74	7.33	100	2.28
23	39.31	49	21.81	75	7.01	101	1.79
24	38.59	50	21.11	76	6.69	102	1.30
25	37.86	51	20.39	77	6.40	103	0.83

By the above table it will be perceived, that the average age to which 100 persons will live from birth will be $38\frac{72}{100}$ years, and that 100 persons having attained the age of 35 years will live on an average 31 years longer; and the average time which 100 persons will continue to live, after the age of 100 years, is $2\frac{28}{100}$ years. Taking the above table as the basis of their calculations, various life-insurance companies have been formed, whose tables or rates of insurance vary according as they charge a greater or less per cent. on their capital. We have selected two tables from those used by the numerous corporations which have been formed in the United States. The first is the one adopted by the Massachusetts Hospital Life Insurance Company, and the other by the New York Life Insurance Company.

Age.	Massachusetts.			New York.		
	1 Year	7 Years.	10 Years.	1 Year.	7 Years.	For Life.
14	.82	.84	.85	.72	.86	1.53
15	.83	.85	.86	.77	.88	1.56
16	.84	.86	.87	.84	.90	1.62
17	.85	.87	.88	.86	.91	1.65
18	.86	.88	.89	.89	.92	1.69
19	.87	.89	.90	.90	.94	1.73
20	.88	.90	.92	.91	.95	1.77
21	.89	.91	.93	.92	.97	1.82
22	.90	.92	.94	.94	.99	1.88
23	.91	.94	.96	.97	1.03	1.93
24	.92	.95	.98	.99	1.07	1.98
25	.93	.97	.99	1.00	1.12	2.04
26	.95	.98	1.01	1.07	1.17	2.11
27	.96	1.00	1.03	1.12	1.23	2.17
28	.98	1.02	1.05	1.20	1.28	2.24
29	1.00	1.04	1.07	1.28	1.35	2.31
30	1.01	1.06	1.09	1.31	1.36	2.36
31	1.03	1.08	1.11	1.32	1.42	2.43
32	1.05	1.10	1.14	1.33	1.46	2.50
33	1.07	1.13	1.16	1.34	1.48	2.57
34	1.09	1.15	1.19	1.35	1.50	2.64
35	1.12	1.18	1.22	1.36	1.53	2.75
36	1.14	1.20	1.25	1.39	1.57	2.81
37	1.16	1.23	1.29	1.43	1.63	2.90
38	1.19	1.27	1.34	1.48	1.70	3.05
39	1.22	1.31	1.39	1.57	1.76	3.11
40	1.25	1.35	1.44	1.69	1.83	3.20
41	1.28	1.40	1.50	1.78	1.88	3.31
42	1.31	1.46	1.58	1.85	1.89	3.40
43	1.35	1.53	1.66	1.89	1.92	3.51
44	1.40	1.61	1.75	1.90	1.94	3.63
45	1.47	1.70	1.85	1.91	1.96	3.73
46	1.54	1.80	1.95	1.92	1.98	3.87
47	1.62	1.90	2.07	1.93	1.99	4.01
48	1.71	2.01	2.20	1.94	2.02	4.17
49	1.81	2.14	2.34	1.95	2.04	4.49
50	1.91	2.27	2.49	1.96	2.09	4.60
51	2.03	2.42	2.65	1.97	2.20	4.75
52	2.16	2.58	2.83	2.02	2.37	4.90
53	2.29	2.75	3.03	2.10	2.59	5.24
54	2.44	2.94	3.24	2.18	2.89	5.49
55	2.60	3.14	3.47	2.32	3.21	5.78
56	2.78	3.36	3.72	2.47	3.56	6.05
57	2.96	3.61	4 00	2.70	4.20	6.27
58	3.17	3.87	4.29	3.14	4.31	6.50
59	3.39	4.17	4.62	3.67	4.63	6.75
60	3.64	4.48	4.97	4.35	4.91	7.00

The preceding table shows at what rate a hundred dollars may be insured on a person's life for one year, for seven years, ten years, and for life, at any age from 14 to 60 years. Thus, a man who is 42 years old, and who wishes to obtain a life insurance in Massachusetts for one year, for $ 100, must pay $1.31, and in the same proportion for a larger sum. And, if he would obtain a life insurance in New York, he must pay annually $ 3.40 on every hundred dollars for which he obtains insurance.

EXAMPLES.

1. What premium will the Massachusetts Hospital Life Insurance Company require for the insurance of a life for 1 year for $1728, the person being 30 years of age? Ans. $17.45.

OPERATION.

$ 1728 × .0101 = $ 17.45 Ans.

NOTE.—As the premium is $ 1.01 on $ 100, the sum insured must be multiplied by $\frac{1.01}{100}$ = .0101.

2. What amount of premium must James Kimball pay at the above office to effect an insurance on his life for 7 years for $ 8000, his age being 53 years? Ans. $ 220.

OPERATION.

$ 8000 × .0275 = $ 220 annually, Ans.

3. John Smith, 60 years of age, wishes to engage in a very profitable speculation, and, being destitute of the necessary funds, he effects an insurance on his life for 10 years, for $ 78,000, at the office of the above Company. Required the amount of the annual premium. Ans. $ 3876.60.

4. What will be the premium per annum for insuring a person's life, who is 15 years old, for $ 2000 for 7 years, at the New York Life Insurance Company? Ans. $ 17.60.

5. A gentleman 45 years of age, being bound on a long and dangerous voyage, and wishing to secure a competence for his family, obtains an insurance for life at the above office in New York, for $ 12,000. By an act of Providence, he dies before the end of the third year. What is the net gain to his family? Ans. $ 10,657.20.

6. John Swan, 20 years old, effects an insurance for life at New York, for $10,000, for which he pays an annual premium of $\frac{1.77}{100}$ per cent. If the Insurance Company loan the premium at 6 per cent. compound interest, and Swan should die at the age of 60 years, who will gain by the insurance?

Ans. Company will gain $ 17,392.86+.

NOTE.—All premiums are paid annually and in advance.

SECTION LXXII.

POSITION.

POSITION is a method of performing such questions as cannot be resolved by the common direct rules, and is of two kinds, called *single* and *double*.

SINGLE POSITION.

SINGLE POSITION teaches us to resolve those questions whose results are proportional to their suppositions.

RULE. — *Take any number and perform the same operations with it as are desirable to be performed in the question. Then say, as the result of the operation is to the position, so is the result in the question to the number required.*

EXAMPLES.

1. A schoolmaster being asked how many scholars he had, replied, that if he had as many more as he now had, and half as many more, he should have 200; of how many did his school consist?

Suppose he had	60	As 150 : 200 : : 60
Then, as many more	60	60
Half as many more	30	150) 12000 (80 scholars, Ans.
	150	12000

By analysis. — By having as many more, and half as many more, he must have $2\frac{1}{2}$ times the original number; therefore, by dividing 200 by $2\frac{1}{2}$, we obtain the answer, 80, as before.

NOTE. — Having performed all the following questions by *position*, the student should then perform them by *analysis*.

2. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money had \$60 left; what had he at first? Ans. \$144.

3. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the sum shall be 125? Ans. 60.

4. A's age is double that of B, and B's is triple that of C, and the sum of all their ages is 140. What is each person's age? Ans. A's 84, B's 42, C's 14 years.

5. A person lent a sum of money at 6 per cent., and at the end of 10 years received the amount \$560. What was the sum lent? Ans. \$350.

6. Seven eighths of a certain number exceed $\frac{1}{5}$ by 81; what is the number? Ans. 120.

7. What number is that whose $\frac{3}{5}$ exceed $\frac{4}{7}$ by $2\frac{17}{35}$? Ans. 87.

DOUBLE POSITION.*

DOUBLE POSITION teaches to resolve questions, by making two suppositions of false numbers.

Those questions in which the results are not proportional to their positions belong to this rule.

RULE. — *Take any two convenient numbers, and proceed with each according to the conditions of the question. Find how much the results are different from the result in the question. Multiply each of the errors by the contra supposition, and find the sum and difference of the products. If the errors are alike, divide the difference of the products by the difference of the errors, and the quotient will be the answer. If the errors are unlike, divide the sum of the products by the sum of the errors, and the quotient will be the answer.*

NOTE. — The errors are said to be alike when they are both too great, or both too small; and unlike when one is too great and the other too little.

EXAMPLES.

1. A lady purchased a piece of silk for a gown at 80 cents per yard, and lining for it at 30 cents per yard; the gown and lining contained 15 yards, and the price of the whole was \$7.00. How many yards were there of each?

Suppose 6 yards of silk, value	\$4.80
She must then have 9 yards of lining, value	2.70
Sum of their values,	\$7.50
Which should have been	7.00
So the first error is 50 too much,	+.50
Again; suppose she had 4 yards of silk, value	\$3.20
Then she must have 11 yards of lining, value	3.30
Sum of their values,	\$6.50
Which should have been	7.00
So that the second error is 50 too little,	—.50

* This rule is founded on the supposition, that the first error is to the second as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this is not the case, the exact answer to the questions cannot be found by this rule.

First supposition multiplied by last error,	$6 \times 50 = 3.00$
Last supposition multiplied by first error,	$4 \times 50 = 2.00$
Add the products, because *unlike*,	\$5.00

$500 \div \overline{50 + 50} = 5$ yards of silk, } Ans. $5 \times 80 = \$4.00$
$15 - 5 = 10$ yards of lining, } $10 \times 30 = 3.00$
Proof \$7.00

By Analysis. — As the silk and lining contain 15 yards, and cost \$7.00, the average price per yard is $46\frac{2}{3}$; and this taken from 80 leaves $33\frac{1}{3}$; and 30 taken from $46\frac{2}{3}$ leaves $16\frac{2}{3}$; and as the quantity of lining will be to that of the silk as $33\frac{1}{3}$ to $16\frac{2}{3}$, it is therefore evident that the quantity of lining is twice the quantity of silk. Wherefore, if 15, the number of yards, be divided into three parts, two of those parts (10) will be the number of yards for the lining, and the other part (5) will be the yards for the silk, as before.

NOTE. — The student should perform each question by analysis.

2. A and B invested equal sums in trade; A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost \$225; then A's money was double that of B's. What did each invest? Ans. \$600.

3. A person being asked the age of each of his sons, replied, that his eldest son was 4 years older than the second, his second 4 years older than the third, his third 4 years older than the fourth, or youngest, and his youngest half the age of the oldest. What was the age of each of his sons?
Ans. 12, 16, 20, and 24 years.

4. A gentleman has two horses and a saddle worth \$50. Now, if the saddle be put on the first horse, it will make his value double that of the second horse; but if it be put on the second, it will make his value triple that of the first. What was the value of each horse? Ans. The first \$30, second \$40.

5. A gentleman was asked the time of day, and replied, that $\frac{2}{3}$ of the time past from noon was equal to $\frac{8}{33}$ of the time to midnight. What was the time? Ans. 12 minutes past 3.

6. A and B have the same income. A saves $\frac{1}{12}$ of his, but B, by spending \$100 per annum more than A, at the end of 10 years finds himself \$600 in debt. What was their income?
Ans. \$480.

7. A gentleman hired a laborer for 90 days on these conditions: that for every day he wrought he should receive 60 cents, and for every day he was absent he should forfeit 80

cents. At the expiration of the term he received $.33. How many days did he work, and how many days was he idle?

Ans. He labored 75 days, and was idle 15 days.

The following question, with some variation in the language, is taken from Fenn's Algebra, page 62. It is believed, however, that Sir Isaac Newton was the author of it.

8. If 12 oxen eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen would it require to eat 24 acres in 18 weeks, the grass to be growing uniformly?

Ans. 36 oxen.

OPERATION BY ANALYSIS.

Each ox eats a certain quantity in each week, which we may suppose to be 100 pounds; and of the whole quantity eaten in each case, a part must have already grown during the time of eating.

Then, by the first conditions of the question,

$12 \times 4 \times 100 = 4800$lbs. $=$ whole quantity on $3\frac{1}{3}$ acres for 4 weeks.

$4800 \div 3\frac{1}{3} = 1440$lbs. $=$ whole quantity on 1 acre for 4 weeks.

By the second conditions of the question,

$21 \times 9 \times 100 = 18900$lbs. $=$ whole quantity on 10 acres for 9 weeks.

$18900 \div 10 = 1890$lbs. $=$ whole quantity on 1 acre for 9 weeks.

$1890 - 1440 = 450$lbs. $=$ the quantity grown on an acre for $9 - 4 = 5$ weeks.

$450 \div \overline{9-4} = 90$lbs. $=$ the quantity which grows on each acre for 1 week.

$90 \times 3\frac{1}{3} \times 4 = 1200$lbs. $=$ quantity grown on $3\frac{1}{3}$ acres for 4 weeks.

$4800 - 1200 = 3600$lbs. $=$ original quantity of grass on $3\frac{1}{3}$ acres.

$3600 \div 3\frac{1}{3} = 1080$lbs. $=$ original quantity on 1 acre.

Then, by the last condition of the question,

$24 \times 1080 = 25920$lbs. $=$ original quantity on 24 acres.

$24 \times 90 \times 18 = 38880$lbs. $=$ quantity which grows on 24 acres in 18 weeks.

$25920 + 38880 = 64800$lbs. $=$ whole quantity on 24 acres for 18 weeks.

$64800 \div 18 = 3600$lbs. $=$ quantity to be eat from 24 acres each week.

$3600 \div 100 = 36 =$ number of oxen required to eat the whole, and the answer to the question.

9. There is a fish whose head weighs 15 pounds, his tail weighs as much as his head and $\frac{1}{3}$ as much as his body, and his body weighs as much as his head and tail. What was the weight of the fish? Ans. 72lbs.

10. Suppose a clock to have an hour-hand, a minute-hand, and a second-hand, all turning on the same centre. At 12 o'clock all the hands are together and point at 12.

(1.) How long will it be before the second-hand will be between the other two hands, and at equal distances from each?

Ans. $60\frac{780}{1427}$ seconds.

(2.) Also before the minute-hand will be equally distant between the other two hands? Ans. $61\frac{683}{697}$ seconds.

(3.) Also before the hour-hand will be equally distant between the other two hands? Ans. $59\frac{13}{73}$ seconds.

Section LXXIII.

EXCHANGE.

Exchange is the act of paying or receiving the money of one country for its equivalent in the money of another country, by means of Bills of Exchange. This operation, therefore, comprehends both the reduction of moneys and the negotiation of bills. It determines the comparative value of the currencies of all nations, and shows how foreign debts are discharged, loans and subsidies paid, and other remittances made from one country to another, without the risk, trouble, or expense of transporting specie or bullion.

BILLS OF EXCHANGE.

A Bill of Exchange is a written order for the payment of a certain sum of money, at an appointed time. It is a mercantile contract, in which four persons are mostly concerned; viz.

1. The *drawer*, who receives the value, and is also called the *maker* and *seller* of the bill.

2. The person upon whom the bill is drawn is called the *drawee*. He is also called the *acceptor*, when he accepts the bill, which is an engagement to pay it when due.

3. The person who gives value for the bill, who is called the *buyer*, *taker*, and *remitter*.

4. The person to whom it is ordered to be paid, who is called

the *payee*, and who may, by indorsement, pass it to any other person.

Most mercantile payments are made in Bills of Exchange, which generally pass from hand to hand, until due, like any other circulating medium; and the person who at any time has a bill in his possession is called the *holder*.

When the holder of a bill disposes of it, he writes his name on the back, which is called *indorsing;* and the payee should be the first indorser. If the bill be indorsed in favor of any particular person, it is called a *special indorsement;* and the person to whom it is thus made payable is called the *indorsee*, who must also indorse the bill if he negotiates it. Any person may indorse a bill, and every indorser (as well as the acceptor, or payee) is a security for the bill, and may therefore be sued for payment.

The *term* of a bill varies according to the agreement between the parties, or the custom of countries. Some bills are drawn at sight; others, at a certain number of days, or months, after sight or after date; and some, at *usance*, which is the customary or usual term between different places.

Days of grace are a certain number of days granted to the acceptor, after the term of a bill is expired. Three days are usually allowed.

In reckoning when a bill, payable after date, becomes due, the day on which it is dated is not included; and if it be a bill payable after *sight*, the day of presentment is not included. When the term is expressed in months, calendar months are understood; and when a month is longer than the preceding, it is a rule not to go in the computation into a third month.

Thus, if a bill be dated the 28th, 29th, 30th, or 31st of January, and payable one month after date, the term equally expires on the last day of February, to which the days of grace must, of course, be added; and therefore the bill becomes due on the 3d of March.

Form of a Bill of Exchange.

Boston, September 25, 1835.

Exchange for £5,000 sterling.

At ninety days' sight of this, my first Bill of Exchange (second and third of the same tenor and date unpaid), pay to James Ayer, or order, five thousand pounds sterling, with or without further advice.

John L. French.

Messrs. Dana & Hyde,
Merchants, Liverpool.

ACCEPTING BILLS.

When a bill is presented for acceptance, it is generally left till the next day; and the common way of accepting is for the drawee to write his name at the bottom, or across the body of the bill, with the word *accepted*.

When two or more persons are in partnership, the acceptance of one binds all the others, if the bill concerns their joint trade; but if it should be made known to the person who receives the bill that it concerns the acceptor only, in a distinct interest, he alone, as acceptor, can be sued.

A clerk or servant may accept a bill for his master, when he has authority for that purpose; or if he usually transacts business of this nature for him; and his acceptance binds the master. But if the bill be drawn nominally on the servant, directing him to place it to the account of his master, and if the servant should accept it generally, without specifying that he does it for his master's account, the acceptance binds the servant only, and not his employer.

When a bill is drawn for the account of a third person, and is accepted as such, and he fails without making provision for its payment, the acceptor must discharge the bill, and can have no recourse against the drawer.

A bill may be accepted to be paid at a longer period than is mentioned in the bill, or to pay a part of the sum only; such an acceptance is binding on him who made it; but the holder is at liberty to take it as it is offered, or to act as if acceptance had been entirely refused.

INDORSING BILLS.

Bills payable to bearer are transferred by simple delivery, and without any indorsement; but in order to transfer a bill payable to order, the holder must express his order of paying to another person, which is always done by an indorsement.

An indorsement may be blank or special. A *blank indorsement* consists only of the indorser's name, and the bill becomes then transferable by simple delivery; a *special indorsement* orders the money to be paid to some particular person, or to his order; a blank indorsement may always be filled up with any person's name, so as to make it special.

An indorsement may take place at any time after the bill is issued, even after the day of payment is elapsed.

A person who receives a bill with a blank indorsement may take it as indorsee, negotiate it again, or demand payment on his own account, or he may receive the money as agent, or for the account of the indorser; and the latter, notwithstanding his indorsement, may still appear as holder in an action against the drawer or acceptor.

A special indorsement need not contain the words *to order*, and the bill is negotiable; it may also be restrictive, giving authority to the indorsee to receive the money for the indorser, but not to transfer the bill again to another.

An indorsement for part of the money only is not valid, except with regard to him who makes it. The drawer and acceptor are not bound by it.

When the holder of a bill dies, his executors may indorse it; but by so doing they become answerable to their indorsee personally, and not as executors.

PROTESTING BILLS.

When acceptance or payment has been refused, the holder of the bill should give regular and immediate notice to all the parties to whom he intends to resort for payment; and if, on account of unnecessary delay, a loss should be incurred by the failure of any of the parties, the holder must bear the loss.

With respect to the manner in which notices of non-acceptance or non-payment are to be given, a difference exists between inland and foreign bills.

For foreign bills a protest is indispensably necessary; thus, a public notary is to appear with the bill, and to demand either acceptance or payment; and, on being refused, he is to draw up an instrument, called a *protest*, expressing that acceptance or payment has been demanded and refused, and that the holder of the bill intends to recover any damages which he may sustain in consequence. This instrument is admitted in foreign countries as legal proof of the fact.

It is customary, as a precaution against accidents or miscarriage, to draw three copies of a foreign bill, and to send them by different posts. They are denominated the *first*, *second*, and *third of exchange*; and when any one of them is paid, the rest become void and of no value. When the acceptor of a bill becomes insolvent, or absconds before the term of payment is expired, the holder may cause a notary to demand better security, and, on that being refused, to protest the bill for

want of it. In such cases, however, the most general practice is to wait the regular time, till the bill becomes due.

The damages incurred by non-acceptance and non-payment, besides interest, consist usually of the exchange, or reëxchange, commission, and postage together, with the expenses of protest, and interest. The exchange is reckoned according to the course at sight from the place where the protest is made to the place where the bill is to be paid by the drawer; and, if it be not paid there, the exchange is then reckoned from the same place to that where the bill is paid, and also double commission. The interest commences from the day when the demand was made.

RECOVERING BILLS.

The drawer, acceptor, and even indorser of a bill are equally liable to the payment of it; and though the holder can have but one satisfaction, yet, until such satisfaction is actually had, he may sue any of them, or all of them, either at the same time or in succession, and obtain judgment against them all, till satisfaction be made. Proceedings cannot be staid in any action except on payment of the debt and costs, not only in that action, but in all the others in which judgment has not been obtained; and though the principal sum should be paid by one of the parties, still costs may be recovered in the several actions against the others.

When acceptance is refused, and the bill is returned by protest, an action may be commenced immediately against the drawer, though the regular time of payment be not arrived. His debt, in such a case, is considered as contracted the moment the bill is drawn. Thus, if before the bill is returned the drawer should become a bankrupt, the debt was contracted before the commission of bankruptcy took place.

Nothing will discharge an indorser from his engagement but the absolute payment of the money; not even a judgment recovered against the drawer, or any previous indorser, or any execution against any of them, unless the money be paid in consequence.

INLAND EXCHANGE, OR DRAFTS.

By Inland Exchange is understood the act of remitting bills to places in the same country; by which means debts are dis charged more conveniently than by cash remittances.

Suppose, for example, A, of Boston, is creditor to B, of Baltimore, $100, and C, of Boston, debtor to D, of Baltimore, $ 100, both these debts may be discharged by means of one bill. Thus, A draws for this sum on B, and sells his bill to C, who remits it to D, and the latter receives the amount, when due, from B. Here, by a transfer of claims, the Boston debtor pays the Boston creditor; and the Baltimore debtor the Baltimore creditor; and no money is sent from one place to the other. The same would take place if D, of Baltimore, drew on C, of Boston, and sold his bill to B, of Baltimore, who should send it to A, of Boston; the effect, in either case, being merely a transfer of debtors and creditors.

NOTE. — In this operation, A is the *drawer* and *seller*, B the *drawee* and *acceptor*, C the *buyer* and *remitter*, and D the *payee*, if his name be mentioned in the bill; and he is the *holder* when he receives the bill from A. When D, or any other holder, presents the bill for acceptance or payment, he is called the *presenter*.

By the foregoing example, it appears that reciprocal and equal debts due between two places may be discharged without remitting specie; and it may be supposed that such an operation is of equal convenience to all parties concerned; but when the debts are unequal, the advantage must be different, as the obligation of remittance is no longer mutual, because the debtor place must pay its balance, either by sending cash or bills; and as the latter mode is generally preferred, an increased demand for bills must be the consequence, which enhances their price, as it would that of any other article of sale or purchase.

PAR OF EXCHANGE.

The Par of Exchange may be considered under two general heads; viz. the *intrinsic par* and the *commercial par*, each of which admits of subordinate divisions and distinctions.

The *intrinsic par* is the value of the money of one country, compared with that of another, with respect both to weight and fineness.

The *commercial par* is the comparative value of the moneys of different countries, according to the weight, fineness, and market prices of the metals.

Thus, two sums of different countries are *intrinsically* at par, when they *contain* an equal quantity of the same kind of pure metal; and two sums of different countries are commercially at par, when they can purchase an equal quantity of the same kind of pure metal.

COURSE OF EXCHANGE.

The Course of Exchange is the variable price of the money of one country, which is given for a fixed sum of money of another country; the latter is called the *certain*, and the former the *uncertain* price, as before stated.

When the market price of foreign bills is above par, the exchange is said to be favorable to the place that gives the certain for the uncertain.

It should, however, be recollected, that when the exchange is favorable to a place, it is only so to the buyers and remitters of bills, but it is unfavorable to the drawers and sellers.

Thus, the interest of the remitter is identified with that of the place where he purchases the bill, and the interest of the drawer with that of the place where his funds are established, and on which he draws.

It is natural to inquire why such prices are considered favorable or unfavorable, if the drawers and remitters, whose interests are opposite, are natives of the same country. The usual answer is, that when the exchange is against a place, it becomes the interest of remitters to pay their foreign debts in specie instead of bills, and the exportation of the precious metals is often considered a national disadvantage.

The fluctuations of exchange are occasioned by various circumstances, both political and commercial. The principal cause is generally stated to be the balance of trade; that is, the difference between the commercial exports and imports of any one country with respect to another. Experience, however, shows, that the exchange may be unfavorable to a country, when the balance of trade is greatly in its favor; for the demand for bills must chiefly depend on the balance of such debts as come into immediate liquidation; that is to say, on the *balance of payments*.

Besides, it does not follow that large exports are always successful, or quick in their returns; and even should it be the case, the balance of payments may still be unfavorable from political causes.

When any alteration takes place in the coin or currency of a country, the exchange will, of course, vary so as to keep pace or correspond to such alteration. This, however, cannot be considered a change in the price of bills, but in the money in which they are bought or sold.

In times of peace the course of exchange seldom remains

long unfavorable to any country, at least, beyond the expense that might be incurred by the transportation of the precious metals; for bullion is considered the universal currency of merchants, and exchange gives it circulation, and thus tends to maintain the level of money throughout the commercial world.

An unfavorable course of exchange may, therefore, be corrected, either by the exportation of bullion, or the shipment of goods, — and another method sometimes offers, by negotiating bills through several places; but the latter remedy must fail if the exchange be universally unfavorable.

From what has been said of the causes, both commercial and political, which produce the fluctuations of exchange, and which sometimes counteract or balance each other, the following simple conclusion may be drawn; — that bills rise or fall in their prices, like any other salable articles, according to the proportion that exists between the demand and supply.

GREAT BRITAIN

Accounts are kept in Great Britain in pounds, shillings, pence, and farthings, sterling. The principal coins are guineas, sovereigns, crowns, and their fractional parts.

The Guinea	= 21	Shillings,	sterling,	=	$ 5.07,5
" Sovereign	= 20	"	"	=	4.84,6
" Crown	= 5	"	"	=	1.08

The pound sterling, or sovereign, has different values according to circumstances.

The exchange value is $ 4.44$\frac{4}{9}$.

Its legal value at the mint is $ 4.86,6.

It is received at the custom-house in payment of duties at $ 4.84.

Its commercial value is from $ 4.82 to $ 4.86; and its value is often greater in some of the States than in others. The price of foreign coin in our market determines its value.

The commercial value is generally about 9 per cent. more than the exchange value.

Thus, the exchange value being	=	$ 4.44$\frac{4}{9}$
To which we add 9 per cent. premium	=	.40
The commercial value will be	=	$ 4.84$\frac{4}{9}$

EXAMPLES.

1. What must a merchant in Boston pay in dollars and cents

for a bill of 9765£. 15s. 6d. on Liverpool, the premium being 9 per cent.? Ans. $ 47,309.75+.

2. Kimball, Jewett, & Co., of Boston, wish to purchase a bill of 18761£. 10s. on Liverpool, the premium being 8½ per cent.; what will be the cost of the bill? Ans. 20,356£. 4s. 6½d.+.

3. If I pay 9½ per cent. premium, what will be the amount of a bill on London which I can purchase for $ 81,727.75?

Ans.

4. John Jones, of Boston, has consigned a cargo of flour, valued at 17,000£., to Robert Morrison, Liverpool. R. S. Davis, of Boston, who is about to import many valuable books, has purchased of J. Jones a bill of exchange, at 6 per cent. premium, for the value of the above flour. The following is the form of the bill: —

Exchange for 17,000£. Boston, August 2d, 1847.

Ninety days after sight of this my first Bill of Exchange (second and third of the same tenor and date unpaid), pay to Robert S. Davis, or order, seventeen thousand pounds sterling, with or without further advice.

John Jones.

Robert Morrison, Esq.,
Merchant, Liverpool.

What should be paid for the above bill?

Ans. $ 80,088.88+.

FRANCE.

Accounts in France are kept in francs and centimes. The mercantile value of some of the principal coins of France, in United States currency, is as follows: —

The Double Napoleon or Louis, 40 francs,	=	$ 7.44
" Napoleon or Louis, 20 francs,	=	3.72
" Franc, 100 centimes,	=	.18⅗

5. If a merchant in Boston should remit to Paris 172,000 francs, exchange being 1½ per cent., what will be the cost of his bill in United States currency? Ans. $ 32,471.88.

6. What must a merchant in New York pay for a bill on Havre for 76,000 francs, the exchange being 5 francs 8 centimes for a dollar? Ans. $ 14,960.62+.

7. What is the value of a bill on Paris for 79,000 francs, exchange being 2 per cent. below par? Ans. $14,400.12.

8. John Smith, of Boston, remits to Bordeaux $17,280, the exchange being 5 francs 10 centimes for a dollar. Required the amount in francs. Ans. 88,128 francs.

AMSTERDAM.

Accounts are kept in florins, stivers, and pfennings, or in pounds, shillings, and pence Flemish.

16 Pfennings	= 1 Stiver.
20 Stivers	= 1 Florin, or Guilder.
Also, 12 Grotes or Pence Flemish, or 6 Stivers,	= 1 Shilling Flemish.
20 Shillings Flemish, or 6 Florins,	= 1 Pound Flemish.
2½ Florins, or 50 Stivers,	= 1 Rix Dollar.

9. United States on Amsterdam. Reduce 896 florins 10 stivers to U. S. money, exchange at 38 cents per florin.
Ans. $ 340.67.

10. Amsterdam on the United States. Reduce $ 340.67 to the money of Amsterdam, exchange at 38 cents per florin.
Ans. 896 florins 10 stivers.

CONSTANTINOPLE.

Accounts are kept in piasters, paras, and aspers, or in piasters and aspers; sometimes in piasters and half-paras, or in piasters and minas.

3 Aspers	= 1 Para.
40 Paras	= 1 Piaster, or Turkish Dollar.
Also, 80 Half-paras or 100 Minas	= 1 Piaster.

11. United States on Constantinople. Reduce 78 piasters 20 paras to U. S. money, exchange at 40 cents per piaster.
Ans. $ 31.40.

12. Constantinople on the United States. Reduce $ 31.40 to the money of Constantinople. Ans. 78 piasters 20 paras.

COPENHAGEN.

Accounts are kept here in rix dollars, marks, and skillings Danish, but sometimes in rix dollars, marks, and skillings lubs. Pfennings are also occasionally reckoned.

12 Pfennings	= 1 Skilling.
16 Skillings	= 1 Mark.
6 Marks Danish, or 3 Marks Lubs	= 1 Ryksdaler, or Rix Dollar.

13. United States on Copenhagen. Reduce 896 rix dollars 3 marks to U. S. money, exchange at 50 cents per rix dollar.
Ans. $ 448.25.

DANTZIC.

Accounts are computed here in florins, groschen, and pfennings.

3 Pfennings	=	1 Groschen.
30 Groschen	=	1 Florin.
3 Florins	=	1 Rix Dollar.

14. United States on Dantzic. Reduce 196 rix dollars 2 florins to U. S. money, exchange at 17 cents per florin.
Ans. $ 100.30.

15. Dantzic on the United States. Reduce $ 100.30 to the currency of Dantzic. Ans. 196 rix dollars 2 florins.

HAMBURG.

Computations are made in marks, schillings, and pfennings, banco or current; banco bears an agio on currency of from 20 to 25 per cent.

12 Pfennings	=	1 Schilling, or Sol, Lubs.
16 Schillings Lubs	=	1 Mark.
3 Marks	=	1 Rix Dollar.

16. United States on Hamburg. Reduce 675 rix dollars 2 marks to U. S. money, exchange at 30 cents per mark.
Ans. $ 608.10.

17. Hamburg on the United States. Reduce $ 608.10 to the currency of Hamburg, exchange at 30 cents per mark.
Ans. 675 rix dollars, 2 marks.

LEGHORN.

Accounts are kept in pezze, soldi, and denari di pezza.

12 Denari di Pezza	=	1 Soldo di Pezza.
20 Soldi di Pezza	=	1 Pezza.
12 Denari di Lira	=	1 Soldo di Lira.
20 Soldi di Lira	=	1 Lira.

18. United States on Leghorn. Reduce 286 pezze 10 soldi to U. S. money, exchange at 90 cents per pezza.
Ans. $ 257.85.

19. Leghorn on the United States. Change $ 257.85 to the currency of Leghorn. Ans. 286 pezze 10 soldi.

MILAN.

Accounts are kept in lire, soldi, and denari correnti or imperiali.

12 Denari	=	1 Soldo.
20 Soldi	=	1 Lira.
106 Soldi or Lire Imperiali	=	150 Soldi or Lire Correnti.
150 Soldi or Lire Correnti	=	1 Filippo.
117 Soldi Imperiali	=	1 Scudo or Crown.

20. United States on Milan. Change 176 lire 10 soldi to U. S. money, the lira being valued at 20 cents. Ans. $ 35.30.

21. Milan on the United States. Reduce $ 35.30 to the currency of Milan. Ans. 176 lire 10 soldi.

NAPLES.

Computations are made in ducats of 100 grains.

10 Grani	=	1 Carlino.
10 Carlini	=	1 Ducato di Regno.

22. United States on Naples. Change 769 ducati di regno 5 carlini to United States money, the value of the ducato being 80 cents. Ans. $ 615.60.

23. Naples on the United States. Reduce $ 615.60 to the currency of Naples. Ans. 769 ducati di regno 5 carlini.

SICILY.

Accounts are kept here in oncie, tari, and grani; and also in scudi, tari, and grani.

20 Grani	=	1 Taro.
30 Tari	=	1 Oncia.
Also, 12 Tari	=	1 Scudo or Sicilian Crown.
5 Scudi	=	2 Oncie.

24. United States on Sicily. Change 876 oncie 3 scudi to U. S. money, the oncia being valued at $ 2.38,3.

Ans. $ 2090.36,7$\frac{4}{5}$.

25. Sicily on the United States. Change $ 2090.36,7$\frac{4}{5}$ to the currency of Sicily. Ans. 876 oncie 3 scudi.

RUSSIA.

Computations are made here in rubles and kopecks.

10 Kopecks	=	1 Grieve or Grievener.
10 Grieves	=	1 Ruble.

26. United States on Russia. What is the value of 7684 rubles 8 grieves in U. S. money, the value of the ruble in the United States being 75 cents? Ans. $ 5763.60.

27. Russia on the United States. What is the value of $ 5763.60 in Russian currency? Ans. 7684 rubles 8 grieves.

ROME.

Accounts are kept in scudi moneta and baiocchi; or in scudi di stampa soldi and denari d' oro; quattrini and mezzi quattrini are sometimes reckoned.

2 Mezzi Quattrini	=	1 Quattrino.
5 Quattrini	=	1 Baioccho.
10 Baiocchi	=	1 Paolo.
10 Paoli	=	1 Scudo Moneta, or Roman Crown.

28. United States on Rome. Change 7689 scudi moneta to U. S. money, the value of the scudo being $ 1.00,0$\frac{67}{100}$.

Ans. $ 7694.15$\frac{163}{1000}$.

29. Rome on the United States. Change $ 7694.15$\frac{163}{1000}$ to Roman currency. Ans. 7689 scudi moneta.

SPAIN.

The general mode of keeping accounts in Spain is in maravedis and reals.

34 Maravedis	=	1 Real.
8 Reals	=	1 Dollar of Plate.
375 Maravedis	=	1 Ducat of Exchange.
4 Dollars of Plate	=	1 Pistole of Exchange.

Exchanges are generally computed in denominations of plate, which is always understood to be *old plate*, if *new plate* be not mentioned.

There are three principal denominations of these imaginary moneys in which exchanges are generally transacted; viz., dollars, doubloons, and ducats, and they are divided into reals and maravedis of plate, and sometimes converted into vellon and other denominations.

The dollar of exchange, also called *peso* or *piastre de cambio*, or *de plata*, is divided into 8 reals of 34 maravedis of plate each, and sometimes into 16 quartos.

The *doubloon de plata*, or pistole of exchange, is four times the dollar, and therefore contains 32 reals, or 1088 maravedis of plate.

The ducat of plate, also called *ducado de cambio*, contains 11 reals 1 maravedi, or 375 maravedis of plate.

At Alicant, Valencia, and Barcelona, exchanges are transacted in libras of 20 sueldos, or 240 dineros.

The libra of Alicant and Valencia is the dollar of plate. This is sometimes divided into 10 reals of new plate, which are, of course, equal to 8 reals of old plate.

The libra of Barcelona, commonly called *libra Catalan*, is worth $5\frac{5}{7}$ reals of plate; hence 7 of those libras equal 5 dollars of plate, and therefore 28 sueldos Catalan equal 1 dollar.

The hard dollar of 20 reals vellon is occasionally used in exchanges, and is also divided into 12 reals, each of 16 quartos. The current dollar, which is an imaginary money, valued at two thirds of the hard dollar, is divided into 8 reals, and the real into 16 quartos. The two latter are the principal moneys of exchange used at Gibraltar.

30. United States on Spain. What is the value in United States money of 7600 dollars of plate, exchange at 75 cents for a plate dollar? Ans. $ 5700.

31. Spain on the United States. What is the value of $ 5700 in Spanish money, exchange at 75 cents per dollar of plate? Ans. 7600 dollars of plate.

SWEDEN.

Accounts are kept in rix dollars, skillings, and pfennings.

12 Pfennings = 1 Skilling.
48 Skillings = 1 Rix Dollar.

32. United States on Sweden. Reduce 476 rix dollars 24 skillings to U. S. money, the rix dollar being valued at 107 cents. Ans. $ 509.85,5.

33. Sweden on the United States. Change $ 509.85,5 to the currency of Sweden. Ans. 476 rix dollars 24 skillings.

TURIN.

Accounts are kept in lire, soldi, and denari.

12 Denari = 1 Soldo.
20 Soldi = 1 Lira.

34. United States on Turin. Change 462 lire 10 soldi to U. S. money, exchange at 20 cents per lira. Ans. $ 92.50.

35. Turin on the United States. Change $ 92.50 to the currency of Turin, exchange at 20 cents per lira. Ans. 462 lire 10 soldi.

VIENNA.

Computations are made in florins and creutzers, or in rix dollars and creutzers.

4 Pfennings = 1 Creutzer.
60 Creutzers = 1 Florin.
$1\frac{1}{2}$ Florins = 1 Rix Dollar of Account.
2 Florins = 1 " " Specie.

36. United States on Vienna. Reduce 876 rix dollars, specie, 1 florin to U. S. money, the specie rix dollar being equal to 97 cents. Ans. $ 850.20,5.

37. Vienna on the United States. Change $ 850.20,5 to the currency of Vienna. Ans. 876 rix dollars, specie, 1 florin.

EAST INDIES, BENGAL, CALCUTTA, &c.

12 Pice = 1 Anna.
16 Annas = 1 Rupee.
1 Sicca Rupee = 2s. 6d. sterling.

38. United States on Calcutta. Reduce 432 rupees 12 annas to U. S. money, exchange at 50 cents per rupee. Ans. $ 216.37$\frac{1}{2}$.

39. Calcutta on the United States. Reduce $ 216.37$\frac{1}{2}$ to the currency of Calcutta, exchange at 50 cents per rupee. Ans. 432 rupees 12 annas.

BOMBAY.

100 Rees = 1 Quarter.
4 Quarters = 1 Rupee.
1 Rupee = 2s. 4d. sterling.

40. United States on Bombay. Change 678 rupees 2 quarters to U. S. money, the rupee being 50 cents. Ans. $ 339.25.

41. Bombay on the United States. Change $ 339.25 to Bombay money, reckoning the rupee at 50 cents. Ans. 678 rupees 2 quarters.

MADRAS.

Accounts are kept here in pagodas, fanams, and cash.

80 Cash = 1 Fanam.
45 Fanams = 1 Star Pagoda.
1 Star Pagoda = 8s. sterling.

42. United States on Madras. Change 375 star pagodas to U. S. money, the star pagoda being valued at $ 1.77 7/9.
Ans. $ 666.66+.

43. Madras on the United States. Reduce $ 896 to the currency of Madras. Ans. 504 pagodas.

TRIESTE.

Accounts are here kept in pfennings, florins, and creutzers.

4 Pfennings = 1 Creutzer.
60 Creutzers = 1 Florin.
1½ Florins = 1 Rix Dollar.

44. United States on Trieste. What is the value in U. S. money of 769 rix dollars 40 creutzers, the value of the rix dollar being 92 cents? Ans. $707.88⅔.

45. Trieste on the United States. Reduce $ 707.88⅔ to the currency of Trieste. Ans. 769 rix dollars 40 creutzers.

SECTION LXXIV.

VALUE OF GOLD COINS,

ACCORDING TO THE LAWS OF MAY AND JUNE, 1[illegible]

Names of Coins.	Weight.		Former Standard.	Standard of July 31st, 1834.
UNITED STATES.	dwt.	grs.	$ cts. m.	$ cts. m.
Eagle coined before July 31, 1834, Shares in proportion.	11	6	10.00,0	10.66,5
Foreign Gold.				
AUSTRIAN DOMINIONS.				
Souverein, - - - - -	3	14	3.17,6	3.37,7
Double Ducat, - - - - -	4	12	4.29,9	4.58,9
Hungarian Ducat, - - - -	2	5¾	2.15,4	2.29,6
BAVARIA.				
Carolin, - - - - - - -	6	5¼	4.64,6	4.95,7
Max d'Or, or Maximilian, -	4	4	3.11,1	3.31,8
Ducat, - - - - - - - -	2	5¾	2.13,3	2.27,5
BERNE.				
Ducat, double in proportion, -	1	23	1.85,4	1.[illegible],6
Pistole, - - - - - - -	4	21	4.26,2	4.[illegible],2

	dwt.	grs.	$ cts. m.	$ cts. m.
Brazil.				
Johannes, half in proportion, -	18	00	16.00,0	17.06,4
Dobraon, - - - - - -	34	12	30.66,6	32.70,6
Dobra, - - - - - -	18	6	16.22,2	17.30,1
Moidore, half in proportion, - -	6	22	6.14,9	6.55,7
Crusade, - - - - - -		16½	.59,8	.63,5
Brunswick.				
Pistole, double in proportion, - -	4	21½	4.27,1	4.54,8
Ducat, - - - - - -	2	5¾	2.09,2	2.23,0
Cologne.				
Ducat, - - - - - - -	2	5¾	2.12,5	2.26,7
Colombia.				
Doubloons, - - - - -	17	9	14.56,0	15.53,5
Denmark.				
Ducat, current, - - - - -	2	0	1.70,5	1.81,2
Ducat, specie, - - - - -	2	5¾	2.12,5	2.26,7
Christian d'Or, - - - - -	4	7	3.77,0	4.02,1
East Indies.				
Rupee, Bombay, 1818, - -	7	11	6.65,4	7.09,6
Rupee, Madras, 1818, - - -	7	12	6.66,7	7.11,0
Pagoda, Star, - - - - -	2	4¾	1.68,9	1.79,8
England.				
Guinea, half in proportion, - -	5	8½	4.79,9	5.07,5
Sovereign, - - - - -	5	2½	4.57,0	4.84,6
Seven Shilling Piece, - -	1	19	1.60,0	1.69,8
France.				
Double Louis, coined before 1786,	10	11	9.08,7	9.69,7
Louis, coined before 1786, - -	5	5½	4.54,1	4.84,6
Double Louis, coined since 1786,	9	20	8.59,0	9.15,3
Louis, coined since 1786, - -	4	22	4.29,5	4.57,6
Double Napoleon, or 40 francs, -	8	7	7.23,2	7.70,2
Napoleon, or 20 francs, - -	4	3½	3.61,6	3.85,1
Frankfort on the Maine.				
Ducat, - - - - - - -	2	5¾	2.13,7	2.27,9
Geneva.				
Pistole, old, - - - - -	4	7½	3.73,7	3.98,5
Pistole, new, - - - - -	3	15¾	3.23,2	3.44,4
Hamburg.				
Ducat, double in proportion, -	2	5¾	2.13,7	2.27,9
Genoa.				
Sequin, - - - - - -	2	5¾	2.15,8	2.30,2
Hanover.				
Double Geo. d'Or, single in proportion,	8	13	7.48,2	7.87,9

	dwt.	grs.	$ cts. m.	$ cts. m.
Ducat, - - - - - - -	2	5¾	2.15,4	2.29,6
Gold Florin, double in proportion,	2	2	1.57,6	1.67,0
HOLLAND.				
Double Ryder, - - - - -	12	21	11.44,2	12.20,5
Ryder, - - - - - - -	6	9	5.66,5	6.04,3
Ducat, - - - - - - -	2	5¾	2.13,3	2.27,5
Ten Guilder Piece, Five do. in pro.,	4	8	3.78,0	4.03,4
MALTA.				
Double Louis, - - - -	10	16	8.69,9	9.27,8
Louis, - - - - - - -	5	8	4.36,4	4.85,2
Demi Louis, - - - - - -	2	16	2.20,2	2.33,6
MEXICO.				
Doubloon, shares in proportion,	17	9	14.56,0	15.53,5
MILAN.				
Sequin, - - - - - -	2	5¾	2.15,6	2.29,0
Doppia or Pistole, - - - -	4	1½	3.57,2	3.80,7
Forty Livre Piece, 1808, - -	8	8	7.26,1	7.74,2
NAPLES.				
Six Ducat Piece, 1783, - - -	5	16	4.92,5	5.24,9
Two Ducat Piece or Sequin, 1762,	1	20¼	1.51,1	1.59,1
Three Ducat Piece or Oncetta, 1818,	2	10¼	2.34,7	2.49,0
NETHERLANDS.				
Gold Lion or 14 Florin Piece, -	5	7¾	4.73,1	5.04,6
Ten Florin Piece, 1820, - - -	4	7¾	3.76,6	4.01,9
PARMA.				
Quadruple Pistole, double in proportion,	18	9	15.59,6	16.62,8
Pistole or Doppia, 1787, - -	4	14	3.93,5	4.19,4
Pistole or Doppia, 1796, - -	4	14	3.87,5	4.13,5
Maria Theresa, 1818, - -	4	3½	3.62,4	3.86,1
PIEDMONT.				
Pistole, coined since 1785, half in pro.,	5	20	5.07,5	5.41,1
Sequin, half in proportion, - -	2	5	2.13,7	2.28,0
Carlino, coined since 1785, half in pro.,	29	6	25.63,2	27.34,0
Piece of 20 francs, called Marengo,	4	3¼	3.34,1	3.56,4
POLAND.				
Ducat, - - - - - -	2	5¾	2.13,7	2.27,5
PORTUGAL.				
Dobraon, - - - - - - -	34	12	30.66,6	32.70,6
Dobra, - - - - - -	18	6	16.22,2	17.30,1
Johannes, - - - - - -	18	0	16.00,0	17.06,4
Moidore, half in proportion, -	6	22	6.14,9	6.55,7
Piece of 16 Testoons or 1600 Rees,	2	6	1.99,2	2.12,1
Old Crusado of 400 Rees, - -		15	.84,9	.58,5
New Crusado of 480 Rees, - -		16¼	.59,8	.63,5
Milree, coined in 1775, - -		19¾	.73,2	.78,0

	dwt.	grs.	$ cts. m.	$ cts. m.
PRUSSIA.				
Ducat, 1748, - - - - -	2	5¾	2.13,7	2.27,9
Ducat, 1787, - - - - -	2	5¾	2.12,5	2.26,7
Frederick, double, 1769, - -	8	14	7.47,5	7.95,5
" " 1800, - -	8	14	7.45,4	7.95,1
" single, 1778, - -	4	7	3.74,9	3.99,7
" " 1800, - -	4	7	3.72,5	3.97,5
ROME.				
Sequin, coined since 1760, - -	2	4¼	2.10,9	2.25,1
Scudo of Republic, - - -	17	0¼	14.82,8	15.81,1
RUSSIA.				
Ducat, 1796, - - - - -	2	6	2.15,0	2.29,7
Ducat, 1763, - - - - -	2	5¾	2.12,5	2.26,7
Gold Ruble, 1756, - - - -	1	0½	.90,9	.96,7
Gold Ruble, 1799, - - -		18¾	.69,1	.73,7
Gold Poltin, 1777, - - - -		9	.33,1	.35,5
Imperial, 1801, - - - -	7	17¼	7.34,9	7.83,9
Half Imperial, 1801, - - - -	4	3½	3.68,9	3.93,3
SARDINIA.				
Carlino, half in proportion, -	10	7½	8.88,1	9.47,2
SAXONY.				
Ducat, 1784, - - - - -	2	5¾	2.12,5	2.26,7
Ducat, 1797, - - - - -	2	5¾	2.13,7	2.27,9
Augustus, 1754, - - - - -	4	6½	3.68,5	3.92,5
Augustus, 1784, - - - -	4	6⅔	3.72,5	3.97,4
SICILY.				
Ounce, 1751, - - - - -	2	20½	2.35,1	2.50,4
Double Ounce, 1758, - - -	5	17	4.72,7	5.04,4
SPAIN.				
Doubloon, 1772, parts in proportion,	17	8½	15.03,0	16.02,8
Doubloon, - - - -	17	9	14.56,0	15.53,5
Pistole, - - - - - - -	4	8½	3.64,0	3.88,4
Coronilla, Gold Dol. or Vintem, 1801,	1	3	.92,1	.98,3
SWEDEN.				
Ducat, - - - - - - - -	2	5	2.09,7	2.23,5
SWITZERLAND.				
Pistole of the Helvetic Republic, 1800,	4	21½	4.27,9	4.56,0
TREVES.				
Ducat, - - - - - - - -	2	5¾	2.02,5	2.26,7
TURKEY.				
Sequin Fonducli of Constan'ple, 1773,	2	5¾	1.74,9	1.86,8
Sequin Fonducli of Constan'ple, 1789,	2	5¾	1.73,3	1.84,8
Half Misseir, 1818, - - -		18½	.49,1	.52,1
Sequin Fonducli, - - -	2	5	1.71,7	1.83,0
Yeermeeblekblek, - - - - -	3	1¾	2.84,0	3.02,8

	dwt.	grs.	$ cts. m.	$ cts. m.
Tuscany.				
Zechino or Sequin, - - -	2	5¾	2.16,6	2.31,8
Ruspone of the Kingdom of Etruria,	6	17½	6.50,5	6.93,8
Venice.				
Zechino or Sequin, shares in proportion,	2	6	2.16,0	2.31,0
Wurtemberg.				
Carolin, - - - - - -	6	3¼	4.59,4	4.89,8
Ducat, - - - - - -	2	5	2.09,7	2.23,5
Zurich.				
Ducat, double and half in proportion,	2	5¾	2.12,5	2.26,7

Section LXXV.

GEOMETRY.

DEFINITIONS.

1. A point is that which has position, but no magnitude nor dimensions; neither length, breadth, nor thickness.

2. A line is length, without breadth or thickness.

3. A surface or superficies is an extension, or a figure of two dimensions, length and breadth, but without thickness.

4. A body or solid is a figure of three dimensions; viz. length, breadth, and thickness.

5. Lines are either right or curved, or mixed of these two.

6. A right or straight line lies all in the same direction between its extremities, and is the shortest distance between two points. When a line is mentioned simply, it means a right line.

7. A curve continually changes its direction between its extreme points.

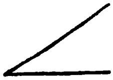
8. Lines are either parallel, oblique, perpendicular, or tangential.

9. Parallel lines are always at the same perpendicular distance, and never meet, though ever so far produced.

10. Oblique lines change their distance from each other, and would meet if produced on the side of the least distance.

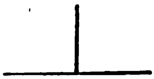
11. One line is perpendicular to another, when it inclines not more on the one side than the other, or when the angles on both sides of it are equal.

12. An angle is the inclination, or opening of two lines having different directions, and meeting in a point.

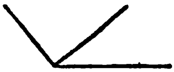


13. Angles are right or oblique, acute or obtuse.

14. A right angle is that which is made by one line perpendicular to another; or, when the angles on each side are equal to one another, they are right angles.



15. An oblique angle is one which is made by two oblique lines, and is either less or greater than a right angle.



16. An acute angle is less than a right angle.

17. An obtuse angle is greater than a right angle.

18. Superficies are either plane or curved.

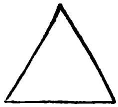
19. A plane superficies, or plane, is that with which a right line may every way coincide; or, if the line touch the plane in two points, it will touch it in every point; but if not, it is curved.

20. Plane figures are bounded either by right lines or curves.

21. Plane figures, that are bounded by right lines, have names according to the number of their sides, or of their angles; for they have as many sides as angles, the least number being three.

22. A figure of three sides and angles is called a triangle; and it receives particular denominations from the relations of its sides and angles.

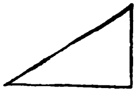
23. An equilateral triangle is that whose three sides are equal.



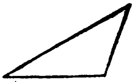
24. An isosceles triangle is that which has two sides equal.

25. A scalene triangle is that whose three sides are unequal.

26. A right-angled triangle is that which has one right angle.



27. Other triangles are oblique-angled, and are either acute or obtuse.



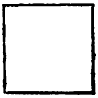
28. An obtuse-angled triangle has one obtuse angle.

29. An acute-angled triangle has its three angles acute.

30. A figure of four sides and angles is called a quadrangle, or a quadrilateral.

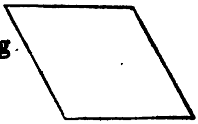
31. A parallelogram is a quadrilateral, which has both its pairs of opposite sides parallel; and it takes the following particular names; viz. rectangle, square, rhombus, and rhomboid.

32. A rectangle is a parallelogram, having a right angle.



33. A square is an equilateral rectangle, having its length and breadth equal.

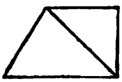
34. A rhomboid is an oblique-angled parallelogram, whose opposite sides are equal.



35. A rhombus is a parallelogram, having all its sides equal, but its angles oblique.

36. A trapezium is a quadrilateral, which has neither two of its opposite sides parallel.

37. A trapezoid has only one pair of its opposite sides parallel.



38. A diagonal is a line joining any two opposite angles of a quadrilateral.

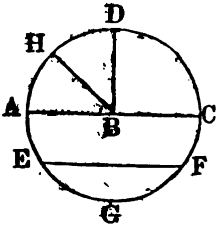
39. Plane figures that have more than four sides are in general called polygons; and they receive other particular names according to the number of their sides or angles. Thus,

40. A pentagon is a polygon of five sides; a hexagon, of six sides; a heptagon, of seven; an octagon, of eight; a nonagon, of nine; a decagon, of ten; an undecagon, of eleven; and a dodecagon, of twelve sides.

41. A regular polygon has all its sides and all its angles equal. If they are not both equal, the polygon is irregular.

42. An equilateral triangle is also a regular figure of three sides, and the square is one of four; the former being also called a trigon, and the latter a tetragon.

43. Any figure is equilateral when all its sides are equal; and it is equiangular when all its angles are equal. When both these are equal, it is a regular figure.



44. A circle is a plane figure, bounded by a curve line, called the circumference, which is everywhere equidistant from a certain point, called its centre. The circumference itself is often called a circle, and also the periphery.

45. The radius of a circle is a line drawn from the centre to the circumference, as A B.

46. The diameter of a circle is a line drawn through the centre and terminating at the circumference on both sides, as A C.

47. An arc of a circle is any part of the circumference, as A D.

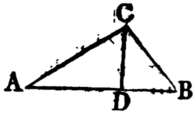
48. A chord is a right line joining the extremities of an arc, as E F.

49. A segment is any part of a circle bounded by an arc and its chord, as E F G.

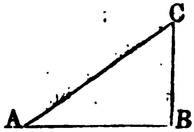
50. A semicircle is half the circle, or segment cut off by a diameter. The half circumference is sometimes called the semicircle, as A G C.

51. A sector is any part of the circle bounded by an arc and two radii drawn to its extremities, as A B H.

52. A quadrant, or quarter of a circle, is a sector, having a quarter of its circumference for its arc, and its two radii are perpendicular to each other. A quarter of the circumference is sometimes called a quadrant, as A B D.



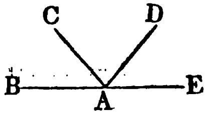
53. The height or altitude of a figure is a perpendicular, let fall from an angle, or its vertex, to the opposite side, called the base, as C D.



54. In a right-angled triangle the side opposite to the right angle is called the hypothenuse, and the other two sides are called the legs, and sometimes the base and perpendicular; thus,

A B is the base, B C the perpendicular, and A C the hypothenuse.

55. When an angle is denoted by three letters, of which one stands at the angular point, and the other two on the two sides, that which stands at the angular point is read in the middle. Thus, the angle contained by the lines B A and A D is called the angle B A D, or D A B.



56. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 minutes, each minute into 60 seconds, and so on. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

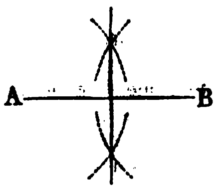
GEOMETRICAL PROBLEMS.

PROBLEM I.

To divide a line, A B, *into two equal parts.*

NOTE. — It would be useful for the pupil to be furnished with a pair of dividers and a rule, and to be required to draw the diagrams here given.

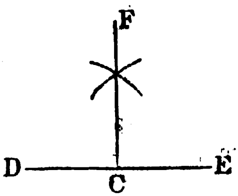
Set one foot of the dividers in A, and, opening them beyond the middle of the line, describe arches above and below the line; with the same extent of the dividers, set one foot in the point B, and describe arches crossing the former; draw a line from the intersection above the line to the intersection below the line, and it will divide the line A B into two equal parts.



PROBLEM II.

To erect a perpendicular on the point C, *in a given line.*

Set one foot of the dividers in the given point C, extend the other foot to any distance at pleasure, as to D, and with that extent make the marks D and E. With the dividers, one foot in D, at any extent above half the distance D E, describe an arch above the line, and with the same extent, and one foot in E, de-



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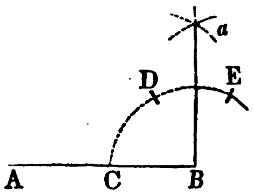
scribe an arch crossing the former; draw a line from the intersection of the arches to the given point C, which will be perpendicular to the given line in the point C.

PROBLEM III.

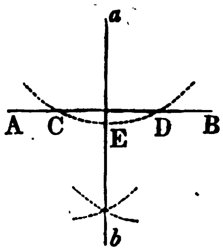
To erect a perpendicular upon the end of a line.



Set one foot of the dividers in the given point B, open them to any convenient distance, and describe the arch C D E; set one foot in C, and with the same extent cross the arch at D; with the same extent cross the arch again from D to E; then with one foot of the dividers in D, and, with any extent above the half of D E, describe an arch *a*; take the dividers from D, and, keeping them at the same extent, with one foot in E, intersect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to A B.

PROBLEM IV.

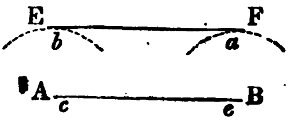
From a given point, a, to let fall a perpendicular to a given line A B.



Set one foot of the dividers in the point *a*, extend the other so as to reach beyond the line A B, and describe an arch to cut the line A B in C and D; put one foot of the dividers in C, and, with any extent above half C D, describe an arch *b*; keeping the dividers at the same extent, put one foot in D, and intersect the arch *b* in *b*; through which intersection, and the point *a*, draw *a* E, the perpendicular required.

PROBLEM V.

To draw a line parallel to a given line A B.

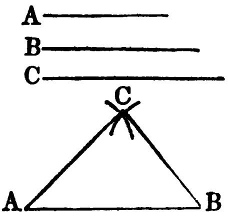


Set one foot of the dividers in any part of the line, as at *c*; extend the dividers at pleasure, unless a distance be assigned, and describe an arch *b*; with the same

extent in some other part of the line A B, as at *e*, describe the arch *a*; lay a rule to the extremities of the arches, and draw the line E F, which will be parallel to the line A B.

PROBLEM VI.

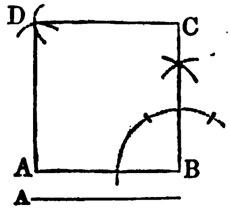
To make a triangle whose sides shall be equal to three given lines, any two of which are longer than the third.



Let A B C be the three given lines; draw a line, A B, at pleasure; take the line C in the dividers, set one foot in A, and with the other make a mark at B; then take the given line B in the dividers, and, setting one foot in A, draw the arch C; then take the line A in the dividers, and, setting one foot in B, intersect the arch C in C; lastly, draw the lines A C and B C, and the triangle will be completed.

PROBLEM VII.

To make a square whose sides shall be equal to a given line.

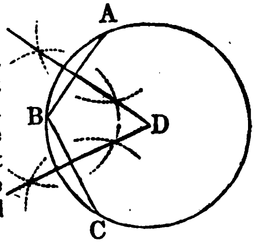


Let A be the given line; draw a line, A B, equal to the given line; from B raise a perpendicular to C, equal to A B; with the same extent, set one foot in C, and describe the arch D; also, with the same extent, set one foot in A, and intersect the arch D; lastly, draw the line A D and C D, and the square will be completed.

In like manner may a parallelogram be constructed, only attending to the difference between the length and breadth.

PROBLEM VIII.

To describe a circle, which shall pass through any three given points, not in a straight line.



Let the three given points be A B C, through which the circle is to pass. Join the points A B and B C with right lines, and bisect these lines; the point D, where the bisecting lines cross each other, will be the centre of the circle required. Therefore, place one foot of the dividers in D, extending the other to either of the given points, and

the circle described by that radius will pass through all the points.

Hence it will be easy to find the centre of any given circle; for, if any three points are taken in the circumference of the given circle, the centre will be found as above. The same may also be observed when only a part of the circumference is given.

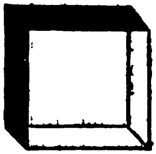
MENSURATION OF SOLIDS.

DEFINITIONS.

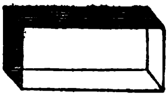
1. Solids are figures having length, breadth, and thickness.

2. A prism is a solid, whose ends are any plane figures which are equal and similar, and its sides are parallelograms.

NOTE.—A prism is called a *triangular prism*, when its ends are triangles; *a square prism*, when its ends are squares; *a pentagonal prism*, when its ends are pentagons; and so on.



3. A *cube* is a square prism, having six sides, which are all squares.

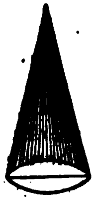


4. A *parallelopiped* is a solid, having six rectangular sides, every opposite pair of which is equal and parallel.

5. A *cylinder* is a round prism, having circles for its ends.

6. A *pyramid* is a solid, standing on a triangular, square, or polygonal basis, and its sides are triangles, whose vertices meet in a point at the top, called the *vertex* of the pyramid.

7. A *cone* is a solid figure, having a circle for its base, and its top terminated in a point or vertex.



8. A *sphere* is a solid, bounded by one continued convex surface, every point of which is equally distant from a point within, called the *centre*. The sphere may be conceived to be formed by the revolution of a semicircle about its diameter, which remains fixed.

A *hemisphere* is half a sphere.

9. The *segment* of a pyramid, sphere, or any other solid, is a part cut off the top by a plane parallel to the base of that figure.

10. A *frustum* is the part that remains at the bottom after the segment is cut off.

11. The *sector of a sphere* is composed of a segment less than a hemisphere, and of a cone having the same base with the segment, and its vertex in the centre of the sphere.

12. The *axis of a solid* is a line drawn from the middle of one end to the middle of the opposite end; as between the opposite ends of a prism. The axis of a sphere is the same as a diameter, or a line passing through the centre, and terminating at the surface on both sides.

13. The *height* or altitude of a solid is a line drawn from its vertex, or top, perpendicular to its base.

MENSURATION OF SUPERFICIES AND SOLIDS.

PROBLEM I.

To find the area of a square or parallelogram.

RULE. — *Multiply the length by the breadth, and the product is the superficial contents.*

1. What are the contents of a board 15 feet long and 2 feet wide? Ans. 30 feet.

2. The State of Massachusetts is about 128 miles long and 48 miles wide. How many square miles does it contain? Ans. 6144 miles.

3. The largest of the Egyptian pyramids is square at its base, and measures 693 feet on a side. How much ground does it cover? Ans. 11 acres 4 poles.

4. What is the difference between a floor 40 feet square, and 2 others, each 20 feet square? Ans. 800 feet.

5. There is a square of 3600 yards area; what is the side of a square, and the breadth of a walk along each side of the square, and each end, which may take up just one half of the square? Ans. { 42.42+ yards, side of the square. 8.78+ yards, breadth of the walk.

PROBLEM II.

To find the area of a rhombus or rhomboid.

RULE.—*Multiply the length of the base by the perpendicular height.*

6. The base of a rhombus being 12 feet, and its height 8 feet, required the area. Ans. 96 feet.

PROBLEM III.

To find the area of a triangle.

RULE.—*Multiply the base by half the perpendicular height; or, add the three sides together; then take half of that sum, and out of it subtract each side severally; multiply the half of the sum and these remainders together, and the square root of this product will be the area of the triangle.*

7. What are the contents of a triangle, whose perpendicular height is 12 feet, and the base 18 feet? Ans. 108 feet.

8. There is a triangle, the longest side of which is 15.6 feet, the shortest side 9.2 feet, and the other side 10.4 feet. What are the contents? Ans. 46.139+ feet.

PROBLEM IV.

Having the diameter of a circle given, to find the circumference.

RULE.—*Multiply the diameter by* 3.141592.

NOTE.—The exact ratio of the diameter of a circle to its circumference has never yet been ascertained. Nor can a square, or any other right-lined figure, be found, that shall be exactly equal to a given circle. This is the famous problem, called *the squaring the circle*, which has exercised the abilities of the greatest mathematicians for ages, and has been the occasion of so many endless disputes. Van Ceulen, a Dutchman, was the first who ascertained this ratio to any great degree of exactness, which he extended to thirty-six places of decimals;* and it was effected by

* This is said to have been thought so curious a performance, that the numbers were cut on his tombstone, in St. Peter's church-yard, at Leyden.

means of the continual bisection of an arc of a circle. This process was exceedingly troublesome and laborious; but since the invention of *Fluxions* and the *Summation of Infinite Series*, there have been several methods found for doing the same thing with less labor and trouble, and far more expedition. If, therefore, the diameter of a circle be 1 inch, the circumference will be 3.1415926535897932384626433832795028841971693 9937510582097494459230781640628620899862803482534211706798214808-65132823066470938446460955051822317253594081284802 inches nearly.

9. If the diameter of a circle is 144 feet, what is the circumference? Ans. 452.389248 feet.

10. If the diameter of the earth be 7964 miles, what is its circumference? Ans. 25019.638688+ miles.

PROBLEM V.

Having the diameter of a circle given, to find the area.

RULE. — *Multiply half the diameter by half the circumference, and the product is the area; or, which is the same thing, multiply the square of the diameter by* .785398, *and the product is the area.*

Demonstration. — If we suppose a circle to be divided into an infinite number of triangles, by lines drawn from the centre of the circle to the circumference, we may find the contents of each triangle by multiplying its perpendicular height by half its base, but its perpendicular height is half the diameter of the circle, and half its base is half a certain portion of the circumference; and all the bases of all the triangles united form the whole circumference.

Again, if multiplying half the circumference by half the diameter give the area of a circle, it is evident that the area will be obtained by multiplying one fourth the circumference by the whole diameter; and, as the circumference of a circle, whose diameter is 1, is 3.141592, therefore by multiplying 1 by one fourth of 3.141592 we shall obtain the area of a circle whose diameter is 1. Thus, $3.141592 \div 4 = .785398$. And as circles are to each other as the squares of their diameters (see page 246), therefore, if we wish to obtain the area of a circle whose diameter is 20 feet, we make the following statement.

$$\text{As } 1^2 \text{ foot} : 20^2 \text{ feet} :: .785398 : 314.1592 \text{ feet, Ans.}$$

And this process is equivalent to multiplying the square of the diameter of the given circle by .785398. Q. E. D.

11. If the diameter of a circle be 761 feet, what is the area? Ans. 454840.475158 feet.

12. There is a circular island, three miles in diameter; how many acres does it contain? Ans. 4523.89+ acres.

PROBLEM VI.

Having the diameter of a circle given, to find the side of an equal square.

RULE. — *Multiply the diameter by .886227, and the product is the side of an equal square.*

Demonstration. — We have seen in Problem V. that the area of a circle, whose diameter is 1, is .7853981633397; if, therefore, we extract the square root of this number, we shall obtain the side of a square of a circle whose diameter is 1. Thus, $\sqrt{.785398163397} = .886227$. And since, as we have before stated, the diameters of circles are to each other as the sides of their similar inscribed figures, therefore, as 1, the diameter of the given circle, is to the diameter of the required circle, so is .886227, the side of a square equal to the given circle, to the side of a square equal to the required circle. If, therefore, the diameter of a circle were 20 feet, and it was required to find the side of a square that would contain that quantity, we should make the following statement: —

As 1 foot : 20 feet : : .886227 : 17.72454 feet, Ans.

We see, from this process, that multiplying the diameter of the required circle by .886227 gives the side of an equal square. Q. E. D.

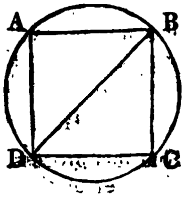
13. I have a round field, 50 rods in diameter; what is the side of a square field, that shall contain the same area?

Ans. 44.31135+ rods.

PROBLEM VII.

Having the diameter of a circle given, to find the side of a square inscribed.

RULE. — *Multiply the diameter by .707106, and the product is the side of a square inscribed.*



Demonstration. — Let A B C D represent a square inscribed in a circle whose diameter is 1. D B is the diameter of the circle, and it is also the diagonal of the square. As D A B is a right-angled triangle, the squares of D A and A B are equal to the square of B D; but A D is equal to A B, therefore the square of D A is equal to half the square of B D. The square of B D is 1, therefore the square of D A is .5; and the square root of .5 is $\sqrt{.5} = .707106 =$ A D, the side of the square, whose diameter is 1. Q. E. D. There-

fore, to find the side of a square inscribed in any circle, we say, as 1 is to the diameter of any required circle, so is .707106 to the side of a square inscribed in the required circle.

14. I have a piece of timber 30 inches in diameter; how large a square stick can be hewn from it?

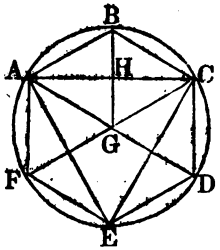
Ans. 21.21+ inches square.

15. Required the side of a square, that may be inscribed in a circle 80 feet in diameter. Ans. 56.56848+ feet.

PROBLEM VIII.

In a given circle to describe a hexagon and an equilateral triangle, and to find the length of one of the sides of the inscribed triangle.

RULE. — *Multiply the diameter by .8660254 and the product is the side of an inscribed equilateral triangle.*



Demonstration. — Let ABCDEF be the given circle, and G the centre. From the point B in the circumference apply the radius BG six times to the circumference, and join BC, CD, DE, EF, FA, and AB, and the figure ABCDEF, thus formed, is an equilateral inscribed hexagon.

Join the alternate angles AE, EC, and CA, and the figure AEC, thus formed, is an equilateral triangle inscribed. It is equilateral because the three sides subtend the equal arches of the circumference.

ABCG is a rhombus, and the diagonal BG is equal to either side of the rhombus. If, therefore, the diameter of the circle AD is 1, the semidiameter AG or BG will be .5; and BH, which is half of BG, will be .25. AHB is a right-angled triangle, and therefore AH is equal to the square root of the difference of the squares of AB and BH. Thus $AH = \sqrt{AB^2 - BH^2} = \sqrt{.5^2 - .25^2} = \sqrt{.25 - .0625} = \sqrt{.1875} = .4330127$. Now if AH be .4330127, AC, which is twice AH, will be .8660254. But AC is the side of the equilateral triangle inscribed; and as we have before shown, page 246, that all similar figures are in proportion to their homologous sides, it will therefore follow, that as the diameter of the given circle, which is 1, is to the side of its inscribed equilateral triangle

.8660254, so is the diameter of any other circle to the side of its inscribed equilateral triangle.

16. Required the side of an equilateral triangle, that may be inscribed in a circle 80 feet in diameter. Ans. 69.28+ feet.

17. Required the side of an equilateral triangle, that may be inscribed in a circle 50 inches in diameter. Ans. 43.3+in.

18. There is a certain piece of round timber 30 inches in diameter; required the side of an equilateral triangular beam, that may be hewn from it. Ans. 25.98+ inches.

PROBLEM IX.

Having the circumference of a circle given, to find the diameter.

RULE. — *Multiply the circumference by* .3183098, *and the product is the diameter.*

Rationale. — As we have seen in Problem IV., the ratio of the circumference of a circle to its diameter is as 3.141592 to 1; and as the ratio of all circumferences of circles to their diameters is the same, therefore, if the circumference of a circle be 1, its diameter may be found by the following proportion: —

As 3.141592 : 1 : : 1 : : .3183098.

Wherefore, if we multiply the circumference of any circle by .3183098, the product is the diameter. Q. E. D.

19. If the circumference of a circle be 25,000 miles, what is its diameter? Ans. 7957.74+ miles.

20. If the circumference of a round stick of timber be 50 inches, what is its diameter? Ans. 15.91549+ inches.

PROBLEM X.

Having the circumference of a circle given, to find the side of an equal square.

RULE. — *Multiply the circumference by* .282094, *and the product is the side of an equal square.*

We have demonstrated in a previous problem, that, when the diameter of a circle is 1, the side of an equal square is .886227; but when the circumference is 1, the side of an equal square must be as much less than this number, as 1 is less than 3.141592; that is, it will be equal to the number of times .886227 will contain 3.141592; .886227 ÷ 3.141592 = .282094; therefore, if we multiply the circumference of any circle by .282094, the product is the side of an equal square. Q. E. D.

21. I have a circular field 360 rods in circumference; what must be the side of a square field, that shall contain the same quantity? Ans. 101.55+ rods.

22. John Smith had a farm, which was 10,000 rods in circumference, and which he sold at $71.75 per acre, and he purchased another farm containing the same quantity of land in the form of a square; required the length of one of its sides. Ans. 2820.94+ rods.

PROBLEM XI.

Having the circumference of a circle given, to find the side of an equilateral triangle inscribed.

RULE. — *Multiply the circumference by* .2756646, *and the product is the side of an equilateral triangle inscribed.*

Rationale. — We have seen in Problem VIII. that the ratio of the diameter of a circle to its inscribed triangle is as 1 to .8660254; but the ratio of the circumference of a circle to its diameter is as 3.141592 to 1; therefore, the ratio of the circumference of a circle to its inscribed equilateral triangle is as 3.141592 to .8660254; that is, it will be equal to the number of times .8660254 will contain 3.141592. Thus .8660254 ÷ 3.141592 = .2756646. Therefore, by multiplying the circumference of any circle by .2756646, we obtain the side of an equilateral triangle inscribed. Q. E. D.

23. How large an equilateral triangle may be inscribed in a circle, whose circumference is 5000 feet? Ans. 1378.323ft.

24. Required the side of an equilateral triangular beam, that may be hewn from a round piece of timber 80 inches in circumference. Ans. 22.05+ inches.

PROBLEM XII.

Having the circumference of a circle given, to find the side of an inscribed square.

RULE. — *Multiply the circumference by* .225079, *and the product is the side of a square inscribed.*

Rationale. — We have seen in Problem VII. that the ratio of the diameter of a circle to its inscribed square is as 1 to .707106; we have also seen in Problem IV. that the ratio of the circumference of a circle to its diameter is as 3.141592 to 1; therefore, the ratio of the circumference of a circle to its inscribed square is as 3.141592 to .707106. That is, it will be equal to the number of times .707106 will contain 3.141592. Thus

.707106 + 3.141592 = .225079; therefore, if the circumference of any circle be multiplied by .225079, the product is the side of a square inscribed. Q. E. D.

25. I have a circular field, whose circumference is 5000 rods, what is the side of a square field that may be made in it?

Ans. 1125.395+ rods.

26. How large a square stick may be hewn from a piece of round timber 100 inches in circumference?

Ans. 22.5+ inches square.

NOTE.—If we wish to find the circumference of a tree, which will hew any given number of inches square, we divide the given side of the square by .225079, and the quotient is the circumference required.

27. What must be the circumference of a tree that will make a beam 10 inches square? Ans. 44.42+ inches.

28. What must be the circumference of a tree, that, when hewn, it may be 18 inches square? Ans. 79.97+ inches.

29. I have a garden which is 20 rods square; required the circumference of a circle, in feet, that will inclose this garden.

Ans. 1466.15+ feet.

PROBLEM XIII.

To find the contents of a cube or parallelopipedon.

RULE.—*Multiply the length, height, and breadth continually together, and the product is the contents.*

30. How many cubic feet are there in a cube, whose side is 18 inches? Ans. $3\frac{3}{8}$ feet.

31. What are the contents of a parallelopipedon, whose length is 6 feet, breadth $2\frac{1}{2}$ feet, and altitude $1\frac{3}{4}$ feet?

Ans. $26\frac{1}{4}$ feet.

32. How many cubic feet in a block of marble, whose length is 3 feet 2 inches, breadth 2 feet 8 inches, and depth 2 feet 6 inches? Ans. $21\frac{1}{9}$ feet.

PROBLEM XIV.

To find the solidity of a prism.

RULE.—*Multiply the area of the base, or end, by the height.*

33. What are the contents of a triangular prism, whose length is 12 feet, and each side of its base $2\frac{1}{2}$ feet?

Ans. 32.47+ feet.

34. Required the solidity of a triangular prism, whose length is 10 feet, and the three sides of its triangular end, or base, are 5, 4, and 3 feet. Ans. 60 feet.

PROBLEM XV.

To find the solidity of a cone or pyramid.

RULE. — *Multiply the area of the base by ⅓ of its height.*

35. What is the solidity of a cone, whose height is 12½ feet, and the diameter of the base 2½ feet? Ans. 20.45+ feet.

36. What are the contents of a triangular pyramid, whose height is 14 feet 6 inches, and the sides of its base 5, 6, and 7 feet? Ans. 71.035+ feet.

QUESTIONS TO EXERCISE THE ABOVE PROBLEMS.

37. I have a round garden, containing 75 square rods; how large a square garden can be made in it?
Ans. 47.7464+ square rods.

38. I have a circular garden containing 75 square rods; what must be the side of a square field that would contain it?
Ans. 9.772+ rods.

39. There is a small circular field, 25 rods in diameter; what is the difference of the areas of the inscribed and circumscribed squares, and how much do they differ from the areas of the field?

Ans. 312.5 rods, the difference of the squares; 134.12625+ rods, the circumscribed square, more than the area; 178.373 rods, inscribed square less than the area.

PROBLEM XVI.

To find the surface of a cone.

RULE. — *Multiply the circumference of the base by half its slant height.*

40. What is the convex surface of a cone, whose side is 20 feet, and the circumference of its base 9 feet? Ans. 90 feet.

PROBLEM XVII.

To find the solidity of the frustum of a cone.

RULE. — *Multiply the diameters of the two bases together, and to the product add ⅓ of the square of the difference of the diameters; then multiply this sum by .785398, and the product will be the mean area between the two bases; lastly, multiply the mean area by the length of the frustum, and the product will be the solid contents. Or, find the area of the two ends of the frustum, multiply those two areas together, and extract the square root of their product. To this root add the two areas, and*

multiply their sum by one third of the altitude or length of the frustum, and the product will be the solidity of the frustum of a cone, or pyramid.

NOTE. — These are the exact rules for measuring round timber, and should be adopted. See page 330.

41. What are the contents of a stick of timber, whose length is 40 feet, the diameter of the larger end 24 inches, and the smaller end 12 inches? Ans. $73\frac{1}{3}$ feet, nearly.

PROBLEM XVIII.

To find the solidity of a sphere or globe.

RULE. — *Multiply the cube of the diameter by* .5236.

42. What is the solidity of a sphere, whose axis or diameter is 12 inches? Ans. 904.78+ inches.

43. Required the contents of the earth, supposing its circumference to be 25,000 miles.

Ans. 263858149120.06886875 miles.

PROBLEM XIX.

To find the convex surface of a sphere, or globe.

RULE. — *Multiply its diameter by its circumference.*

44. Required the convex surface of a globe, whose diameter or axis is 24 inches. Ans. 1809.55+ inches.

45. Required the surface of the earth, its diameter being $7957\frac{3}{4}$ miles, and its circumference 25,000 miles.

Ans. 198943750 square miles.

PROBLEM XX.

To find how large a cube may be cut from any given sphere, or be inscribed in it.

RULE. — *Square the diameter of the sphere, divide that product by* 3, *and extract the square root of the quotient for the answer.*

Demonstration. — It is evident, that, if a cube be inscribed in a sphere, its corners or angles will be in contact with the surface of the sphere, and that a line passing from the lower corner of the cube to its opposite upper corner will be the diameter of the sphere; and that the square of this oblique line is equal to the sum of the squares of three sides of the inscribed cube is evident, from the fact that the square of any two sides of the cube (suppose two sides at the base) is equal to the square of the diagonal across the base; and that the square of

this diagonal (which we have just proved to be equal to the square of two sides at the base) and the square of the height of the cube are equal to the square of the diagonal line, which passes from the lower corner of the square to the opposite upper corner, which line is the diameter of the sphere. Therefore, the square of the diameter of any sphere is equal to the sum of the squares of any three sides of an inscribed cube; or $\frac{1}{3}$ of the square of the diameter of any sphere is equal to the square of one of the sides of an inscribed cube. Q. E. D.

46. How large a cube may be inscribed in a globe 12 inches in diameter? Ans. 6.928+in. in the side of the cube.

$$\frac{12 \times 12}{3} = 48; \sqrt{48} = 6.928+ \text{ Answer.}$$

47. How large a cube may be inscribed in a sphere 40 inches in diameter? Ans. 23.09+ inches.

48. How many cubic inches are contained in a cube, that may be inscribed in a sphere 20 inches in diameter?
Ans. 1539.6+ inches.

Section LXXVI.

GAUGING.

Gauging is the art of finding the contents of any regular vessel, in gallons, bushels, &c.

PROBLEM I.

To find the number of gallons, &c., in a square vessel.

Rule. — *Take the dimensions in inches; then multiply the length, breadth, and height together; divide the product by* 282 *for ale gallons,* 231 *for wine gallons, and* 2150.42 *for bushels.*

1. How many wine gallons will a cubical box contain, that is 10 feet long, 5 feet wide, and 4 feet high?
Ans. $1496\frac{8}{77}$gal.

2. How many ale gallons will a trough contain, that is 12 feet long, 6 feet wide, and 2 feet high? Ans. $882\frac{18}{47}$gal.

3. How many bushels of grain will a box contain, that is 15 feet long, 5 feet wide, and 7 feet high? Ans. 421.8bu.

PROBLEM II.

To find the contents of a cask.

RULE. — *Take the dimensions of the cask in inches; namely, the diameter of the bung and head, and the length of the cask. Note the difference between the bung diameter and the head diameter.*

If the staves of the cask be much curved between the bung and the head, multiply the difference by .7; if not quite so much curved, by .65; if they bulge yet less, by .6; and if they are almost straight, by .55; add the product to the head diameter; the sum will be a mean diameter by which the cask is reduced to a cylinder.

Square the mean diameter thus found, then multiply it by the length; divide the product by 359 for ale or beer gallons, and by 294 for wine gallons.

4. Required the contents in wine gallons of a cask, whose bung diameter is 35 inches, head diameter 27 inches, and length 45 inches.

$35 - 27 \times .7 = 5.6 \qquad 32.6 \times 32.6 \times 45 = 47824.20$

$27 + 5.6 = 32.6$

$$\frac{47824.20}{294} = 162.66 \text{ wine gallons.}$$

5. What are the contents of a cask in ale gallons, whose bung diameter is 40 inches, head diameter 30 inches, and length 50 inches? Ans. 185.55+ ale gallons.

PROBLEM III.

To find the contents of a round vessel, wider at one end than the other.

RULE. — *Multiply the greater diameter by the less; to this product add ⅓ of the square of their difference, then multiply by the height, and divide as in the last rule.*

6. What are the contents in wine measure of a tub, 40 inches in diameter at the top, 30 inches at the bottom, and whose height is 50 inches? Ans. 209.75 wine gallons.

SECTION LXXVII.

TONNAGE OF VESSELS.

CARPENTER'S RULE. — For single-decked vessels, multiply the length, breadth at the main beam, and depth in the hold together, and divide the product by 95, and the quotient is the tons

But for a double-decked vessel, take half of the breadth of the main beam for the depth of the hold, and proceed as before.

1. What is the tonnage of a single-decked vessel, whose length is 65ft., breadth 20ft., and depth 10ft.? Ans. $136\frac{16}{19}$ tons.

2. What is the tonnage of a double-decked vessel, whose length is 70 feet, and breadth 24 feet? Ans. $212\frac{4}{19}$ tons.

GOVERNMENT RULE. — If the vessel be double-decked, take the length thereof from the fore part of the main stem to the after part of the stern-post above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel, and then deduct from the length $\frac{3}{5}$ of the breadth; multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95, the quotient whereof shall be deemed the true contents or tonnage of such ship or vessel; and if such ship or vessel be single-decked, take the length and breadth, as above directed, deduct from the said length $\frac{3}{5}$ of the breadth, and take the depth from the under side of the deck-plank to the ceiling in the hold, and then multiply and divide as aforesaid, and the quotient shall be deemed the tonnage.

3. What is the government tonnage of a single-decked vessel, whose length is 70 feet, breadth 30 feet, and depth in the hold 9 feet? Ans. $147\frac{15}{19}$ tons.

4. What is the government tonnage of a single-decked vessel, whose length is 75 feet, breadth 22 feet, and depth in the hold 12 feet? Ans. $171\frac{351}{475}$ tons.

5. What is the government tonnage of a double-decked vessel, which has the following dimensions: length 98 feet, breadth 35 feet? Ans. $496\frac{17}{38}$ tons.

6. Required the government tonnage of a double-decked vessel, whose length is 180 feet, and breadth 40 feet.

Ans. $1313\frac{13}{19}$ tons.

7. Required the government tonnage of a single-decked vessel, whose length is 78 feet, width 21 feet, and depth 9 feet.

Ans. $130\frac{53}{475}$ tons.

8. What is the government tonnage of a double-decked vessel, whose length is 159ft., and width 30ft.? Ans. $667\frac{17}{19}$ tons.

9. What is the government tonnage of Noah's ark, admitting its length to have been 479 feet, its breadth 80 feet, and its depth 48 feet. Ans. $17421\frac{9}{19}$ tons.

10. What is the government tonnage of a vessel, whose length is 200 feet, and breadth 35 feet? Ans. $1154\frac{3}{38}$ tons.

11. The new ship Montezuma is 280 feet in length, and 40 feet in breadth. Required the government tonnage.

Ans. $2155\frac{15}{19}$ tons.

Section LXXVIII.

MENSURATION OF LUMBER.

PROBLEM I.

To find the contents of a board.

Rule. — *Multiply the length of the board, taken in feet, by its breadth taken in inches, divide this product by* 12, *and the quotient is the contents in square feet.*

1. What are the contents of a board 24 feet long, and 8 inches wide? Ans. 16 feet.

2. What are the contents of a board 30 feet long, and 16 inches wide? Ans. 40 feet.

PROBLEM II.

To find the contents of joists.

Rule. — *Multiply the depth and width together, taken in inches, and their product by the length in feet; divide the last product by* 12, *and the quotient is the contents in feet.*

3. How many feet are there in 3 joists, which are 15 feet long, 5 inches wide, and 3 inches thick? Ans. $56\frac{1}{4}$ feet.

4. How many feet in 20 joists, 10 feet long, 6 inches wide, and 2 inches thick? Ans. 200 feet.

PROBLEM III.

To measure round timber.

We have inserted below the rule usually adopted by surveyors of lumber; but it is a very *unjust* rule, if it is intended to give only 40 cubic feet of timber for a ton. For, if a stick of round timber be 40 feet long, and its circumference be 48 inches, it is considered by surveyors to contain one ton, or 40 feet; whereas, it in reality contains, according to the following correct process, $50\frac{93}{100}$ cubic feet.

OPERATION.

$48 \times .31831 = 15.27888$; $15.27888 \div 2 = 7.63944$; $7.63944 \times 24 = 183.34656$; $183.34656 \times 40 = 7333.8624$; $7333.8624 \div 144 = 50\frac{93}{100}$, that is, it contains as many cubic feet as a stick $50\frac{93}{100}$ feet long and 1 foot square.

Rule. — *Multiply the length of the stick, taken in feet, by the square*

of ¼ the girth, taken in inches; divide this product by 144, *and the quotient is the contents in cubic feet.*

NOTE. — The girth is usually taken about ⅓ of the distance from the larger to the smaller end.

5. How many cubic feet in a stick of timber, which is 30 feet long, and whose girth is 40 inches? Ans. $20\frac{5}{6}$ feet.

6. If a stick of timber is 50 feet long, and its girth is 56 inches, what number of cubic feet does it contain? Ans. $68\frac{1}{18}$ feet.

7. What are the contents of a log 90 feet long, and whose circumference is 120 inches? Ans. $562\frac{1}{2}$ feet.

SECTION LXXIX.

PHILOSOPHICAL PROBLEMS.*

PROBLEM I.

To find the time in which pendulums of different lengths would vibrate; that which vibrates seconds being 39.2 inches.

The time of the vibrations of pendulums are to each other as the square roots of their lengths; or their lengths are as the squares of their times of vibrations.

RULE. — *As the square of one second is to the square of the time in seconds in which a pendulum would vibrate, so is* 39.2 *inches to the length of the required pendulum.*

EXAMPLES.

1. Required the length of a pendulum that vibrates once in 8 seconds.

$$1^2 : 8^2 :: 39.2\text{in.} : 2508.8\text{in.} = 209\tfrac{1}{15} \text{ feet, Ans.}$$

2. Required the length of a pendulum that shall vibrate 4 times a second. Ans. $2\frac{9}{20}$ inches.

3. Required the length of a pendulum that shall vibrate once a minute. Ans. 3920 yards.

4. How often will a pendulum vibrate whose length is 100 feet? Ans. once in 5.53+ seconds.

PROBLEM II.

To find the weight of any body, at any assignable distance above the earth's surface.

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* For demonstrations of the following problems, the student is referred to Enfield's Philosophy, or to the Cambridge Mathematics.

The gravity of any body above the earth's surface *decreases*, as the squares of its distance in semidiameters of the earth from its centre *increases*.

RULE. — *As the square of the distance from the earth's centre is to the square of the earth's semidiameter, so is the weight of the body on the earth's surface to its weight at any assignable distance above the surface of the earth, and vice versâ.*

5. If a body weigh 900 pounds at the earth's surface, what would it weigh 2000 miles above its surface? Ans. 400lbs.

6. Admitting the semidiameter of the earth to be 4000 miles, what would be the weight of a body 20,000 miles above its surface, that on its surface weighed 144 pounds? Ans. 4lbs.

7. How far must a body be raised to lose half its weight?
Ans. 1656.85+ miles.

8. If a man at the earth's surface could carry 150 pounds, how much would that burden weigh at the earth, which he could sustain at the distance of the moon, whose centre is 240,000 miles from the earth's centre? Ans. 540,000lbs.

9. If a body at the surface of the earth weigh 900 pounds, but being carried to a certain height weighs only 400 pounds, what is that height? Ans. 2000 miles.

PROBLEM III.

By having the height of a tide on the earth given, to find the height of one at the moon.

RULE. — *As the cube of the moon's diameter, multiplied by its density, is to the cube of the earth's diameter, multiplied by its density, so is the height of a tide on the earth to the height of one at the moon.*

10. The moon's diameter is 2180 miles, and its density 494; the earth's diameter is 7964 miles, and its density 400. If, then, by the attraction of the moon, a tide of 6 feet is raised at the earth, what will be the height of a tide raised by the attraction of the earth at the moon? Ans. 236.8+ feet.

NOTE. — The above question is on the supposition that the moon has seas and oceans similar to those on the earth, but astronomers at the present time doubt their existence at this secondary planet.

PROBLEM IV.

To find the weight of a body at the sun and planets, having its weight given at the earth.

If the diameters of two globes be equal, and their densities different, the weight of a body on their surfaces will be as their densities.

If their densities be equal, and their diameters different, the weight of a body will be as $\frac{1}{3}$ of their circumference.

If their diameters and densities be both different, the weight of a body will be as $\frac{2}{3}$ of their semidiameters, multiplied by their densities.

Therefore, having the weight of a body on the surface of the earth given, to find its weight at the surface of the sun and the several planets, we adopt the following

RULE. — *As $\frac{2}{3}$ of the earth's semidiameter, multiplied by its density, is to $\frac{2}{3}$ of the sun's or planet's semidiameter, multiplied by its density, so is the weight of a body at the surface of the earth to the weight of a body at the surface of the sun or planet.*

11. If the weight of a man at the surface of the earth be 170 pounds, what will be his weight at the surface of the sun, and the several planets, whose densities, &c., are as in the following table?

	Density.	Diameter.	Semidiameter.	$\frac{2}{3}$ Semidiameter.
Sun,	100	883246	441623	294415
Jupiter,	94.5	89170	44585	29723
Saturn,	67	79042	39521	26347
Earth,	400	7964	3982	2654
Moon,	494	2180	1090	726

Ans. Weight at the sun 4714.6+lbs., at Jupiter 449.7+lbs., at Saturn 282.6+lbs., at the moon 57.4+lbs.

PROBLEM V.

To find how far a heavy body will fall in a given time, near the surface of the earth.

Heavy bodies near the surface of the earth fall 16 feet in one second of time; and the velocities they acquire in falling are as the squares of the times; therefore, to find the distance any body will fall in a given time, we adopt the following

RULE. —*As the square of 1 second is to the square of the time in seconds that the body is falling, so is 16 feet to the distance in feet that the body will fall in the given time.*

12. How far will a leaden bullet fall in 8 seconds?

$$1^2 : 8^2 : : 16\text{ft.} : 1024\text{ft.} = \text{Answer.}$$

13. How far would a body fall in 1 minute?

Ans. 10 miles 1600 yards.

14. How far would a body fall in 1 hour?

Ans. 39,272 miles 1280 yards.

15. How far would a body fall in 9 days?

Ans. 1,832,308,363 miles 1120 yards.

PROBLEM VI.

The velocity given, to find the space fallen through to acquire that velocity.

RULE. — *Divide the velocity by* 8, *and the square of the quotient will be the distance fallen through to acquire that velocity.*

16. The velocity of a cannon-ball is 660 feet per second. From what height must it fall to acquire that velocity?

Ans. 6806¼ feet.

17. At what distance must a body have fallen to acquire the velocity of 1000 feet per second? Ans. 2 miles 5065 feet.

PROBLEM VII.

The velocity given per second, to find the time.

RULE. — *Divide the velocity by* 8, *and a fourth part of the quotient will be the time in seconds.*

18. How long must a body be falling to acquire a velocity of 200 feet per second? Ans. 6¼ seconds.

19. How long must a body be falling to acquire a velocity of 320 feet per second? Ans. 10 seconds.

PROBLEM VIII.

The space through which a body has fallen given, to find the time it has been falling.

RULE. — *Divide the square root of the space in feet fallen through by* 4, *and the quotient will be the time in seconds in which it was falling.*

20. How long would a body be falling through the space of 40,000 feet? Ans. 50 seconds.

21. How long would a ball be falling from the top of a tower, that was 400 feet high, to the earth? Ans. 5 seconds.

PROBLEM IX.

The weight of a body and the space fallen through given, to find the force with which it will strike.

RULE. — *Multiply the space fallen through by* 64, *then multiply the square root of this product by the weight, and the product is the momentum, or force with which it will strike.*

22. If the rammer for driving the piles of Warren Bridge weighed 1000 pounds, and fell through a space of 16 feet, with what force did it strike the pile?

$\sqrt{16 \times 64} = 32$ $\quad$ $32 \times 1000 = 32{,}000$lbs. Answer.

23. Bunker Hill Monument is 220 feet in height; what would be the momentum of a stone, weighing 4 tons, falling from the top to the ground? Ans. 1,063,184.6+ lbs.

Section LXXX.

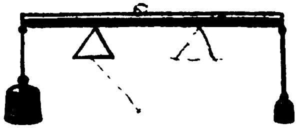
MECHANICAL POWERS.

That body which communicates motion to another is called the *power*.

The body which receives motion from another is called the *weight*.

The mechanical powers are six, the Lever, the Wheel and Axle, the Pulley, the Inclined Plane, the Screw, and the Wedge.

THE LEVER.



The *lever* is a bar, movable about a fixed point, called its *fulcrum* or *prop*. It is in theory considered as an inflexible line, without weight. It is of three kinds; the first, when the prop is between the weight and the power; the second, when the weight is between the prop and the power; the third, when the power is between the prop and the weight.

A power and weight acting upon the arms of a lever will balance each other, when the distance of the point at which the power is applied to the lever from the prop is to the distance of the point at which the weight is applied as the weight is to the power.

Therefore, to find what weight may be raised by a given power, we adopt the following

Rule. — *As the distance between the body to be raised, or balanced, and the fulcrum or prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will balance.*

1. If a man weighing 170 pounds be resting upon a lever 10 feet long, what weight will he balance on the other end, the prop being one foot from the weight? Ans. 1530lbs.

2. If a weight of 1530 pounds were to be raised by a lever 10 feet long, and the prop fixed one foot from the weight, what power applied to the other end of the lever would balance it? Ans. 170lbs.

3. If a weight of 1530 pounds be placed one foot from the prop, at what distance from the prop must a power of 170 pounds be applied to balance it? Ans. 9 feet.

4. At what distance from a weight of 1530 pounds must a prop be placed, so that a power of 170 pounds, applied 9 feet from the prop, may balance it? Ans. 1 foot.

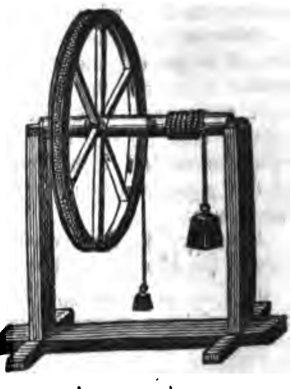
5. Supposing the earth to contain 4,000,000,000,000,000,000,000 cubic feet, each foot weighing 100 pounds, and that the earth was suspended at one end of a lever, its centre being 6000 miles from the fulcrum or prop, and that a man at the other end of the lever was able to pull, or press with a force of 200 pounds; what must be the distance between the man and the fulcrum, that he might be able *to move the earth?*

Ans. 12,000,000,000,000,000,000,000,000 miles.

6. Supposing the man in the last question to be able to move *his* end of the lever 100 feet per second, how long would it take him to raise the earth *one inch?*

Ans. 52,813,479,690y. 17d. 14h. 57m. $46\frac{2}{3}$sec.

THE WHEEL AND AXLE.



The *wheel and axle* is a wheel turning round together with its axle; the power is applied to the circumference of the wheel, and the weight to that of the axle by means of cords.

An equilibrium is produced in the wheel and axle, when the weight is to the power as the diameter of the wheel to the diameter of its axle.

To find, therefore, how large a power must be applied to the wheel to raise a given weight on the axle, we adopt the following

RULE. — *As the diameter of the wheel is to the diameter of the axle, so is the weight to be raised by the axle to the power that must be applied to the wheel.*

7. If the diameter of the axle be 6 inches, and the diameter of the wheel 4 feet, what power must be applied to the wheel to raise 960 pounds at the axle? Ans. 120lbs.

8. If the diameter of the axle be 6 inches, and the diameter of the wheel 4 feet, what power must be applied to the axle to raise 120 pounds at the wheel? Ans. 960lbs.

9. If the diameter of the axle be 6 inches, and 120 pounds applied to the wheel raise 960 pounds at the axle, what is the diameter of the wheel? Ans. 4 feet.

10. If the diameter of the wheel be 4 feet, and 120 pounds applied to the wheel raise 960 pounds at the axle, what is the diameter of the axle? Ans. 6 inches.

THE PULLEY.

The *pulley* is a small wheel, movable about its axis by means of a cord, which passes over it.

When the axis of a pulley is fixed, the pulley only changes the direction of the power; if movable pulleys are used, an equilibrium is produced when the power is to the weight as one to the number of ropes applied to them. If each movable pulley has its own rope, each pulley will be double the power.

To find the weight that may be raised by a given power.

RULE. — *Multiply the power by the number of cords that support the weight, and the product is the weight.*

11. What power must be applied to a rope, that passes over one movable pulley, to balance a weight of 400 pounds? Ans. 200lbs.

12. What weight will be balanced by a power of 10 pounds, attached to a cord that passes over 3 movable pulleys? Ans. 60lbs.

13. What weight will be balanced by a power of 144 pounds, attached to a cord that passes over 2 movable pulleys? Ans. 576lbs.

14. If a cord, that passes over two movable pulleys, be attached to an axle 6 inches in diameter, whose wheel is 60 inches in diameter, what weight may be raised by the pulley, by applying 144 pounds to the wheel? Ans. 5760lbs.

THE INCLINED PLANE.

An *inclined plane* is a plane which makes an acute angle with the horizon.

The motion of a body descending an inclined plane is uniformly accelerated.

The force with which a body descends an inclined plane, by the force of attraction, is to that with which it would descend freely, as the elevation of the plane to its length; or, as the size of its angle of inclination to radius.

To find the power that will draw a weight up an inclined plane.

RULE. — *Multiply the weight by the perpendicular height of the plane, and divide this product by the length.*

15. An inclined plane is 50 feet in length, and 10 feet in perpendicular height; what power is sufficient to draw up a weight of 1000 pounds? Ans. 200lbs.

16. What weight, applied to a cord passing over a single pulley at the elevated part of an inclined plane, will be able to sustain a weight of 1728 pounds, provided the plane was 600 feet long, and its perpendicular height 5 feet? Ans. $14\frac{2}{5}$lbs.

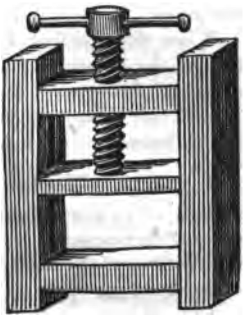
17. A certain railroad, one mile in length, has a perpendicular elevation of 50ft.; what power is sufficient to draw up this elevation a train of cars weighing 20,000lbs.? Ans. $189\frac{13}{33}$.

18. An inclined plane is 300 feet in length, and 30 feet in perpendicular height; what power is sufficient to draw up a weight of 2000 pounds? Ans. 200lbs.

19. An inclined plane is 1000 feet in length, and 100 feet in perpendicular height; what power is sufficient to draw up this elevation a weight of 5000 pounds? Ans. 500lbs.

20. What weight applied to and passing over a single pulley, at the elevated part of an inclined plane, will be able to sustain a weight of 7000lbs., provided the plane is 300 feet long and its perpendicular height 30 feet? Ans. 700lbs.

THE SCREW.



The *screw* is a cylinder, which has either a prominent part or a hollow line passing round it in a spiral form, so inserted in one of the opposite kind that it may be raised or depressed at pleasure, with the weight upon its upper, or suspended beneath its lower surface.

In the *screw* the equilibrium will be produced, when the power is to the weight as the distance between the two contiguous threads, in a direction parallel to the axis of the screw, to the circumference of the circle described by the power in one revolution.

To find the power that should be applied to raise a given weight.

RULE. — *As the distance between the threads of the screw is to the circumference of the circle described by the power, so is the power to the weight to be raised.*

NOTE. — One third of the power is lost in overcoming friction.

21. If the threads of a screw be 1 inch apart, and a power of 100 pounds be applied to the end of a lever 10 feet long, what force will be exerted at the end of the screw?

Ans. 75,398.20+lbs.

22. If the threads of a screw be ½ an inch apart, what power must be applied to the end of a lever 100 inches in length to raise 100,000 pounds? Ans. 79.5774+lbs.

23. If the threads of a screw be ½ an inch apart, and a power of 79.5774+ pounds be applied to the end of a lever 100 inches in length, what weight will be raised?

Ans. 100,000lbs.

24. If a power of 79.5774+ pounds be applied to the end of a lever 100 inches long, and by this force a weight of 100,000 pounds be raised, what is the distance between the threads of the screw? Ans. ½ an inch.

25. If a power of 79.5774+ pounds be applied to the end of a lever, raising by this force a weight of 100,000 pounds, what must be the length of the lever, if the threads of the screw be ½ an inch apart? Ans. 100 inches.

WEDGE.

The *wedge* is composed of two inclined planes, whose bases are joined.

When the resisting forces and the power which acts on the wedge are in equilibrio, the weight will be to the power as the height of the wedge to a line drawn from the middle of the base to one side, and parallel to the direction in which the resisting force acts on that side.

To find the force of the wedge.

RULE.—*As half the breadth or thickness of the head of the wedge is to one of its slanting sides, so is the power which acts against its head to the force produced at its side.*

26. Suppose 100 pounds to be applied to the head of a wedge that is 2 inches broad, and whose slant is 20 inches long, what force would be affected on each side?

Ans. 2000lbs.

27. If the slant side of a wedge be 12 inches long, and its head $1\frac{1}{2}$ inches broad, and a screw whose threads are $\frac{3}{4}$ of an inch asunder be applied to the head of this wedge, with a power of 200 pounds at the end of the lever, 16 feet long, what would be the force exerted on the sides of the wedge?

Ans. 5147184.3+lbs.

SECTION LXXXI.

SPECIFIC GRAVITY.*

To find the specific gravity of a body.

RULE. — *Weigh the body both in water and out of water, and note the difference, which will be the weight lost in water; then, as the weight lost in water is to the whole weight, so is the specific gravity of water to the specific gravity of the body. But if the body whose specific gravity is required is lighter than water, affix to it another body heavier than water, so that the mass compounded of the two may sink together. Weigh the dense body and the compound mass separately, both in water and out of it; then find how much each loses in water by subtracting its weight in water from its weight in air; and subtract the less of these remainders from the greater; then say, as the last remainder is to the*

* The specific gravity of a body is its weight compared with water; the water being considered 1000.

weight of the body in air, so is the specific gravity of water to the specific gravity of the body.

NOTE. — A cubic foot of water weighs 1000 ounces.

1. A stone weighed 10 pounds, but in water only $6\frac{1}{8}$ pounds. Required the specific gravity. Ans. 2608.6+.

2. Suppose a piece of elm weigh 15 pounds in air, and that a piece of copper, which weighs 18 pounds in air and 16 pounds in water, is affixed to it, and that the compound weighs 6 pounds in water. Required the specific gravity of the elm. Ans. 600.

SECTION LXXXII.

STRENGTH OF MATERIALS.

THE force with which a solid body resists an effort to separate its particles or destroy their aggregation can only become known by experiment.

There are four different ways in which the strength of a solid body may be exerted; first, by resisting a longitudinal tension; secondly by its resisting a force tending to break the body by a transverse strain; thirdly, in resisting compression, or a force tending to crush the body; and, fourthly, in resisting a force tending to wrench it asunder by torsion. We shall, however, only consider the strength of materials as affected by a transverse strain.

When a body suffers a transverse strain, the mechanical action which takes place among the particles is of a complicated nature. The resistance of a beam to a transverse strain is in a compound ratio of the strength of the individual fibres, the area of the cross section, the distance of the centre of gravity of the cross section from the points round which the beam turns in breaking.

The following are the facts and principles on which mechanics make their calculations.

1. A stick of oak one inch square and twelve inches long, when both ends are supported in a horizontal position, will sustain a weight of 600 pounds; and a bar of iron of the same dimensions will sustain 2190 pounds.

2. The strength of similar beams varies inversely as their

lengths; that is, if a beam 10 feet long will support 1000 pounds, a similar beam 20 feet long would support only 500 pounds.

3. The strength of beams of the same length and depth is directly as their width; that is, if there be two beams, each 20 feet long and 6 inches deep, and one of them is 6 inches wide and the other but 3 inches, the former will support twice the weight of the latter.

4. The strength of beams of the same length and width is as the squares of their depths; that is, if there be two beams, each of which is 20 feet long and 4 inches wide, but one is 6 inches deep and the other is 3 inches deep, their strength is as the squares of these numbers. Thus, $6 \times 6 = 36$; $3 \times 3 = 9$; that is, the strength of the former is to the latter as 36 to 9. It will, therefore, sustain four times the weight of the latter. Thus, $36 \div 9 = 4$.

5. To compare the strength of two beams of the same length, but of different breadth and depth, we multiply their widths by the squares of their depths, and their products show their comparative strength. Thus, if we wish to ascertain how much stronger is a joist that is 2 inches wide and 8 inches deep, than one of the same length that is 4 inches square, we multiply 2 by the square of 8, and 4 by the square of 4; thus, $2 \times 8 \times 8 = 128$; $4 \times 4 \times 4 = 64$; $128 \div 64 = 2$. Thus, we see that although the quantity of material in one joist is the same as in the other, yet the former will sustain twice the weight of the latter. Hence "deep joists" are much stronger than square ones, which have the same area of a transverse section.

6. To compare the strength of two beams of different lengths, widths, and depths, we multiply their widths by the squares of their depths, and divide their products by their lengths, and their quotients will show their comparative strength. Therefore, if we wish to ascertain how much stronger is a beam that is 20 feet long, 8 inches wide, and 10 inches deep, than one 10 feet long, 6 inches wide, and 5 inches deep, we adopt the following formulas: —

$$\frac{8 \times 10 \times 10}{20} = 40; \quad \frac{6 \times 5 \times 5}{10} = 15.$$

The strength of the former, therefore, is to the latter as 40 to 15; that is, if the first beam would sustain a weight of 40cwt., the latter would sustain only 15cwt.

7. Having all the dimensions of one beam given, to find

another, part of whose dimensions are known, that will sustain the same weight. We multiply the width of the given beam by the square of its depth, and divide this product by the length, and the result we call the *reserved quotient;* then, if we have the length and breadth of the required beam given to find the depth, we multiply the *reserved quotient* by the length of the required beam, and divide the product by its width, and the quotient is the square of the depth of the required beam. If the length and depth of the required beam were given to find the width, we multiply the *reserved quotient* by the length of the required beam, and divide this product by the square of the depth of the required beam, and the quotient is the breadth. But if the width and depth of the required beam were given to find the length, we multiply the width of the required beam by the square of the depth, and divide this product by the *reserved quotient*, and he result is the length of the required beam.

8. A triangular beam will sustain twice the weight with its edge *up* that it will with its edge *down*. Hence split-rails have twice the strength with the narrow part upward, which they have with the narrow part downward.

9. In making the above calculations, we have not noticed the *weight* of the beam itself, and in short distances it is of but little consequence; but where a long beam is required, its weight is of importance in the calculation.

10. A beam supported at one end will sustain only one fourth part the weight which it would if supported at both ends.

11. The tendency to produce fracture in a beam by the application of a weight is greatest in the centre, and decreases towards the points of support; and this ratio varies as the square of half the length of the beam to the product of any two parts where the weight may be applied. Hence the tendency of a weight to break a bar 8 feet long, when applied to the centre, to that of the same weight, when applied 3 feet from one end, is as $4 \times 4 = 16$ to $3 \times 5 = 15$.

QUESTIONS TO BE PERFORMED BY THE PRECEDING RULES.

1. If a stick of oak 1 inch square and 12 inches long, when both ends are supported in a horizontal position, will sustain a weight of 600 pounds, how many pounds would a similar stick sustain, that was 36 inches long? Ans. 200 pounds.

2. If a beam 4 inches square and 12 feet long would support a weight of 1000 pounds, how many pounds would a similar beam support, that was 3 feet long? Ans. 4000 pounds.

3. If a beam 20ft. long, 4in. wide, and 6in. deep, will sustain a weight of 2000lbs., how many pounds would a similar beam sustain, that was 3in. wide? Ans. 1500lbs.

4. If a beam 10 feet long, 4 inches wide, and 3 inches deep, will sustain a weight of 1000 pounds, how many pounds would a similar beam support, that was 6 inches deep?
Ans. 4000 pounds.

5. If a beam 10 feet long, 2 inches wide, and 4 inches deep, will sustain a weight of 1000 pounds, how many pounds would a beam, that is 10 feet long, 4 inches wide, and 6 inches deep, sustain? Ans. 4500 pounds.

6. If a beam 2 feet long, 2 inches wide, and 3 inches deep, will sustain a weight of 4000 pounds, what will a beam, that is 4 feet long, 3 inches wide, and 6 inches deep, sustain?
Ans. 12000 pounds.

7. If a beam that is 10 feet long, 4 inches wide, and 6 inches deep, will sustain a weight of 4 tons, what weight will a beam sustain, that is 20 feet long, 8 inches wide, and 10 inches deep?
Ans. $11\frac{1}{9}$ tons.

8. If a beam 6 inches square and 8 feet long will support a weight of 2000 pounds, what weight will a beam 10 feet long and 10 inches square sustain? Ans. $7407\frac{11}{27}$ pounds.

9. If a beam 15 feet long and 5 inches square will sustain a weight of 1200 pounds, required the length of a beam, that is 8 inches square, that will sustain a weight of 2000 pounds.
Ans. $36\frac{108}{125}$ feet.

10. If a beam 8 feet long and 7 inches square will sustain a weight of 3000 pounds, how many inches square must be the beam, that is 6 feet long, that will sustain 2000 pounds?
Ans. 5.5+ inches.

11. If a bar of iron 10 feet long, 2 inches wide, and 3 inches deep, will sustain 10 tons, what must be the depth of a bar, that is 12 feet long and 3 inches wide, that will sustain 30 tons?
Ans. 4.64+ inches.

12. If it require 1000 pounds to break a certain beam, that is 24 feet long, when placed in its centre, required the weight necessary to break it, when placed within 4 feet of one end of the beam? Ans.

13. If a beam 6 inches square and 10 feet long will sustain a weight of 2000 pounds from its centre, what weight would a beam of the same material sustain that is 10 inches square and 12 feet long, if the weight were suspended 2 feet from the centre? Ans.

Section LXXXIII.

ASTRONOMICAL PROBLEMS.

PROBLEM I.

To find the dominical letter for any year in the present century, and also to find on what day of the week January will begin.

Rule. — *To the given year add its fourth part, rejecting the fractions; divide this sum by 7; if nothing remains, the dominical letter is A; but if there be a remainder, subtract it from 8, and the residue will show the dominical letter, reckoning 0 = A, 2 = B, 3 = C, 4 = D, 5 = E, 6 = F, 7 = G. These letters will also show on what day of the week January begins. For when A is the dominical letter, January begins on the Sabbath; when B is the dominical letter, January begins on Saturday; C begins it on Friday; D, on Thursday; E, on Wednesday; F, on Tuesday; G, on Monday.*

Note. — If it be required to find the dominical letter for the last century, proceed as above; only, if there be a remainder after division, subtract it from 7, and the remainder shows the dominical letter, reckoning 1 = A, 2 = B, 3 = C, 4 = D, 5 = E, 6 = F, 0 = G.

1. Required the dominical letter for 1835.

OPERATION.

$$\begin{array}{r} 4)1835 \\ \underline{458} \\ 7)\overline{2293} \\ \overline{327-4} \end{array}$$

8 — 4 = 4 = D = dominical letter.

As D is the dominical letter, January began on Thursday, and the fourth day was the Sabbath.

2. Required the dominical letter for 1836.

OPERATION.

$$\begin{array}{r} 4)1836 \\ 459 \\ 7)\overline{2295} \\ \overline{327}-6 \end{array}$$

8 — 6 = 2 = B and C = dominical letters.

In leap years there are two dominical letters. The last letter, C, is for January and February, and B for the remainder of the year. As C is the dominical letter, January began on Friday, and the third day was the Sabbath.

3. Required the dominical letter for 1841? Ans. C.
4. Required the dominical letter for 1899. Ans. A.
5. Required the dominical letters for 1896. Ans. D and E.
6. What is the dominical letter for 1786? Ans. A.
7. What is the dominical letter for 1837? Ans. A.

PROBLEM II.

To find on what day of the week any given day of the month will happen.

RULE. — *Find by the last problem the dominical letter for the given year, and on what day in January will be the first Sabbath; and the corresponding day in the succeeding months will be as follows: — Wednesday for February; Wednesday for March; Saturday for April; Monday for May; Thursday for June; Saturday for July; Tuesday for August; Friday for September; Sabbath for October; Wednesday for November; Friday for December. Having found the day of the week for any day in the month, any other day may be easily obtained, as may be seen in the following example.*

8. Let it be required to ascertain on what day of the week was the 25th day of September, 1838.

The dominical letter for 1838 is G; therefore, the 7th of January was the Sabbath, and, by the above rule, the 7th of February was Wednesday, the 7th of March was Wednesday, the 7th of April was Saturday, the 7th of May was Monday, the 7th of June was Thursday, the 7th of July was Saturday, the 7th of August was Tuesday, the 7th of September was Friday. If the 7th was Friday, the 14th, the 21st, and the 28th were Fridays. And if the 21st was Friday, the 22d was Saturday, the 23d was the Sabbath, the 24th was Monday, and the 25th, the day required, was Tuesday.

The following distich will assist the memory in finding the corresponding days of the month: —

At Dover Dwells George Brown Esquire,
Good Christian Friend, And David Friar.

It will be recollected, that the initial A is for the Sabbath, B for Monday, C for Tuesday, D for Wednesday, E for Thursday, F for Friday, and G for Saturday.

NOTE. — When it is leap year, the days of the week, after February, will be one day later than on other years.

9. Required the day of the week for the 4th of July, 1836.

We find the dominical letters to be B and C, the 3d day of January therefore was on the Sabbath, and by the above rule the 3d day of July would have been on Saturday; but as it was leap year, the 3d of July was one day later; that is, it was on the Sabbath; and, if the 3d was the Sabbath, the 4th of July was on Monday.

10. On what day of the week will be Dec. 8, 1849?

Ans. Saturday.

11. On what day will happen July 4, 1857? Ans. Saturday.

12. On what day will March begin in the year 1890?
Ans. Saturday.

13. On what day of the week was our Independence declared? Ans. Thursday.

14. There will be a "Transit of Venus," December 8, 1874; on what day of the week will it happen? Ans. Tuesday.

15. On what day of the week were you born?

Section LXXXIV.

MISCELLANEOUS QUESTIONS.

1. What number must $7\frac{3}{5}$ be multiplied by, that the product may be $6\frac{2}{3}$? Ans. $\frac{50}{57}$.

2. What number is that, which, being multiplied by half itself, the product shall be $4\frac{1}{2}$? Ans. 3.

3. What fraction is that, which, being divided by $11\frac{3}{7}$, the quotient shall be 5? Ans. $57\frac{1}{7}$.

4. What part of $7\frac{3}{4}$ is $9\frac{4}{7}$? Ans. $\frac{268}{217}$ or $1\frac{51}{217}$.

5. Reduce $\dfrac{7}{19\frac{3}{5}}$ to a simple fraction. Ans. $\frac{5}{14}$.

6. Add $\frac{4}{7}$ of a ton to $\frac{9}{10}$ of a cwt. Ans. 12cwt. 1qr. $8\frac{4}{5}$lb.

7. If the earth make one complete revolution in 23 hours 56 minutes 3 seconds, in what time does it move one degree?
Ans. 3m. 59″ 20‴.

8. Multiply $\dfrac{\frac{3}{4}}{\frac{7}{8}}$ of $9\dfrac{\frac{1}{7}}{\frac{5}{6}}$ by $\frac{4}{5}$ of $\dfrac{\frac{7}{8}}{\frac{1}{5}}$ of $8\frac{3}{4}$. Ans. $240\frac{3}{4}$.

9. Divide $12\dfrac{\frac{5}{12}}{7}$ of $\dfrac{\frac{2}{5}}{\frac{9}{10}}$ of 100 by $\dfrac{\frac{3}{8}}{\frac{11}{12}}$ of $7\frac{1}{5}$.
Ans. $181\frac{14821}{15309}$.

10. What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added the sum will be 1? Ans. $\frac{53}{63}$.

11. A certain gentleman, at the time of his marriage, agreed to give his wife $\frac{2}{3}$ of his estate, if, at the time of his death, he left only a daughter, and, if he left only a son, she should have $\frac{1}{2}$ of his property; but, as it happened, he left a son and a daughter, by which the widow received in equity \$2400 less than if there had been only a daughter. What would have been his wife's dowry if he had left only a son? Ans. \$2100.

12. A gentleman being asked what o'clock it was, said that

it was between 5 and 6; but, to be more particular, he said that the minute hand had passed as far beyond the 6 as the hour hand wanted of being to the 6; that is, that the hour and minute hands made equal acute angles with a line passing from the 12 through the 6. Required the time of day. Ans. 32m. 18$\frac{6}{13}$sec. past 5.

13. A, B, and C are to share $ 100,000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively; but C's part being lost by his death, it is required to divide the whole sum, properly, between the other two. Ans. A's part is $ 57,142$\frac{6}{7}$, and B's $ 42,857$\frac{1}{7}$.

14. A father devised $\frac{7}{18}$ of his estate to one of his sons, and $\frac{7}{18}$ of the residue to the other, and the remainder to his wife. The difference of his sons' legacies was found to be 257£. 3s. 4d. What money did he leave for his widow?

Ans. 635£. 0s. 10$\frac{30}{49}$d.

15. In the walls of Balbec, in Turkey, the ancient Heliopolis, there are three stones laid end to end, now in sight, that measure 61 yards in length; one of which is 63 feet long, 12 feet thick, and 12 feet broad; what is its weight, supposing its specific gravity to be 3 times that of water? Ans. 759$\frac{3}{8}$ tons.

16. A burden of 200lbs., suspended on a pole 4ft. in length, the point of suspension being 6in. from the middle, is carried by two men, the ends of the pole resting on their shoulders; how much of this load is borne by each man? Ans. 125lbs. and 75lbs.

17. The new court-house in Boston has 8 pillars of granite, each 25ft. 4in. in length, 4ft. 5in. in diameter at one end, and 3ft. 5in. in diameter at the other end. How many cubic feet do they contain, and what is their weight, allowing a cubic foot to weigh 3000 ounces? Ans. 2455.03 cubic feet, 205.4 tons.

18. A father, dying, left his son a legacy, $\frac{1}{4}$ of which he spent in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer; after which he had only $ 410 left. What did his father bequeathe him? Ans. $ 956.66$\frac{2}{3}$.

19. A butcher, wishing to buy some sheep, asked the owner how much he must give him for 20; on hearing his price, he said it was too much; the owner replied, that he should have 10, provided he would give him a cent for each different choice of 10 in 20, to which he agreed. How much did he pay for the 10 sheep, according to the bargain? Ans. $ 1847.56.

20. A merchant sold goods to a certain amount, on a commission of 4 per cent., and, having remitted the net proceeds to the owner, he received $\frac{1}{2}$ per cent. for prompt payment, which amounted to $ 15.60. What was the amount of his commission? Ans. $ 260.00.

21. A, of Boston, remits to B, of New York, a bill of exchange on London, the avails of which he wishes to be invested in goods on his account. B having disposed of the bill at $7\frac{1}{2}$ per cent. advance, he received \$ 9675.00, and having reserved for himself $\frac{1}{4}$ per cent. on the sale of the bill, and 2 per cent. for commission, what will remain for investment, and for how much was the bill drawn?

Ans. For investment, \$9461.58$\frac{3}{34}$; the bill was £2025.

22. Bunker Hill Monument is 30ft. square at its base, and 15ft. square at its top; its height is 220ft. From the bottom to the top, through its centre, is a conical avenue 15ft. in diameter at the bottom and about 11ft. at the top. How many cubic feet are there in the monument? Ans. 86,068.518+ft.

23. A hired a house for one year for \$ 300; at the end of 4 months he takes in M as a partner, and at the end of 8 months he takes in P. At the end of the year what rent must each pay?

Ans. A pays \$ $183\frac{1}{3}$; M pays \$ $83\frac{1}{3}$; P pays \$ $33\frac{1}{3}$.

24. A merchant receives on commission three kinds of flour; from A he receives 20 barrels, from P 25 barrels, and from C 40 barrels. He finds that A's flour is 10 per cent. better than B's, and that B's is 20 per cent. better than C's. He sells the whole at \$ 6 per barrel. What in justice should each man receive?

Ans. A receives \$ $139\frac{161}{241}$; B, \$ $158\frac{172}{241}$; C, \$ $211\frac{149}{241}$.

25. Bought 100 barrels of flour, at \$ 5 per barrel, and immediately sold it on a credit of 6 months. The note which I received for pay, I got discounted at the Suffolk Bank, and, on examining my money, I found that I had gained 20 per cent. on my purchase. What did I receive per barrel for the flour?

Ans. \$ $6.18\frac{1698}{1939}$.

26. Purchased for a cloak $5\frac{1}{4}$ yards of broadcloth, that was $1\frac{7}{8}$ yards wide; to line this, I purchased flannel that was $\frac{7}{8}$ yard wide, but on being wet it shrunk 1 nail in width, and 1 yard in every 20 yards in length. How many yards of flannel was it necessary to buy? Ans. $12\frac{186}{247}$ yards.

27. How many bricks would it require to build the walls of a house 40 feet long, 30 feet wide, and 20 feet high, admitting the walls to be a foot thick, and that each brick was 8 inches long, 4 inches wide, and 2 inches thick? Ans. 73,440 bricks.

28. How many feet of boards would it require to cover a house, that was 40 feet square, and whose height to the top of the plate was 20 feet, the roof projecting 1 foot over the plate, and coming to a point over the centre of the house, 15 feet above the garret floor? Ans. 5367.7+ feet.

29. Lent a friend \$700, which he kept 20 months. Some years after, I borrowed of him \$300; how long should I keep it to balance the favor? Ans. $46\frac{2}{3}$ months.

30. John Lee gave half of his estate to his wife, $\frac{1}{5}$ of the remainder to his oldest son, and $\frac{1}{4}$ of the residue to his oldest daughter, and $\frac{1}{2}$ of what then remained, which was \$1500, to be equally distributed among his other children, who received \$150 each; required the number of his children, and the value of his estate. Ans. 12 children; estate, \$10,000.

31. A and B set out to travel round a certain island, which is 80 miles in circumference. A travels 5 miles a day, and B 7 miles a day. How far must B travel to overtake A?
Ans. 280 miles.

32. If 24.4 cubic inches of lead weigh 16 pounds, required the number of feet of lead pipe that can be made from 80 pounds of lead, the diameter of the pipe to be 1 inch, and the thickness of it $\frac{1}{4}$ of an inch. Ans. 124.26+ feet.

33. How long a tube can be made from a cylinder of lead 8 inches long and 2 inches in diameter, and through the centre of which is a hole $\frac{3}{4}$ of an inch in diameter; the tube or pipe to be $\frac{3}{4}$ of an inch in diameter, and $\frac{3}{8}$ of an inch in thickness?
Ans. 16.29+ feet.

34. Four men, A, B, C, and D, bought a stack of hay, containing 8 tons, for \$100. A is to have 12 per cent. more of the hay than B, and B is to have 10 per cent. more than C, and C is to have 8 per cent. more than D. Each man is to pay in proportion to the quantity he receives. The stack is 20 feet high, and 12 feet wide at its base, it being an exact pyramid; and it is agreed that A shall take his share first from the top of the stack, B is to take his share the next, and then C and D. How many feet of the perpendicular height of the stack shall each take, and what sum shall each pay?

Ans. A takes 13.22+ft., and pays \$$28.93\frac{12987}{28741}$; B takes 3.14+ft., and pays \$$25.83\frac{11297}{28741}$; C takes 2.06+ft. and pays \$$23.48\frac{16132}{28741}$; D takes 1.58+ft., and pays \$$21.74\frac{17066}{28741}$.

35. Suppose the rails of a railroad to be 5 feet 4 inches apart at the place of the wheels bearing, and on a curve line of 1200 feet radius for the outer rail. Suppose the wheels of the car running on the rails to be firmly fixed to the axle, and that it is 5 feet from the outside of the flange of one wheel to the outside of the flange of the opposite wheel; that from the outer side of one wheel to the outer side of its opposite wheel is 5 feet 8 inches; that the diameter of each wheel at the outside

of the flange is 3 feet; the face of the wheels to be so bevelled that at the outside of each wheel the diameter of the wheel is 2 feet 11.5 inches, and that the axle will always be in a position square across the two rails. In what part, between the two wheels, must the centre of gravity of the load be placed, so that the weight of the load shall bear equally on each rail?

OPERATION.

5ft. 4in. — 5ft. = 4in.; and 4in. ÷ 2 = 2in. = the place, beyond the flange, on the face of the wheel, where the wheel bears on the rail. The perpendicular width of the face of the wheel is 4 inches, and in that distance the wheel diminishes in diameter 3ft. — 2ft. 11.5in. = .5in. Then, as 4in. : .5in. : : 2in. : .25in.; and so the diameter of the wheel at the bearing is 3ft. — .25in. = 2ft. 11.75in. As the radius of the curve of the outer rail is to that of the inner rail, so is the diameter of the outer wheel at the place of bearing to that of the inner wheel; viz. as 1200ft. : $\overline{\text{1200ft.} - \text{5ft. 4in.}}$ = 1194ft. 8in. : : 2ft. 11.75in. : 2ft. $11.59\dot{1}$in. Then the difference of the diameters of the two wheels at the places of bearing is 35.75in. — $35.59\dot{1}$ = $.15\dot{8}$in. Then, to see how much on the perpendicular face of the wheel this difference in diameter will vary the bearing; as $\overline{\text{3ft.} - \text{2ft. 11.5in.}}$ = .5in. : 4in. : : $.15\dot{8}$in. : $1.27\dot{1}$in. But this difference must be applied half to each wheel, viz. $1.27\dot{1}$in. ÷ 2 = $.63\dot{5}$in., which brings the place of bearing .635in. nearer to the flange on the outer wheel, and .635in. further from the flange on the inner wheel. And the middle point between the two bearings will be .635in. from the centre of the axle towards the inner curve; and in that place must the centre of gravity of the whole load be placed, to make the weight of the load bear equally on each rail; viz. .635in. from the middle of the axle towards the inner rail, Answer.

36. The dimensions of a bushel measure are 18½ inches wide, and 8 inches deep; what should be the dimensions of a similar measure that would contain 8 bushels?

Ans. 37in. wide, 16in. deep.

37. What is the weight of a hollow spherical iron shell 5in. in diameter, the thickness of the metal being 1in., and a cubic inch of iron weighing $\frac{128}{500}$ of a pound? Ans. 13.2387lbs.

38. At a certain time between 2 and 3 o'clock, the minute hand was between 3 and 4. Within an hour after, the hour

hand and minute hand had exactly changed places with each other. What was the precise time when the hands were in the first position? Ans. 2h. 15m. $56\frac{92}{143}$sec.

39. Required the contents of a cube, that will contain a globe 20 inches in diameter; also of a cube, that may be inscribed in said globe.

Ans. Larger cube 8000 cub. in.; smaller, 1539 cub. in.

40. If in a pair of scales a body weigh 90 pounds in one scale, and only 40 pounds in the other, required the true weight, and the proportion of the lengths of the two arms of the balance-beam on each side of the point of suspension.

Ans. the weight 60lbs., and the proportions 3 to 2.

41. In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one; the wheels were each 4 feet high; and supposing them fixed at the distance of 5 feet asunder on the axletree, what was the circumference of the track described by the outer wheel? Ans. 62.83+ feet.

42. The ball on the top of St. Paul's church is 6 feet in diameter. What did the gilding of it cost, at $3\frac{1}{2}$d. per square inch? Ans. 237£. 10s. 1d.

43. There is a conical glass, 6 inches high, 5 inches wide at the top, and which is $\frac{1}{5}$ part filled with water. What must be the diameter of a ball, let fall into the water, that shall be immersed by it? Ans. 2.445+in.

44. A certain lady, the mother of three daughters, had a farm of 500 acres, in a circular form, with her dwelling-house in the centre. Being desirous of having her daughters settled near her, she gave to them three equal parcels, as large as could be made in three equal circles, included within the periphery of her farm, one to each, with a dwelling-house in the centre of each; that is, there were to be three equal circles, as large as could be made within a circle that contained 500 acres. How many acres did the farm of each daughter contain, how many acres did the mother retain, how far apart were the dwelling-houses of the daughters, and how far was the dwelling-house of each daughter from that of the mother?

Ans. Each daughter's farm contained 107 acres 2 roods 31.22+ rods. The mother retained 176 acres 3 roods 26.34+ rods. The distance from one daughter's house to the other was 148.119817+ rods. The mother's dwelling-house was distant from her daughters' 85.51+ rods.

45. Required the cube of $\frac{35\frac{1}{7}}{52\frac{5}{7}}$. Ans. $\frac{8}{27}$.

46. Required the cube root of $\frac{78\frac{2}{5}}{264\frac{3}{5}}$. Ans. $\frac{2}{3}$.

47. Multiply the cube root of $\frac{66\frac{2}{11}}{1034\frac{1}{11}}$ by the square of $\frac{69\frac{5}{7}}{87\frac{1}{7}}$.
Ans. $\frac{32}{125}$.

48. Add $\frac{14\frac{4}{9}}{21\frac{2}{3}}$ of $\frac{65\frac{5}{8}}{9\frac{3}{8}}$ to $\frac{13\frac{1}{2}}{31\frac{1}{2}}$ of $\frac{231\frac{1}{4}}{37}$. Ans. $7\frac{29}{84}$.

49. Three carpenters, J. Smith, J. Carleton, and John Jones, agree with T. Jenkins to build him a house and find the materials for \$ 1000, of which \$ 600 were paid in advance, and the remainder when the work was finished. Carleton and Jones took \$ 50 each of the first payment. When the work was completed, it appeared by Smith's account, who received the money and paid the bills, for which he was allowed a compensation of \$ 10, that he had paid \$ 648.95, exclusive of the payments to Carleton and Jones, and that he had labored 63 days. Carleton worked 51 days, and he was allowed \$ 20 for the use of his shop, &c. Jones worked 60 days, and his bill for boarding the men they hired was \$ 68.75. Smith, on settling with Jenkins and allowing him \$ 23.15 charged to Carleton, and \$ 17.48 charged to Jones, receives the balance in cash, and on exhibiting his statement of the business to Carleton and Jones, he pays to each the balance due. How much did they make per day, and how was the last payment disposed of?

Ans. They received \$ 1.45 per day, and Carleton received \$ 20.80, Jones received \$ 88.27, and Smith received \$ 250.30.

50. A and B engaged to reap a field for 90 shillings; and as A could reap it in 9 days, they promised to complete it in 5 days. They found, however, that they were obliged to call in C, an inferior workman, to assist them for the last two days, in consequence of which B received 3s. 9d. less than he otherwise would have done. In what time could B and C each reap the field? Ans. B could reap it in 15 days, and C in 18 days.

51. Samuel Jenkins and James Betton, who have each an apprentice, engage to build a small house for \$ 630. By agreement between them, Jenkins's apprentice is to be allowed \$ 0.62½ per day, and Betton's \$ 1.00. When the work was finished, it appeared that Jenkins had worked 120 days, and his apprentice 100. Betton worked 96 days, and his apprentice

$135\frac{1}{4}$ days. While doing the work, they received each \$210 What is each man's share of the remaining payment?

Ans. Due to Jenkins \$92.50; to Betton \$117.50.

52. A merchant tailor bought 40 yards of broadcloth, $2\frac{1}{4}$ yards wide; but on sponging it, it shrunk in length upon every 4 yards half a quarter, and in width, one nail and a half upon every $1\frac{1}{2}$ yards. To line this cloth, he bought flannel 5 quarters wide, which, being wet, shrunk the whole width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of flannel used in lining the cloth. Ans. $71\frac{7}{13}$ yards.

53. What is the square of $.\dot{1}$?

54. If a stick of timber, 6 inches square and 10 feet long, will support from its centre 1000 pounds, how many pounds would a similar stick, that is 10 inches square and 20 feet long, support, if the weight were suspended 4 feet from the centre of the stick?

55. A certain gentleman has an elegant lamp, to which are appended 72 brilliants, each of which had occupied a *particular* place. His daughter, one day, in preparing the lamp, displaced the brilliants, and in replacing them, found that she had put some of them in the wrong situation, and was at a loss to know how to correct the mistake. At length she told her father that, after dinner, she would begin and place the brilliants in all the situations they would admit of, and then she would be sure of finding the correct way of adjusting them, and that she would not take her tea until she had effected it. Now, supposing she were to place them in all the various ways they would admit, how long would she be obliged to wait for her tea, provided she could make one change each minute?

Ans. 61234458376886086861524070385274672740778091784697328983823014963978384987221689274204160000000000000000 minutes.

56. I have a garden in the form of an equilateral triangle, whose sides are 200 feet. At each corner stands a tower; the height of the first is 30 feet, the second 40, and the third 50. At what distance from the base of each tower must a ladder be placed, that it may just reach the top of each? and what is the length of the ladder, the garden being a horizontal plane?

Ans. The foot of the ladder from the base of the first tower 118.811+ feet; second tower, 115.827+ feet; third tower, 111.875+ feet. Length of the ladder, 122.535+ feet.*

* The operation of this example may be found on page 153 of the Key.

APPENDIX.

WEIGHTS AND MEASURES,

WITH AN HISTORICAL ACCOUNT OF STANDARDS

In commerce and in science, all bodies are estimated by number, weight, or measure. When reckoned by number, they are referred to a unit as a standard of comparison, and when estimated by weight or measure, there is always a reference to some certain fixed quantity, as a pound, a gallon, a mile, to which the quantity measured or weighed bears a specified and definite proportion.

Weights are used to ascertain the gravity of bodies, and measures to determine their magnitude, or the space which they occupy.

Standards of weights or measures are certain quantities of gravity or extension, which are fixed upon as those with which all other quantities of objects reckoned by weight or measure are to be compared; and such standards have always been found necessary, and have existed in every age and nation. It has, however, been only in a highly civilized state of society that they have been such as to secure an accurate and equitable result in the transaction of business; and few things are more indicative of a cultivated age and people, than the exactness with which science provides and adjusts the standards of comparison, required by the sale and interchange of commodities.

In the early stages of society, the ordinary standards of weight and measure were some simple objects or ideas, with which all in the community were supposed to be familiar. Thus, all measures of length were sometimes reckoned by comparing them with the human foot; or, for the sake of greater definiteness, as all human feet were not of the same length, they were referred to the *king's* foot as a standard. In some cases the length of the arm was used, and in other instances the length of a grain or corn of wheat or barley. In land measure, an acre was what could be ploughed by a yoke of oxen in a day, traces of which notion we have in the Hebrew and Latin words used to denote an acre, which properly signify a yoke or pair, — that is, a yoke or pair of oxen.

In early ages, also, weights as well as distances were meas-

ured by grains of corn,* and hence, in England and some other European countries, the lowest denomination of weight is still a grain. Originally, 32 of these grains were reckoned to a pennyweight, but in later times the number was fixed at 24.

A scruple meant originally a small stone, which was regarded as equal to 20 grains, and a dram (Greek *drachma*) was literally what one could hold in his hand,— the word being derived from a verb signifying to grasp with the hand.

With standards thus variable and uncertain, it is evident that no people could advance far in commerce, and as facilities for commercial intercourse opened, they must adopt some more satisfactory methods of ascertaining the quantity and value of what they bought and sold. Some fixed and permanent standards of comparison were needed, to which all might refer, and in which all should have confidence. Accordingly, kings and legislators early gave attention to this subject, and endeavoured, not only to provide such standards, but to preserve them from alteration. In Rome, the model weights and measures were kept for safety in the temple of Jupiter; and among the Jews, they were preserved first in the Tabernacle, and afterwards in the Temple, and their custody committed to the sacerdotal family. In England, the standard yard, to which, as we shall see, all the legal measures and even weights of the kingdom are ultimately referred, has for ages been kept with the greatest care by the government. This yard,† as history informs us, was introduced or revived as a standard of measure by Henry I., who lived at the beginning of the 12th century, and who ordered that the ulna or ancient ell, answering to the modern yard, should be of the exact length of his own arm, and all other measures founded upon it. This is the historical origin of the present English standard of measure, and it is said to have been preserved to the present time without any sensible variation.

In 1742, the Royal Society caused a yard to be made from a very careful comparison of the standard ells or yards of the reigns of Henry VII. and Elizabeth, which had been kept at the Exchequer. In 1758, a committee of the House of Commons recommended that a rod which had been made by their order from that of the Royal Society, and marked "Standard Yard, 1758," should be declared the legal standard of all measures of length. This rod consisted of a solid brass bar a little

* Corn in the English sense, meaning wheat, barley, &c., and not our Indian corn or maize.

† The word *yard* meant originally a rod or shoot, and cannot be supposed to have had any very definite length.

more than an inch square, and 39.06 inches long. At about an inch and a half from each end, there was inserted a gold pin or stud, and in these pins, at the distance of 36 inches from each other, are two points, which are intended to designate the exact length of the yard. The subject was further considered by another committee, and a second rod made by the same man, and as an exact copy of the above, which is known as the "Standard Yard of 1760," and which was declared by Parliament in the "Act of Uniformity," in 1824, which took effect January 1, 1826, to be the legal standard of measure for Great Britain. As the distance between the two fixed points will vary with the temperature of the rod, the Act provides that it shall be at the temperature of 62° Fahrenheit.

This standard yard, which was to be denominated the "Imperial Yard," was intended to afford a fixed standard of measure, which could only be lost with the destruction or mutilation of the standard rod. But this, it was foreseen, might easily occur, as in fact it did happen in 1834, at the burning of the two Houses of Parliament, when the imperial model shared the fate of the other valuable relics which were consumed in that ancient pile; and to provide against such an accident, it was declared that the imperial or standard yard, as compared with a pendulum vibrating seconds in the latitude of London, should bear the proportion of 36 to 39.1393 inches. To procure further accuracy, this pendulum was to move in a vacuum, at the level of the sea, and at the temperature of 62° Fahrenheit. And thus was fixed an absolute and invariable standard of linear measure, by which, according to the Act above named, all measures of extension are determined, whether the same be lineal, superficial, or solid. A third part of this standard yard is a legal foot, a twelfth part of the foot a legal inch, 5½ such yards a pole, 220 a furlong, and 1760 a mile. A legal rood of land contains 1210 square yards, and an acre 4840 square yards, or 160 square poles.

By the same Act, all measures of capacity are determined by the imperial gallon, which contains 277.274 cubic inches. Two such gallons make a peck, one fourth of a gallon a quart, &c. As this standard gallon is determined by cubic inches, we see that it is ultimately referred to the standard yard as the basis on which it is founded. The Act provides, however, that it may also be determined by weight, in which case the measure to contain a gallon must be of a capacity to hold 10 pounds, avoirdupois weight, of distilled water, weighed in air,

at the temperature of 62° Fahrenheit, the barometer being at 30 inches.

For a standard of weights, the law of England makes the pound Troy* to contain 5760 grains, — one cubic inch of distilled water weighed as above, weighing 252.458 such grains, — and the pound Avoirdupois to contain 7000 such grains.

The Act of Parliament which brought the standards of English weights and measures to their present state of comparative perfection was passed in 1835. This Act prohibits the use of the well-known Winchester bushel, and also of heaped measure. The subject of weights and measures is still under consideration by scientific men in England, and further improvements have been proposed, and will probably be introduced. It has even been suggested by commissioners appointed by government, that the system of *coinage* be changed, and one having decimal proportions adopted.

The system of weights and measures established by law in the United States is essentially the same as the English; but there is not, even at this day, that uniformity of practice in this country, which the interests of extensive trade require. At the organization of the Federal Government, authority was given to Congress to regulate this important matter, but no laws have as yet been enacted by that body to secure a uniform system through all the States. By an order of Congress, in June, 1836, a set of standard weights and measures was prepared for the use of each custom-house, and for each State, and these constitute the legal standards of the nation, so far as any national standard can be said to exist. The standards thus provided were similar to those used in England prior to the Act of 1824, and they lack that accuracy and scientific character which are exceedingly desirable in a country so extensive and so commercial as the United States. Many of the States of the Union have attempted to reduce their weights and measures to a uniform system, and not a little legislation has been directed to this end; but generally with very little effect. And until Congress shall take up the matter in good earnest, and with the aid of scientific men, but little can be done to secure a result so important.

* The origin of the words Troy and Avoirdupois has been variously given, but the most probable explanation is this: that the former is derived from *avoirs* (*averia*), the ancient name for goods or chattels, and *poids* (weight.) The word Troy has probably no reference to a town in France, as was formerly supposed, but, as applied to weight, originated in the monkish name of Troy Novant, which was given to London, and founded on an ancient legend; so that *Troy* weight is properly *London* weight.

To France belongs the credit of having established a system of weights and measures more simple and uniform than that possessed by any other nation. This system, which was instituted in 1795, is based upon the length of a quadrant of a meridian, or the distance from the equator to the poles of the earth. This distance being ascertained with the greatest care and exactness, a ten-millionth part of it was assumed as the *mètre* or unit of length, and this is the standard for all lineal measures. The *mètre* was found to correspond very nearly with the old French yard, being equal to 3.07844 French feet, or 3.281 English feet, and by accidental similarity is very nearly equal to the length of the pendulum vibrating seconds.

The unit of the French WEIGHT is the *gramme*, which is equal in weight to a cubic *centimètre*, that is, the 100th part of a *mètre* of pure water, at the temperature of melting ice, which is equal to 15.434 English Troy grains.

One object which was kept in view in the formation of the French metrical system was, that all quantities might be expressed by whole numbers and decimals, without the necessity of separating them into classes of units distinct from the local values which the numeral figures have in the ordinary arithmetical notation; and this was accomplished by forming the classes of units of weight and measure according to the decimal scale. Thus, the *mètre* being the original unit, the next superior unit is the *decamètre*, which is equal to 10 *mètres;* the next above this is the *hectomètre*, equal to 100 *mètres*, &c., as will be seen in the table of French weights and measures. This rise of successive units in a decuple progression is generally expressed by Greek prefixes, as above, *deca* (Greek *deka*), meaning ten, and *hecto* (Greek *hekaton*), one hundred, while to express the classes of units inferior to the *mètre*, Latin prefixes are adopted, as *décimètre*, the 10th part of a *mètre*, *centimètre*, the 100th part, &c., *decem* and *centum* being the Latin words for ten and a hundred.

Besides the metrical system, which is thus accurate and convenient, there is in France what is called the *Système Usuel*, or common system, which was established in 1812 for the purposes of retail trade, on account of the aversion of the common people to the innovation of the metrical system. This latter system "tolerates the names of the old measures necessary in the inferior departments of trade, while, by a slight alteration, the value of these measures is so fixed as to have certain definite proportions to the metrical system." Its divisions are not decimal, but chiefly binary, from the convenience of the latter scale in small business transactions.

The following tables express the French measures of length, and also the French weights, according to both the metrical and common systems

DECIMAL SYSTEM.

LONG MEASURES.

French.		English.
Millimètre	=	0.03937 inches
Centimètre	=	0.39371 "
Décimètre	=	3.93710 "
Mètre	=	39.37100 "
Decamètre	=	32.80916 feet.
Hectomètre	=	328.09167 "
Kilomètre	=	1093.63890 yards.
Myriamètre	=	10936.38900 "
		or 6 miles 1 furlong 28 poles.

WEIGHTS.

Milligramme = 0.0154 grains.
Centigramme = 0.1543 "
Décigramme = 1.5434 "
Gramme = 15.4340 "
Décagramme = 154.3402 " or 5.64 drams Avoirdupois.
Hectogramme = 3.2134oz. Troy, or 3.527oz. Avoirdupois.
Kilogramme = 2lb. 8oz. 3dwt. 2gr. Troy, or 2lb. 3oz. 4.428dr. Avoirdupois.
Myriagramme = 26.795lb. Troy, or 22.0485lb. Avoirdupois
Quintal = 1cwt. 3qr. 25lb. nearly.
Millier, or Bar = 9 tons 16cwt. 3qr. 12lb.

SYSTÈME USUEL.

LINEAR MEASURES.

Measures Usuelles.	Mètres.		English Measure. Feet.	Inches.	Parts.
Toise Usuelle	2.	=	6	6	9
Pied, or Foot	0.333+	=	1	1	1.5
Inch	0.027+	=	0	1	1.125
Aune	1.2	=	3	11	3
Half	0.6	=	1	11	7.5
Quarter	0.3	=	0	11	9.75
Eighth	0.15	=	0	5	10.875
Sixteenth	0.075	=	0	2	11.0625
One third of an Aune	0.4	=	1	3	9
Sixth	0.2	=	0	7	10.5
Twelfth	0.1	=	0	3	11.25

WEIGHTS.

	Grammes.		Troy Weight. lb.	oz.	dwt.	gr.	Avoirdupois. lb.	oz.	dr.
Kilogramme	1000	=	2	8	3	2	2	3	4.5
Livre Usuelle	500	=	1	4	1	13	1	1	10.25
Half	250	=		8	0	18.5		8	13.125
Quarter	125	=		4	0	9.25		4	6.5
Eighth	62.5	=		2	0	4.5		2	3.25
Once	31.3	=		1	0	2.25		1	1.75
Half	15.6	=			10	1.125			8.875
Quarter	7.8	=			5	0.5			4.5
Gros	3.9	=			2	12.25			2.25

THE END.

27
78
―――
9

= 6th power of

364 – ans to the [illegible] on 2 [illegible]

[illegible]

16 [illegible]